# REACTIVE POWER PRICING USING AVERAGE PARTICIPATION FACTORS 

# (Case Study: Southern Region Power System Network, Ethiopia) 

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#### Abstract

In the deregulated electricity market environment as generation, transmission and distribution are separate entities; reactive power flow in transmission lines is a question of great importance. Due to inductive load characteristic, reactive power is inherently flowing in transmission line. Hence under restructured electricity market environment this reactive power allocation is necessary. Hence in this work Newton Raphson (N-R) load flow based scheme is used to obtained reactive power flows. After getting reactive power flows average participation factor method is used to allocate these reactive flows to loads. Further using MVAr-cost method reactive power cost is allocated to the loads. The developed method is tested on 6 bus system, IEEE 14 bus system and 21 bus system Southern Ethiopian region electric power networks.


Keywords: Reactive power pricing; average participation factor; marginal participation method.

## 1. INTRODUCTION

Major objective for the Thermal power generation is to minimize fuel consumption by allocating optimal power generation from each unit subject to equality and inequality constraints. In most of cases fuel cost consists of active power cost only however reactive power is very essential for secure and reliable operation of power systems. However, reactive power production by a generator will reduce its capability to produce active power. Hence, provision of reactive power by generator will result in reduction of its active power production, so the reactive power pricing is equally important with real power pricing [1].

Nevertheless, it would suffice to say at this point that reactive power is a key ancillary service that is intimately linked to maintaining acceptable voltage level - an important measure of quality of supply. Production and transmission of reactive power is linked closely with that of real power (MW). Reactive power supply may come from generators but also from other
sources such as capacitor banks and other devices, with investment decisions on the latter often being in the purview of a transmission system operator. Depending upon the source, provision of reactive power can be "slow" or "fast". Reactive power can serve two purposes: Maintaining voltages in an acceptable band under normal operation, and arresting the collapse of voltage under extreme system conditions. Voltage stability is one of the key security criteria that a power system has to observe in real time and to ensure the generation/load growth of the system is matched with adequate investment to secure longer term supply of reactive power. In practice, real power and reactive power have generally been handled separately by most power system operators. Typically, real power dispatch is carried out using a linear programming model associated with an Economic Load Dispatch (ELD) calculation that maximizes social welfare, while guaranteeing that system security constraints are met [21]. Reactive power, on the other hand, is dispatched
based on power flow studies (N-R load flow) and operational experience.

Almost all bulk electric power is generated, transmitted and consumed in alternating current (AC) networks. Elements of AC systems supply (or produce) and consume (or absorb or lose) two kinds of power; real power and reactive. Real power accomplishes useful work (e.g, runs motors and light lamps). Reactive power supports the voltages that must be controlled for system reliability and induction motor [2]. Conventional reactive power pricing methods are based on power factor. These methods are not suitable for the restructured power systems. Because in these systems the costs of each reactive power support services must be paid separately. In addition, current tariffs only consider local costs and calculate the reactive power consumption respect variables which cannot judge the full customers usage $[3,4,5]$. Development of reactive power pricing service causes to nodal pricing theory definition [6-8]. More research work based on the nodal pricing has been presented in [9]. This method is sensitive to operating conditions and system constraints. Also, its computing time is considerable and due to nonlinearly its convergence is not acceptable. The proportional sharing techniques [ 5,6$]$ provide an efficient computational method for loss allocation, starting from the output of a solved load flow. But this concept is neither provable nor disprovable. Among the circuit-based loss allocation methods [7, 8], Z-bus matrix and modified Y-bus matrix methods are popular. These methods are based on a solved power flow, and all its computations are based on the admittance matrix. In the paper by vishaka et. al., [9], transmission charge allocation based on relative electrical distance (RED) concept is presented. Most of the above referred methods [3-9] consider that the transmission loss charges are an integral part of transmission usage charge and so its separate computation is unnecessary. Hogan [11], [12], [13], [14] extended the spot price theory of the MIT school to an AC power system and affirms the basic idea of charging reactive power spot prices derived from an OPF model. He particularly emphasizes the fact that reactive power consumption is critical for high voltage
lines that tend to be voltage limited. This phenomenon, he argues, is a particular form of congestion and reactive power spot prices are a good way of charging customers contributing towards it. This definition of the reactive spot price was a natural extension of the real power spot price theory proposed by Schweppe et. al., [16] for a power system (DC approximation model). Baughman et. al., [15] show that this pricing approach captures the spatial and chronological variation of the reactive power prices across the (electrical) nodes and across different loading conditions over the day. They also argue that this pricing scheme is a remedy to the inefficient power factor penalty scheme - an improvement that was envisaged by Berg et. al., [10]. Read et. al., [17] went a step further and evaluated the reactive power and voltage prices at the PQ and PV buses.

