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ESTIMATION OF POPULATION MEAN THROUGH A GENERALIZED REGRESSION TYPE ESTIMATOR USING AUXILIARY INFORMATION

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Abstract: A new improved regression type generalized estimator using auxiliary information is proposed for estimating finite population mean. Its properties in terms of bias and mean squared error are studied and its comparative study with some of the well known estimators is also carried out. An empirical study is also carried out in the support of the theoretical results.

Keywords: Auxiliary Variable, Taylor's Series Expansion, Bias, Mean Squared Error and Efficiency.

1. INTRODUCTION

In the theory of sample surveys it is well known that the proper use of auxiliary information results in substantial improvement in the precision of the estimators of the population parameters. Using auxiliary information, it is possible to increase the efficiency of the usual estimators of population parameters of the study variable which are available in the literature. The auxiliary information may be known in advance or it may be collected while the survey is going on without increasing the cost or less increased. The information so collected on the auxiliary character x may be used at the time of estimation or selection.

Let $(y_i, x_i), i=1,2...,n$ be the n pair of sample observations for the study variable and auxiliary variable respectively drawn from the population of size N using simple random sampling without replacement.

Let us denote by $\overline{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i$ be thepopulation mean of study variable y, $\overline{X}=\frac{1}{N}\sum_{i=1}^{N}X_i$ be the population mean of auxiliary variable x.

$\overline{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$ be the sample mean of y and $\overline{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ be the sample mean of auxiliary variable X.

$S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$ be thepopulation variance of study variable y and

$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$ be the population variance of auxiliary variable *x*.

$\rho = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)}{S_Y S_X}$ be the population correlation coefficient between *y* and *x*.

Also let $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^r\left(X_i - \bar{X}\right)^s$, $C_Y^2 = \frac{S_Y^2}{\bar{Y}^2}$, $C_X^2 = \frac{S_X^2}{\bar{X}^2} = \frac{\mu_{02}}{\bar{X}^2}$, $\rho = \frac{\mu_{11}}{S_Y S_X}$

$\beta_2 = \frac{\mu_{04}}{\mu_{02}^2}$, $\beta_1 = \frac{\mu_{03}^2}{\mu_{02}^3}$, $\gamma_1 = \sqrt{\beta_1}$ and $\beta = \frac{S_{YX}}{S_X^2}$.

For simplicity, it is assumed that N is large enough as compared to n so that finite population correction terms may be ignored. A generalized estimator represented by $\bar{y}_g$ for estimating the population mean is proposed as

$$\bar{y}_g = g\left(\bar{y}, b, \bar{x}, s_y^2\right) \tag{1.1}$$

where, b is an estimate of the change in y when x is increased by unity and $g\left(\bar{y}, b, \bar{x}, s_y^2\right)$ is a bounded function such that at the point $\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right)$

(i) $g\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right) = \bar{Y}$

(ii) $g_0 = \frac{\partial}{\partial \bar{y}} g\left(\bar{y}, b, \bar{x}, s_y^2\right)\Big]_{\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right)} = 1$

(iii) $g_1 = \frac{\partial}{\partial b} g\left(\bar{y}, b, \bar{x}, s_y^2\right)\Big]_{\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right)} = 0$

(iv) $g_2 = \frac{\partial}{\partial \bar{x}} g\left(\bar{y}, b, \bar{x}, s_y^2\right)\Big]_{\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right)} = -\beta$

(v) $g_{00} = \frac{\partial}{\partial \bar{y}^2} g\left(\bar{y}, b, \bar{x}, s_y^2\right)\Big]_{\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right)} = 0$

(vi) $g_{22} = \frac{\partial}{\partial \bar{x}^2} g\left(\bar{y}, b, \bar{x}, s_y^2\right)\Big]_{\left(\bar{Y}, \beta, \bar{X}, S_Y^2\right)} = 0$

