IJCTA, 9(37) 2016, pp. 897-909

Optimal Switching Strategy Method Performance in the Design of UPFC Controllers*

Yathisha L[**] and S Patil Kulkarni[***]

Abstract: The Unified Power Flow Controller (UPFC) combines the properties of shunt and series compensations and effectively alters power system parameters in a way that increases power transfer capability and stabilizes system. The selection of the state feedback gains for the UPFC controllers is most commonly designed from the optimal controllers. But parameters of optimal controllers are usually tuned based on the trial-and-error approaches and they are incapable to obtain a good dynamic performance under a wide range of operating conditions. To solve this problem, switching strategy method is proposed for two optimal Linear Quadratic Regulator (LQR) controllers to show the improved dynamic performances compared to individual LQR and other optimization techniques. A Single-Machine Infinite Bus (SMIB) power system installed with a UPFC is considered as case study.

Keywords: UPFC, LQR, SMIB, Power System and Phillips Heffron Model.

1. INTRODUCTION

Recent years have witnessed an enormous growth of interest in dynamic systems that are characterised by a mixture of both continuous and discrete dynamics (Douglas et. al., 2010). Such systems are commonly found in engineering practice and are referred to as hybrid or switching systems. The widespread application of such systems is motivated by ever increasing performance requirements, and by the fact that high performance control systems can be realised by switching between relatively simple LTI systems.

The advantage of switching between different feedback structures is to combine the useful properties of each structure and to introduce new properties that are not present in any of the structures used. The switching surface is actually composed of two subspaces which intersect on the null space of the switching matrix. Analysis of each of the these surfaces shows that sliding might be present on one of them. However, if both state feedbacks are designed using optimal quadratic regulators with different weights in the control effort, sliding motion has not been detected in any of the experiments performed.

The main problems in linear switched systems are stability and poor transient responses, caused by switching between different controllers. Hence, improving the switched systems responses is of prime concern. A switched system consists of linear time invariant (LTI) subsystems and a regulated switching law. (Zhi Hong Huang et. al., 2007; Cheng Xiang and Hai Lin et. al.,2007) derived a necessary and sufficient condition for stability of arbitrarily switched second order LTI systems with marginally stable subsystem. It turns out that the condition for the marginally stable case is similar with the one for asymptotically stable except boundary conditions are included. In (Keith R Santarelli et. al.,2008) the authors made a comparison of a switching controller to two LTI controllers for a class of LTI Plants.

In (Aravena et. al., 2006; Devarakonda et. al., 2006) proposed a performance based switching algorithm for LTI systems, based on Lyapunov stability criteria. In (Keith R Santarelli et. al., 2011) has proposed a stabilization switching algorithm for LTI systems, where state trajectory is driven to (n-1) dimensional stable hyperplane.

* The Authors wish to thank the management of Shree Jayachamarajendra College of Engineering for its financial support.
** Research Scholar, E & C Department, SJCE, Mysore, India. Email: yathisha.171@gmail.com
*** Professor, E & C Department, SJCE, Mysore, India. Email: sudarshan_pk@sjce.ac.in

The unified power ow controller (UPFC) is regarded as one of the most versatile devices in the FACTS-device family (H. Shayeghi et. al., 2009; H. A. Shayanfar et. al., 2009) which has the ability to control of the power flow in the transmission line, improve the transient stability, mitigate system oscillation and provide voltage support. It performs this through the control of the in-phase voltage, quadrate voltage and shunts compensation due to its mains control strategy (Al. Awami et. al., 2007) & (Tambey N et. al., 2003). The application of the UPFC to the modern power system can therefore lead to the more flexible, secure and economic operation (Vilathgamuwa et. al., 2000; Zhu X et. al., 2000). When the UPFC is applied to the interconnected power systems, it can also provide significant damping effect on line power oscillation through its supplementary control.

Design of control strategies using FACTS devices such as UPFC for optimal power flow with improved performance is a major research concern of power system control community. (H. F. Wang et. al., 2000) has presented a modified linearised Phillips-Heron model of a power system installed with UPFC and addressed basic issues pertaining to design of UPFC based power oscillation damping controller along with selection of input parameters of UPFC to be modulated in order to achieve desired damping. Wang has not presented a systematic approach for designing the damping controllers. Further, no effort seems to have been made to identify the most suitable UPFC control inputs, in order to arrive at a robust damping controller for optimal performance of all the state variables. However, in recent times, researchers are working on the selection of UPFC control parameter for the design of UPFC damping controller by applying different control techniques like Phase Compensation, Fuzzy Logic, optimal control techniques like Linear Quadratic Regulator (LQR), H-infinity, particle swarm optimization etc (N. Tambey et. al., 2003; M. L . Kothari et. al., 2003), (Amin Safari et. al., 2009), (R K Pandey et. al., 2010), (M. Shaoba et. al., 2010) & (H. Shayeghi et. al., 2009). Some of the examples are described here. In (N. Tambey et. al., 2003) authors have shown the control inputs and to provide robust performance when compared to the other damping controllers by applying a phase compensation control technique with respect to state space variable speed. In (Amin Safari et. al., 2009) have presented iterative particle swarm optimization (IPSO) based UPFC controller to achieve improved robust performance and to provide superior damping in comparison with the conventional particle swarm optimization (CPSO) for the control inputs and in (R K Pandey et. al., 2010) has presented multi machine system, where some of the states having larger settling time with conventional LQR are well regulated with multistage LQR.