In this work, authors allocate the reactive power generation cost to the loads. It is because the main reason behind the reactive power flow is the inductive loading due to various types of induction motors. For that purpose average participation factor method is used. After allocation of reactive flows, reactive pricing is done by using MVA-cost method.

## 2. DEVELOPED METHODOLOGY

## A. Model for Reactive Power Flow Allocation

Let consider a simple diagraph G showed in Figure 1.


Figure 1: Graph G
The Kirchhoff matrix of above diagraph is given by

$$
K(G)=\left[\begin{array}{cccc}
1 & 0 & -1 & 0 \\
-1 & 2 & 0 & -1 \\
0 & -1 & 2 & -1 \\
0 & -1 & -1 & 2
\end{array}\right]
$$

Hence from the above example for a simple digraph $G$ of $n$ vertices, an $n$ by $n$ matrix called the Kirchhoff matrix $\mathrm{K}(\mathrm{G})$ or $\mathrm{K}=\left[k_{i j}\right]$ is defined as [19],

$$
\mathrm{K}=\left\{\begin{array}{cc}
d^{-}\left(v_{i}\right) & \text { for } i=j  \tag{1}\\
-x_{i j} & \text { for } i \neq j
\end{array}\right.
$$

This matrix is basis of the developed methodology. In the first step a power flow matrix is constructed from the N-R load flow. This matrix gives a complete overview of reactive power flows in the system. It is formed between nodes of the system. Diagonal elements give net reactive flows at nodes and off diagonal elements give the actual reactive flows and counter flows in the system. The developed matrix is defined as follows:

$$
\operatorname{RPF}_{i j}= \begin{cases}-r \not p f_{i j} & \text { for } i \neq j \text { and } \eta_{j i}>0  \tag{2}\\ \left\langle\not f_{i j}\right. & \text { for } i \neq j \text { and } \nu_{i j}>0 \\ \not p_{\mathrm{T} i} & \text { for } i=j\end{cases}
$$

From the above matrix and using equation (1) the modified reactive Kirchhoff matrix is constructed as follows: Denoting the modified reactive Kirchhoff matrix of power network as $\mathrm{K}_{m}=\left(k_{i j}\right)_{n \times n}$, expression for the elements of modified Kirchhoff matrix is given by (3):

$$
k_{i j}= \begin{cases}-r \not f_{i j} & \text { for } i \neq j \text { and } p_{j i}>0  \tag{3}\\ p_{\mathrm{T} j} & \text { for } i=j \\ 0 & \text { otherwise }\end{cases}
$$

Let $\ln =1$... e represents the total number of lines in the system, $\mathrm{G}_{n}=1 \ldots g$ is total number of generators and $\mathrm{D}=1 \ldots d$ is the total number of loads in the system.

For calculating the loads shares in reactive power flows procedure is as followed [18, 19, 20].

The diagonal load matrix $\mathrm{P}_{\mathrm{LL}}=\operatorname{diag}\left(\mathrm{P}_{\mathrm{L} 1}, \mathrm{P}_{\mathrm{L} 2}, \ldots\right.$, $\left.\mathrm{P}_{\mathrm{Ld}}\right)$ and $\mathrm{RPM}=\mathrm{P}_{\mathrm{LL}}\left(\mathrm{K}_{m}^{-1}\right)^{\mathrm{T}}$, where RPM is defined as reactive power flow matrix.

For calculating the load shares of reactive power to line flows, the procedure is as follows. Reactive power allocated to load situated at bus $i$ share the line $s-b$ is given by,

$$
\begin{equation*}
\mathrm{rp}_{\mathrm{i} \rightarrow \mathrm{~s}-\mathrm{b}}=\mathrm{t}_{\mathrm{is}} \mathrm{rf}_{\mathrm{s}-\mathrm{b}} \tag{4}
\end{equation*}
$$

where, $t_{i s}$ is the elements of reactive power flow matrix RPF. $r f_{s-b}$ is the reactive power flow in $s-b$ line. $p_{i \rightarrow s-b}$ is the reactive power flow allocated to the load situated at load bus $i$.