2. BIAS AND MEAN SQUARED ERROR OF THE PROPOSED ESTIMATOR

In order to obtain bias and mean squared error of the proposed estimator, let us denote by

$$\bar{y} = \bar{Y}(1 + e_0)$$

$$\bar{x} = \bar{X}(1 + e_1)$$

$$s_{yx} = e_2 + S_{YX}$$

$$s_y^2 = e_3 + S_Y^2$$

$$s_x^2 = e_4 + S_X^2 \tag{2.1}$$

so that ignoring finite population correction, for simplicity we have

$$E(e_0) = E(e_1) = E(e_2) = E(e_3) = E(e_4) = 0 \tag{2.2}$$

$$E\left(e_0^2\right) = \frac{\mu_{20}}{n\bar{Y}^2} = \frac{1}{n} C_Y^2$$

$$E\left(e_1^2\right) = \frac{\mu_{02}}{n\bar{X}^2} = \frac{1}{n} C_X^2$$

$$E\left(e_3^2\right) = \left(\frac{\beta_2(y) - 1}{n}\right) S_Y^4 = \frac{\mu_{20}^2}{n}\left(\frac{\mu_{40}}{\mu_{20}^2} - 1\right)$$

$$E\left(e_4^2\right) = \left(\frac{\beta_2(x) - 1}{n}\right) S_X^4 = \frac{\mu_{02}^2}{n}\left(\frac{\mu_{04}}{\mu_{02}^2} - 1\right)$$

$$E(e_0 e_1) = \frac{\mu_{11}}{n\bar{Y}\bar{X}} = \frac{1}{n} \rho C_Y C_X$$

$$E(e_0 e_3) = \frac{\mu_{30}}{n\bar{Y}}$$

$$E(e_1 e_2) = \frac{\mu_{12}}{n\overline{X}}$$

$$E(e_1 e_3) = \frac{\mu_{21}}{n\overline{X}}$$

$$E(e_0 e_4) = \frac{\mu_{12}}{n\overline{Y}}$$

$$E(e_1 e_4) = \frac{\mu_{03}}{n\overline{X}}$$

$$E(e_2 e_4) = \frac{\mu_{13}}{n}$$

$$E(e_2 e_3) = \frac{\mu_{21}}{n}$$

$$E(e_3 e_4) = \frac{\mu_{22}}{n} \qquad (2.3)$$

Expanding $g\left(\bar{y}, b, \bar{x}, s_y^2\right)$ about the point $\left(\overline{Y}, \beta, \overline{X}, S_Y^2\right)$ in the third order Taylor's series, we have

$$\begin{aligned}
\bar{y}_g = \; & g\left(\overline{Y}, \beta, \overline{X}, S_Y^2\right) + \left(\bar{y} - \overline{Y}\right) g_0 + (b - \beta) g_1 + \left(\bar{x} - \overline{X}\right) g_2 + \left(s_y^2 - S_Y^2\right) g_3 \\
& + \frac{1}{2!}\Big\{\left(\bar{y} - \overline{Y}\right)^2 g_{00} + (b - \beta)^2 g_{11} + \left(\bar{x} - \overline{X}\right)^2 g_{22} + \left(s_y^2 - S_Y^2\right)^2 g_{33} \\
& + 2\left(\bar{y} - \overline{Y}\right)(b - \beta) g_{01} + 2\left(\bar{y} - \overline{Y}\right)\left(\bar{x} - \overline{X}\right) g_{02} + 2\left(\bar{y} - \overline{Y}\right)\left(s_y^2 - S_Y^2\right) g_{03} \\
& + 2(b - \beta)\left(\bar{x} - \overline{X}\right) g_{12} + 2(b - \beta)\left(s_y^2 - S_Y^2\right) g_{13} + 2\left(\bar{x} - \overline{X}\right)\left(s_y^2 - S_Y^2\right) g_{23}\Big\} \\
& + \frac{1}{3!}\left\{\left(\bar{y} - \overline{Y}\right)\frac{\partial}{\partial \bar{y}} + (b - \beta)\frac{\partial}{\partial b} + \left(\bar{x} - \overline{X}\right)\frac{\partial}{\partial \bar{x}} + \left(s_y^2 - S_Y^2\right)\frac{\partial}{\partial s_y^2}\right\}^3 \\
& g\left(\bar{y}^*, b^*, \bar{x}^*, s_y^{2*}\right)
\end{aligned} \qquad (2.4)$$

where, $g_0 = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{y}} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)}$, $g_1 = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial b} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)}$,

$$g_2 = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{x}} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},\ g_3 = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial s_y^2} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},$$

$$g_{00} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{y}^2} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},\ g_{01} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{y}\partial b} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},$$

$$g_{02} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{y}\partial \bar{x}} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},\ g_{03} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{y}\partial s_y^2} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},$$

$$g_{12} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial b\partial \bar{x}} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},\ g_{13} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial b\partial s_y^2} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)},$$

$$g_{23} = \left(\frac{\partial g(\bar{y},\ b,\ \bar{x},\ s_y^2)}{\partial \bar{x}\partial s_y^2} \right)_{(\bar{Y}\ \ \beta,\ \bar{X},\ S_Y^2)} \quad \text{and } \bar{y}^* = \bar{Y} + h(\bar{y} - \bar{Y}),$$

$b = \beta + h(b - \beta)$, $\bar{x}^* = \bar{X} + h(\bar{x} - \bar{X})$, $s_y^{2^*} = S_Y^2 + h(s_y^2 - S_Y^2)$ for $0 < h < 1$.