The Objective of this paper is to introduce switching concepts for the linearised SMIB Phillips-Heffron model of power system installled with UPFC. Detailed investigations have been carried out considering the four alternatives UPFC based damping controllers namely modulating index of series inverter (m_E), modulating index of shunt inverter (m_B), phase angle of series inverter (δ_E) and phase angle of the shunt inverter (δ_B) in the following three stages:

1. Firstly, for the state space (phillips heffron) model, optimal controllers are designed from the LQR theory by simply choosing weighting matrices (without tuning based on trial and error approaches) as Q = I & R = 1 (K_1) and Q = I & R = 0.01 (K_2).
2. Secondly, switching strategy is developed to switch between two LQR controllers from the above mentioned controller gains as K_1 (master controller) and K_2 (alternate controller).
3. Finally, all the developed state space models with feedback controllers are simulated using MATLAB/SIMULINK @ platform. Results of individual controllers K_1, K_2 and switching between K_1 & K_2 are compared. To validate the robustness of the proposed technique the results are compared with other optimization techniques and performance index J is tabulated.

This paper is organised as follows: Section 2 introduces linearised Phillips-Heffron model for the Power system installed with UPFC. The switching model for PhilipsHeffron plant with UPFC along with the proposed switching rules in detail in Section 3. Results and analysis of research is given in Section 4 to verify the results proposed in this paper. Discussions and Conclusion follows in the next preceeding sections.

2. MODELLING THE POWER SYSTEM WITH UPFC DAMPING CONTROLLERS

The Single Machine Infinite Bus (SMIB) power system installed with a UPFC as shown in Figure 2 is considered in this study (A. K. Baliarsingh et. al., 2010). The UPFC is installed in one of the two parallel transmission lines. This arrangement, comprising two parallel transmission lines, permits the control of real and reactive power ow through a line. The static excitation system, model type IEEESTIA, has been considered. The UPFC is assumed to be based, on pulse width modulation (PWM) converters. The nominal loading condition and system parameters are given in Appendix.

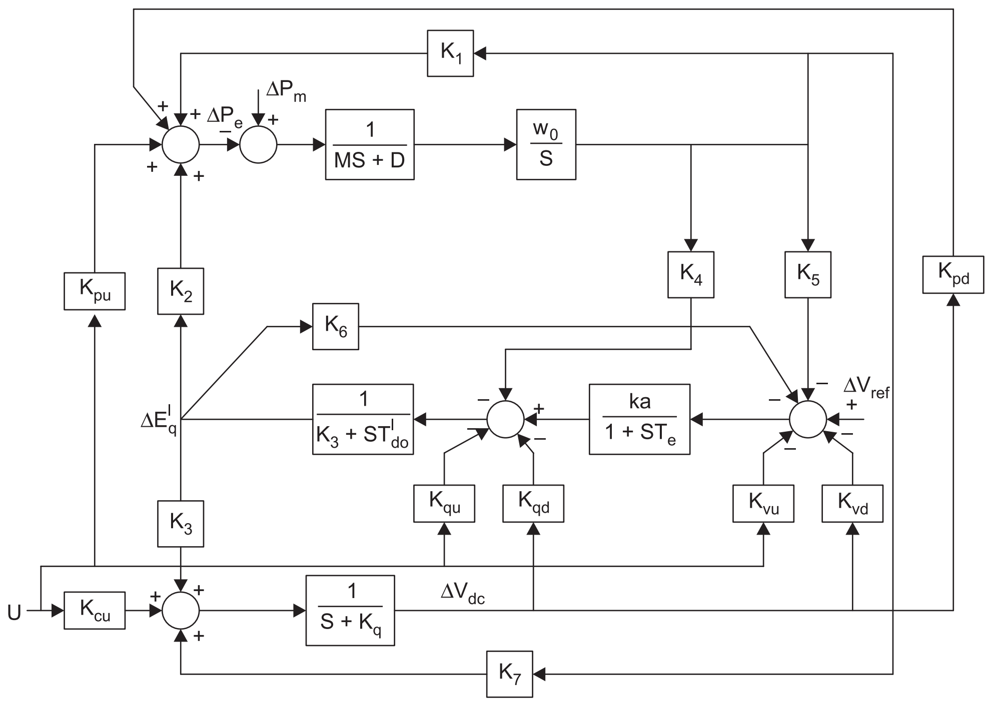

Figure 1: Linearised dynamic model of the SMIB power system with UPFC

The Heffron-Phillips model as shown in Figure 1, of a synchronous machine has successfully been used for investigating the low frequency oscillations and designing power system stabilizers. The parameters of the model are usually calculated using the synchronous generator parameters and some system variables at steady-state conditions. A generating unit is a multivariable system and is well defined in a state space structure. The subspace state space (4SID) identification method is very suitable for the identification of such a system. This method is used to identify the parameters. Since the synchronous generators are nonlinear, the parameters of the identified PhillipsHeffron model would depend on the operating conditions.