## B. Cost Recovery Model

After allocation of reactive power flows MVAr cost method is used for the allocation of reactive power cost to the different loads.

This recovery model provides cost recovery with respect to rated reactive power capacity of transmission line. If the cost of the line is denoted as $\mathrm{TC}_{s-b}$ (in $\mathrm{Rs} / \mathrm{hr}$ ) then reactive power cost allocated to users is given by:

For load $\mathrm{L}_{b}$, transmission reactive power cost allocation is given by $\operatorname{trc}_{s-b}^{\mathrm{L}_{b}}$

$$
\begin{equation*}
\operatorname{trc}_{s-b}^{\mathrm{L}_{j}}=\frac{r p_{j \rightarrow s-b}}{r f_{s-b}} \times \mathrm{TC}_{s-b} \tag{5}
\end{equation*}
$$

Total transmission reactive power cost $\mathrm{TRC}_{p}^{\mathrm{L}_{b}}$ allocated to load $\mathrm{L}_{h}$

$$
\begin{equation*}
\mathrm{TRC}^{\mathrm{L}_{\mathrm{h}}}=\sum_{\ln =1}^{\mathrm{e}} \operatorname{trc}_{\ln }^{\mathrm{L}_{\mathrm{h}}} \tag{6}
\end{equation*}
$$

where, $\sim_{j \rightarrow s-b}$ is the reactive power flow in $s-b$ line allocated to load situated at bus $j . \mathrm{TC}_{s-b}$ is transmission cost of line and $r f_{s-b}$ is the reactive power flow in the line $s-b$.

By using above to equation reactive power flow cost is allocated to loads.

## 3. RESULTS AND DISCUSSION

The developed method is applied to the 6 bus system presented in [4], IEEE 14 bus, and 21-bus Southern Region Ethiopian power system to demonstrate the feasibility and effectiveness of the methodology. It is assume that cost of the line is proportional to the
impedance of the line. A computer program coded in MATLAB is developed.

## A. 6 Bus System

A 6 bus system, which consists of three loads and three generators including slack bus, is used to show the feasibility of the developed method. The single line diagram of the system is shown in appendix. Table 1 shows the reactive power flows in normal condition.

Table 1
Reactive power flows

| Line | Normal Reactive Flow |
| :---: | :---: |
| $1-2$ | 14.2 |
| $1-4$ | 22.7 |
| $1-5$ | 14.9 |
| $2-3$ | 7.5 |
| $2-4$ | 49.6 |
| $2-5$ | 18.5 |
| $2-6$ | 15.3 |
| $3-5$ | 26.9 |
| $3-6$ | 64.5 |
| $4-5$ | 2.3 |
| $5-6$ | 6.3 |

Table 2 shows the allocated reactive power flows to loads with help of average participation factor method.

Table 2
Reactive Power Allocated to Loads

| Line | Flow | Load 4 | Load 5 | Load 6 |
| :---: | :---: | :---: | :---: | :---: |
| $1-2$ | 0.142 | 0.0846 | 0.0771 | 0.0169 |
| $1-4$ | 0.227 | 0.1353 | 0.1232 | 0.0270 |
| $1-5$ | 0.149 | 0.0888 | 0.0809 | 0.0178 |
| $2-3$ | 0.075 | 0.0212 | 0.0130 | 0.0213 |
| $2-4$ | 0.496 | 0.1403 | 0.0860 | 0.1412 |
| $2-5$ | 0.185 | 0.0523 | 0.0321 | 0.0527 |
| $2-6$ | 0.153 | 0.0433 | 0.0265 | 0.0435 |
| $3-5$ | 0.269 | 0.0000 | 0.0793 | 0.1862 |
| $3-6$ | 0.645 | 0.0000 | 0.1902 | 0.4465 |
| $4-5$ | 0.023 | 0.0198 | 0.0012 | 0.0000 |
| $5-6$ | 0.063 | 0.0000 | 0.0612 | 0.0015 |

Table 3 presents the comparative analysis among average participation factor method and marginal participation factor method.