Under the conditions (i) to (vi) given above, the above proposed estimator in terms of e_i's, $i = 0,1,2,3$; up to terms of order $O(1/n)$ reduces to

$$\begin{aligned} \bar{y}_g - \bar{Y} = {} & \bar{Y}e_0 - \beta\bar{X}e_1 + e_3 g_3 + \frac{1}{2!}\Big\{ \bar{Y}^2 e_0^2 g_{00} + \bar{X}^2 e_1^2 g_{22} + e_3^2 g_{33} \\ & + 2\bar{Y}\frac{1}{S_X^2}(e_0 e_2 - \beta e_0 e_4) g_{01} + 2\overline{YX} e_0 e_1 g_{02} + 2\bar{Y}e_0 e_3 g_{03} + 2\bar{X}\frac{1}{S_X^2} \\ & (e_1 e_2 - \beta e_1 e_4) g_{12} + 2\frac{1}{S_X^2}(e_2 e_3 - \beta e_3 e_4) g_{13} + 2\bar{X}e_1 e_3 g_{23} \Big\} \end{aligned} \quad (2.5)$$

Taking expectation on both the sides of (2.5), the bias of $\bar{y}_g$ up to terms of order $O(1/n)$ is given by

$$\text{Bias}\left(\bar{y}_g\right) = \left\{E\left(\bar{y}_g\right) - \bar{Y}\right\}$$

$$= \frac{1}{2n}\left[\left\{\beta_2(y) - 1\right\} S_Y^4 g_{33} + 2\left(\frac{\mu_{21}}{S_X^2} - \beta\frac{\mu_{12}}{S_X^2}\right) g_{01} + 2\mu_{11} g_{02} + 2\mu_{30} g_{03}\right.$$

$$\left. +2\left(\frac{\mu_{12}}{S_X^2} - \beta\frac{\mu_{03}}{S_X^2}\right) g_{12} + 2\left(\frac{\mu_{21}}{S_X^2} - \beta\frac{\mu_{22}}{S_X^2}\right) g_{13} + 2\mu_{21} g_{23}\right] \quad (2.6)$$

Now squaring both sides of (2.5) and taking expectation, the mean square error of $\bar{y}_g$ up to terms of order $O(1/n)$ is given by

$$\text{MSE}\left(\bar{y}_g\right) = \left\{E\left(\bar{y}_g\right) - \bar{Y}\right\}^2$$

$$= \bar{Y}^2 E\left(e_0^2\right) + \beta^2 \bar{X}^2 E\left(e_1^2\right) + g_3^2 E\left(e_3^2\right) - 2\beta\bar{Y}\bar{X} E\left(e_0 e_1\right)$$

$$+ 2\bar{Y} g_3 E\left(e_0 e_3\right) - 2\beta\bar{X} g_3 E\left(e_1 e_3\right)$$

using values of the expectation given in (2.2) and (2.3), we have

$$\text{MSE}\left(\bar{y}_g\right) = \frac{1}{n}\left(\mu_{20} + \beta^2\mu_{02} - 2\beta\mu_{11}\right)$$

$$+ \frac{1}{n}\left[\left\{\beta_2(y) - 1\right\} g_3^2 S_Y^4 - 2\beta\mu_{21} g_3 + 2\mu_{30} g_3\right] \quad (2.7)$$

which attains the minimum for the optimum value

$$g_3 = \frac{\left(\beta\mu_{21} - \mu_{30}\right)}{S_Y^4\left\{\beta_2(y) - 1\right\}} \quad (2.8)$$

Substituting the value of g_3 given by (2.8) in (2.7), we get the minimum mean square error of $\bar{y}_g$ to be

$$\text{MSE}\left(\bar{y}_g\right)_{\min} = \frac{1}{n}\left(\mu_{20} - 2\beta\mu_{11} + \beta^2\mu_{02}\right) - \frac{1}{n}\frac{\left(\beta\mu_{21} - \mu_{30}\right)^2}{S_Y^4\left\{\beta_2(y) - 1\right\}} \quad (2.9)$$

3. EFFICIENCY COMPARISON

(i) **General estimator of mean in case of SRSWOR:** The general estimator of Mean in case of SRSWOR is $\hat{\bar{y}}_{wor} = \bar{y}$ with

$$MSE\left(\hat{\bar{y}}_{wor}\right) = \frac{\mu_{20}}{n} \tag{3.1}$$

It is clear that the proposed estimator is more efficient than the estimator $\hat{\bar{y}}_{wor}$ based on simple random sampling when no auxiliary information is used.

(ii) **Usual regression estimator:** The usual regression estimator is $\bar{y}_{lr} = \bar{y} + b\left(\bar{X} - \bar{x}\right)$ with

$$\text{MSE}\left(\bar{y}_{lr}\right)_{\min} = \frac{1}{n}\left(\mu_{20} + \beta^2\mu_{02} - 2\beta\mu_{11}\right) \tag{3.2}$$

It is clear that the proposed estimator is more efficient than the usual regression estimator of mean where the auxiliary information already is in use.

4. EMPIRICAL STUDY

To illustrate the performance of the proposed estimator, let us consider the following data

Population I: Cochran (1977, Page Number- 181)

y : Paralytic Polio Cases 'placebo' group

x : Paralytic Polio Cases in not inoculated group

μ_{02} = 71.8650173, μ_{20} = 9.889273356, μ_{11} = 19.4349481, μ_{12} = 346.3174191,

μ_{03} = 1453.077703, μ_{40} = 424.1846721, μ_{21} = 94.21286383, μ_{22} = 3029.312542,

μ_{30} = 47.34479951, μ_{04} = 46132.5679, $\bar{y}$ = 2.588235294, $\bar{x}$ = 8.370588235,

S_x = 8.477323711, S_y = 3.144721507, ρ =0.729025009, $\beta_2(y)$ = 4.337367369,

$\beta_2(x)$ = 8.932490454, C_X = 1.012751251, C_Y = 1.215006037, β = 0.270436839,

n = 34.

$MSE\left(\hat{\bar{y}}_{wor}\right)$ = 0.290860981, $MSE\left(\bar{y}_{lr}\right)$ = 0.136274924 and $MSE\left(\bar{y}_g\right)$min = 0.093189164

PRE of the proposed estimator $\bar{y}_g$ over $\hat{\bar{y}}_{wor}$ = 312.1188865.

PRE of the proposed estimator $\bar{y}_g$ over $\bar{y}_{lr}$ = 146.2347322.

Population II: Mukhopadhyay (2012, Page Number - 104)

y : Quality of raw materials (in lakhs of bales)

x : Number of labourers (in thousands)

μ_{02} = 9704.4475, μ_{20} = 90.95, μ_{11} = 612.725, μ_{12} = 93756.3475, μ_{03} = 988621.5173,

μ_{40} = 35456.4125, μ_{21} = 11087.635, μ_{22} = 2893630.349, μ_{30} = 1058.55,

μ_{04} = 341222548.2, $\bar{y}$ = 41.5, $\bar{x}$ = 441.95, S_x = 98.51115419, S_y = 9.536770942,

ρ =0.652197067, $\beta_2(y)$ = 4.286367314, $\beta_2(x)$ = 3.623231573, C_X = 0.22290113,

C_Y =0.229801709, β =0.063138576, n = 20.

$MSE\left(\hat{\bar{y}}_{wor}\right)$ = 4.5475, $MSE\left(\bar{y}_{lr}\right)$ = 2.613170788 and $MSE\left(\bar{y}_g\right)$min = 2.376791909.

PRE of the proposed estimator $\bar{y}_g$ over $\hat{\bar{y}}_{wor}$ = 191.3293285.

PRE of the proposed estimator $\bar{y}_g$ over $\bar{y}_{lr}$ = 109.9452913.