Table 3
Comparison of average participation factor method with marginal participation factor method

|  | Average |  |  |  | Marginal |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Line | Particatiotion Factor |  | Particapation Factor |  |  |  |
|  | Load 4 | Load 5 | Load 6 | Load 4 | Load 5 | Load 6 |
| $1-2$ | 0.0846 | 0.0771 | 0.0169 | 0.0083 | 0.0163 | 0.0312 |
| $1-4$ | 0.1353 | 0.1232 | 0.0270 | 0.0158 | 0.0141 | 0.0053 |
| $1-5$ | 0.0888 | 0.0809 | 0.0178 | 0.0026 | -0.0009 | -0.0022 |
| $2-3$ | 0.0212 | 0.0130 | 0.0213 | -0.0506 | -0.1223 | -0.1956 |
| $2-4$ | 0.1403 | 0.0860 | 0.1412 | -0.0317 | -0.0463 | -0.0679 |
| $2-5$ | 0.0523 | 0.0321 | 0.0527 | -0.0394 | -0.0381 | -0.0694 |
| $2-6$ | 0.0433 | 0.0265 | 0.0435 | -0.0364 | -0.0512 | -0.0379 |
| $3-5$ | 0.0000 | 0.0793 | 0.1862 | -0.0513 | -0.0772 | -0.1305 |
| $3-6$ | 0.0000 | 0.1902 | 0.4465 | -0.0503 | -0.0871 | -0.0984 |
| $4-5$ | 0.0198 | 0.0012 | 0.0000 | -0.0178 | -0.1447 | -0.1296 |
| $5-6$ | 0.0000 | 0.0612 | 0.0015 | -0.0134 | -0.0444 | 0.0280 |

Table 4 shows the allocated reactive power flow cost to demands.

Table 4
Reactive Power Cost Allocated to Loads

| Line | Cost(Birr/hr) | Load 4 | Load 5 | Load 6 |
| :---: | :---: | :---: | :---: | :---: |
| $1-2$ | 223.61 | 133.2212 | 121.4108 | 26.61274 |
| $1-4$ | 206.16 | 122.8786 | 111.8895 | 24.52123 |
| $1-5$ | 310.49 | 185.0437 | 168.5815 | 37.09209 |
| $2-3$ | 254.95 | 72.06587 | 44.19133 | 72.4058 |
| $2-4$ | 111.80 | 31.62407 | 19.38468 | 31.82694 |
| $2-5$ | 316.23 | 89.39908 | 54.87018 | 90.08282 |
| $2-6$ | 211.90 | 59.96908 | 36.70163 | 60.24608 |
| $3-5$ | 286.36 | 0 | 84.41765 | 198.2165 |
| $3-6$ | 101.98 | 0 | 30.07224 | 70.59546 |
| $4-5$ | 447.21 | 384.9895 | 23.3327 | 0 |
| $5-6$ | 316.23 | 0 | 307.1949 | 7.529286 |
| Total |  | 1079.191 | 1002.047 | 619.1289 |

## B. IEEE 14 Bus System

The developed method is also applied on modified IEEE 14 bus system [20]. In modified IEEE 14 bus system there are two generators and remaining twelve loads.

Table 5 shows the allocated reactive power flows to loads with help of average participation factor method.

Table 6 shows the allocated reactive power flow cost to demands.
Table 5

| Line | Flow | L3 | LA | L5 | L6 | L7 | L8 | L9 | L10 | L11 | L12 | L13 | L14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 63.774 | 24.0395 | 12.8888 | 2.1850 | 3.2200 | 0 | 0 | 7.9544 | 2.5260 | 1.0205 | 1.7760 | 3.9481 | 4.2157 |
| 2 | 39.797 | 15.0014 | 8.0430 | 1.3635 | 2.0094 | 0 | 0 | 4.9638 | 1.5763 | 0.6368 | 1.1083 | 2.4637 | 2.6308 |
| 3 | 28.848 | 14.7957 | 5.5619 | 0.4933 | 0.7270 | 0 | 0 | 3.4325 | 0.8517 | 0.2304 | 0.4010 | 0.8914 | 1.4631 |
| 4 | 19.862 | 10.1869 | 3.8294 | 0.3397 | 0.5005 | 0 | 0 | 2.3633 | 0.5864 | 0.1586 | 0.2761 | 0.6137 | 1.0073 |
| 5 | 17.173 | 8.8078 | 3.3109 | 0.2937 | 0.4328 | 0 | 0 | 2.0434 | 0.5070 | 0.1372 | 0.2387 | 0.5306 | 0.8709 |
| 6 | 0.359 | 0.3590 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 8.641 | 1.7808 | 3.5804 | 0 | 0 | 0 | 0 | 2.2097 | 0.3799 | 0 | 0 | 0 | 0.6902 |
| 8 | 15.371 | 3.1677 | 6.3690 | 0 | 0 | 0 | 0 | 3.9307 | 0.6758 | 0 | 0 | 0 | 1.2278 |
| 9 | 10.438 | 2.1511 | 4.3250 | 0 | 0 | 0 | 0 | 2.6692 | 0.4589 | 0 | 0 | 0 | 0.8338 |
| 10 | 33.236 | 3.6432 | 7.3250 | 2.2603 | 3.3310 | 0 | 0 | 4.5207 | 1.9757 | 1.0557 | 1.8373 | 4.0843 | 3.2028 |
| 11 | 6.400 | 0 | 0 | 0 | 1.6032 | 0 | 0 | 0 | 0.5768 | 0.5081 | 0.8843 | 1.9657 | 0.8619 |
| 12 | 2.918 | 0 | 0 | 0 | 0.7310 | 0 | 0 | 0 | 0.2630 | 0.2317 | 0.4032 | 0.8963 | 0.3930 |
| 13 | 8.821 | 0 | 0 | 0 | 2.2097 | 0 | 0 | 0 | 0.7950 | 0.7003 | 1.2188 | 2.7093 | 1.1879 |
| 14 | 0.000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 13.065 | 0 | 0 | 0 | 0 | 0 | 0 | 8.8022 | 1.5133 | 0 | 0 | 0 | 2.7496 |
| 16 | 1.531 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0315 | 0.1773 | 0 | 0 | 0 | 0.3222 |
| 17 | 1.961 | 0 | 0 | 0 | 0 | 0 | 0 | 1.3212 | 0.2271 | 0 | 0 | 0 | 0.4127 |
| 18 | 4.378 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4.3780 | 0 | 0 | 0 | 0 |
| 19 | 1.111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8653 | 0.1708 | 0.0749 |
| 20 | 3.528 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2.4527 | 1.0753 |

Reactive Power Cost Allocated to Loads

| Line | Cost(Birr/br) | L3 | LA | L5 | L6 | L7 | L8 | L9 | L10 | L11 | L12 | L13 | L14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 62.26 | 23.46880029 | 12.58281883 | 2.13312792 | 3.143556935 | 0 | 0 | 7.76556189 | 2.466032552 | 0.996273246 | 1.733837614 | 3.854371782 | 4.115618936 |
| 2 | 229.49 | 86.50579908 | 46.38008066 | 7.86264329 | 11.58723537 | 0 | 0 | 28.62382747 | 9.089757695 | 3.672116793 | 6.391028645 | 14.20696316 | 15.17054783 |
| 3 | 203.47 | 104.3566652 | 39.2290555 | 3.479331357 | 5.127658417 | 0 | 0 | 24.21002409 | 6.007189372 | 1.625051581 | 2.828323281 | 6.28720043 | 10.31950073 |
| 4 | 185.65 | 95.21689583 | 35.79337982 | 3.17517395 | 4.678170627 | 0 | 0 | 22.08975154 | 5.481077434 | 1.48243329 | 2.580705115 | 5.736250378 | 9.415227318 |
| 5 | 182.97 | 93.84284435 | 35.27603639 | 3.129231293 | 4.611274442 | 0 | 0 | 21.77143761 | 5.401839516 | 1.461799569 | 2.543232924 | 5.653286089 | 9.279017819 |
| 6 | 183.69 | 183.69 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 44.18 | 9.104935077 | 18.30599144 | 0 | 0 | 0 | 0 | 11.29782965 | 1.942365698 | 0 | 0 | 0 | 3.528878139 |
| 8 | 209.12 | 43.09605257 | 86.64922777 | 0 | 0 | 0 | 0 | 53.4765457 | 9.194151064 | 0 | 0 | 0 | 16.7040229 |