Population III: Murthy (1967, Page Number - 398)

y : Number of absentees

x : Number of workers

μ_{02} = 1299.318551, μ_{20} = 42.13412655, μ_{11} = 154.6041103, μ_{12} = 5086.694392,

μ_{03} = 32025.12931, μ_{40} = 11608.18508, μ_{21} = 1328.325745, μ_{22} = 148328.4069,

μ_{30} = 425.9735118, μ_{04} = 4409987.245, $\bar{y}$ = 9.651162791, $\bar{x}$ = 79.46511628,

S_x = 36.04606151, S_y = 6.491080538, ρ = 0.660763765, $\beta_2(y)$ = 6.53877409,

$\beta_2(x)$ = 2.612197776, C_X = 0.453608617, C_X = 0.672569791, β = 0.118988612,

n = 43.

$MSE\left(\hat{\hat{y}}_{wor}\right)$ = 0.979863408, $MSE\left(\bar{y}_{lr}\right)$ = 0.552046468 and $MSE\left(\bar{y}_{g}\right)$min = 0.382279464.

PRE of the proposed estimator $\bar{y}_g$ over $\hat{\hat{y}}_{wor}$ = 256.3212257.

PRE of the proposed estimator $\bar{y}_g$ over $\bar{y}_{lr}$ = 144.4091351.

Population IV: Singh and Chaudhary (1997, Page Number - 176)

y : Total number of guava trees

x : Area under guava orchard (in acres)

μ_{02} = 12.50056686, μ_{20} = 187123.9172, μ_{11} = 1377.39858, μ_{12} = 4835.465464,

μ_{03} = 37.09863123, μ_{40} = 1.48935E+11, μ_{21} = 712662.4414, μ_{22} = 8747904.451,

μ_{30} = 100476814.5, μ_{04} = 540.1635491, $\bar{y}$ = 746.9230769, $\bar{x}$ = 5.661538462,

S_x = 3.535614072, S_y = 432.5782209, ρ = 0.900596235, $\beta_2(y)$ = 4.253426603,

$\beta_2(x)$ = 3.456733187, C_X = 0.624497051, C_Y = 0.579146949, β = 110.1868895,

n = 13.

$MSE\left(\hat{\hat{y}}_{wor}\right)$ = 14394.14747, $MSE\left(\bar{y}_{lr}\right)$ = 2719.434771 and $MSE\left(\bar{y}_{g}\right)$min = 2394.080861.

PRE of the proposed estimator $\bar{y}_g$ over $\hat{\hat{y}}_{wor}$ = 601.2389853.

PRE of the proposed estimator $\bar{y}_g$ over $\bar{y}_{lr}$ = 113.5899299.

Population V: Singh and Chaudhary (1997, Page Number: 154-155)

y : Number of milch animals in survey

x : Number of milch animals in census

μ_{02} = 431.5847751, μ_{20} = 270.9134948, μ_{11} = 247.3944637, μ_{12} = 3119.839406,

μ_{03} = 5789.778954, μ_{40} = 154027.4827, μ_{21} = 2422.297374, μ_{22} = 210594.3138,

μ_{30} = 2273.46265, μ_{04} = 508642.4447, $\bar{y}$ = 1133.294118, $\bar{x}$ = 1140.058824,

S_x = 20.77461853, S_y = 16.45945002, ρ = 0.723505104, $\beta_2(y)$ = 2.098635139,

$\beta_2(x)$ = 2.730740091, C_X = 0.018222409, C_Y = 0.014523547, β = 0.573223334,

n = 17.

$MSE\left(\hat{\bar{y}}_{wor}\right)$ = 15.93609, $MSE(\bar{y}_{lr})$ = 7.594189 and $MSE(\bar{y}_g)$min =7.022882678.

PRE of the proposed estimator $\bar{y}_g$ over $\hat{\bar{y}}_{wor}$ = 226.916619.

PRE of the proposed estimator $\bar{y}_g$ over $\bar{y}_{lr}$ = 108.1349283.

5. CONCLUSIONS

From (2.9) it is clear that the proposed new regression type sampling estimator is more efficient than the estimator $\hat{\bar{y}}_{wor}$ based on simple random sampling when no auxiliary information is used and also more efficient than the usual regression estimator $\bar{y}_{lr}$ of mean where the auxiliary information already is in use.

From (2.8) the mean square error MSE $(\bar{y}_g)$ of the estimator $\bar{y}_g$ is minimized for the optimum value

$$g_3 = \frac{(\beta\mu_{21} - \mu_{30})}{S_Y^4\{\beta_2(y) - 1\}} \tag{5.1}$$

The optimum value involving some unknown parameters may not be known in advance for practical purposes; hence the alternative is to replace the unknown parameters of the optimum value by their unbiased estimators giving estimator depending upon estimated optimum value.

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