Table 7
Reactive Power Allocated to Loads

| Line | Flon | LA | L5 | L6 | L7 | L8 | L9 | L10 | L11 | L12 | L13 | L14 | L15 | L16 | L17 | L18 | L19 | L20 | L21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 18.9 | 0.0427 | 0.0743 | 0.7435 | 1.4483 | 2.9049 | 2.5224 | 5.3690 | 0.8451 | 0.6141 | 0.7685 | 0.2692 | 0.2305 | 0 | 3.0675 | 0 | 0 | 0 | 0 |
| 2 | 1.0 | 0.0023 | 0.0039 | 0.0393 | 0.0766 | 0.1537 | 0.1335 | 0.2841 | 0.0447 | 0.0325 | 0.0407 | 0.0142 | 0.0122 | 0 | 0.1623 | 0 | 0 | 0 |  |
| 3 | 0.4 | 0.0009 | 0.0016 | 0.0157 | 0.0307 | 0.0615 | 0.0534 | 0.1136 | 0.0179 | 0.0130 | 0.0163 | 0.0057 | 0.0049 | 0 | 0.0649 | 0 | 0 | 0 | 0 |
| 4 | 21.2 | 0.0479 | 0.0834 | 0.8340 | 1.6245 | 3.2584 | 2.8294 | 6.0223 | 0.9480 | 0.6888 | 0.8620 | 0.3020 | 0.2586 | 0 | 3.4408 | 0 | 0 | 0 | 0 |
| 5 | 51.7 | 0 | 0 | 0 | 0 | 22.2849 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5.8828 | 23.5323 | 0 | 0 | 0 | 0 |
| 6 | 24.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9362 | 14.9004 | 5.7512 | 1.8745 | 0.9376 |
| 7 | 23.2 | 0 | 0 | 0 | 1.8626 | 3.7359 | 3.2440 | 6.9048 | 1.0869 | 0.7897 | 0.9884 | 0.3462 | 0.2964 | 0 | 3.9450 | 0 | 0 | 0 | 0 |
| 8 | 0.4 | 0 | 0 | 0 | 0.0321 | 0.0644 | 0.0559 | 0.1190 | 0.0187 | 0.0136 | 0.0170 | 0.0060 | 0.0051 | 0 | 0.0680 | 0 | 0 | 0 | 0 |
| 9 | 50.2 | 0 | 0 | 0 | 0 | 24.4167 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25.7833 | 0 | 0 | 0 | 0 |
| 10 | 25.7 | 0 | 0 | 0 | 0 | 12.5002 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13.1998 | 0 | 0 | 0 |  |
| 11 | 1.1 | 0 | 0 | 0 | 0 | 0 | 0.2613 | 0.5562 | 0.0875 | 0.0636 | 0.0796 | 0.0279 | 0.0239 | 0 | 0 | 0 | 0 | 0 |  |
| 12 | 0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0.3979 | 0.0626 | 0.0455 | 0.0570 | 0.0200 | 0.0171 | 0 | 0 | 0 | 0 | 0 |  |
| 13 | 0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1326 | 0.0209 | 0.0152 | 0.0190 | 0.0067 | 0.0057 | 0 | 0 | 0 | 0 | 0 |  |
| 14 | 0.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.4714 | 0 | 0 | 0 | 0.1286 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1115 | 0.1396 | 0.0489 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5925 | 0.2075 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 25.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25.2000 | 0 | 0 | 0 | 0 |
| 18 | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9208 | 14.6562 | 5.6569 | 1.8438 | 0.9222 |
| 19 | 25.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3.5838 | 0 | 22.0162 | 0 | 0 |
| 20 | 1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1380 | 0 | 0.8476 | 0.2763 | 0.1382 |

Table 8

| 츤 |  | $\bigcirc$ | $\bigcirc$ | － | $\bigcirc$ |  | $0$ | $\bigcirc$ |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\text { - } \stackrel{\text { İ }}{\infty}$ | $0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ®ิ |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  | $0$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  | $0$ | cis |
| $\frac{2}{3}$ |  | 00 | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\begin{aligned} & \stackrel{1}{i} \\ & \stackrel{y}{0} \\ & \underset{1}{\infty} \underset{\sim}{\infty} \\ & \infty \end{aligned}$ | $0$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |  | $\begin{aligned} & \text { in } \\ & \stackrel{\text { and }}{\text { an }} \\ & \text { ה̀ } \end{aligned}$ |  |
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## C. 21 Bus System Ethiopia Southern Region

Southern region of Ethiopia consist of total 21 buses. This system has three generator buses and remaining 18 load buses. Table 7 shows the allocated reactive power flows to loads with help of average participation factor method. Table 8 shows the allocated reactive power flow cost to demands at normal power flow condition.

## 4. CONCLUSION

In this work reactive power pricing is done with the help of average participation factor method. Further for getting reactive power flows Newton- Raphson load flow is used. After getting reactive power flows by using average participation factor method these flows are allocated to loads because the main reason behind the reactive power flow is the inductive loading at the load end. Hence by using MVAr-cost method the cost of this reactive power flowing is allocated to loads. For showing the feasibility of the developed method sample 6 bus, IEEE 14 bus and practical 21 bus southern region Ethiopia is used.

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