H.F. Wang has presented the following state space model for the modified SMIB linearised phillips heffron power system (H. F.Wang et. al., 2000) & (M. Shoba et. al., 2010).

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

A linear dynamic model is obtained by linearising the nonlinear model round an operating condition.

$$\Delta\dot{\delta} = \omega_0 \Delta\omega \tag{2}$$

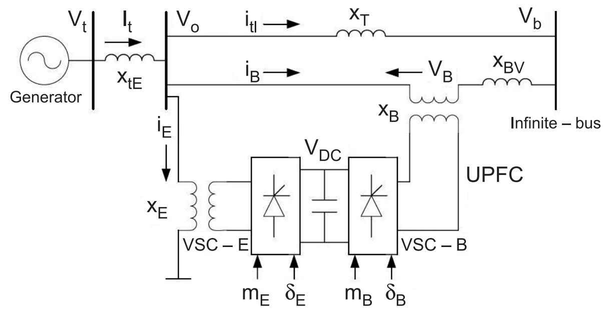

Figure 2: Single-machine infinite-bus power system with UPFC

$$\dot{\Delta\omega} = (-\Delta P_e - \Delta\omega)/M \tag{3}$$

$$\dot{\Delta E}'_q = (-\Delta E_q + \Delta E_{fd})/T'_{do} \tag{4}$$

$$\dot{\Delta E}_{fd} = -\frac{1}{T_A}\Delta E_{fd} - \frac{k_A}{T_A}\Delta V \tag{5}$$

$$\Delta P_e = k_1\Delta\delta + k_2\Delta E'_q + k_{pe}\Delta m_E + k_{p\delta e}\Delta\delta_E + K_{pb}\Delta m_B + k_{p\delta b}\Delta\delta_B \tag{6}$$

$$\Delta E'_q = k_4\Delta\delta + k'_{3q} + k_{qe}\Delta m_E + k_{q\delta e}\Delta\delta_E + K_{qb}\Delta m_B + k_{q\delta b}\Delta\delta_B \tag{7}$$

where, the state variables are the rotor angle deviation ($\Delta\delta$), speed deviation ($\Delta\omega$), q-axis component deviation ($\Delta E'_q$), field voltage deviation (ΔE_{fd}) and input variables are modulating index and phase angle of shunt inverter (m_E, δ_E) and modulating index and phase angle of series inverter (m_B, δ_B). A and B represent the state and control input matrices given by

$$A = \begin{bmatrix} 0 & \omega_o & 0 & 0 \\ -\frac{k_1}{M} & -\frac{D}{M} & -\frac{k_2}{M} & 0 \\ -\frac{k_4}{T'_{do}} & 0 & -\frac{k_3}{T'_{do}} & \frac{1}{T'_{do}} \\ -\frac{k_A k_5}{T_A} & 0 & -\frac{k_A k_6}{T_A} & -\frac{1}{T_A} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{k_{pe}}{M} & -\frac{k_{p\delta e}}{M} & -\frac{k_{pb}}{M} & -\frac{k_{p\delta b}}{M} \\ -\frac{k_{qe}}{T'_{do}} & -\frac{k_{q\delta e}}{T'_{do}} & -\frac{k_{qb}}{T'_{do}} & -\frac{k_{q\delta b}}{T'_{do}} \\ -\frac{k_A k_{ve}}{T_A} & -\frac{k_A k_{v\delta e}}{T_A} & -\frac{k_A k_{vb}}{T_A} & -\frac{k_A k_{v\delta b}}{T_A} \end{bmatrix}$$

$$\dot{x}(t) = \begin{bmatrix} \dot{\Delta\delta} \\ \dot{\Delta\omega} \\ \dot{\Delta E}'_q \\ \dot{\Delta E}_{fd} \end{bmatrix}, \quad u(t) = \begin{bmatrix} \Delta m_E \\ \Delta\delta_E \\ \Delta m_B \\ \Delta\delta_B \end{bmatrix}$$

All the relevant k-constants and variables along with their values used in the experiment are described in the appendix section at the end of paper.

3. PROPOSED SWITCHING STRATEGY

In this section, mathematical modeling of Philips-heffron system with UPFC device as a switched linear systems and the proposed switching algorithm will be explained.

A. Switched Linear Systems

Switching between a number of control structures automatically results in control systems that are no longer constrained by the limitations of linear design. It is therefore not surprising that switching based control strategies can result in algorithms that offer significant performance improvements over traditional linear control.

A switched-linear system model (refer Figure 3) for the current problem is as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) \tag{8}$$

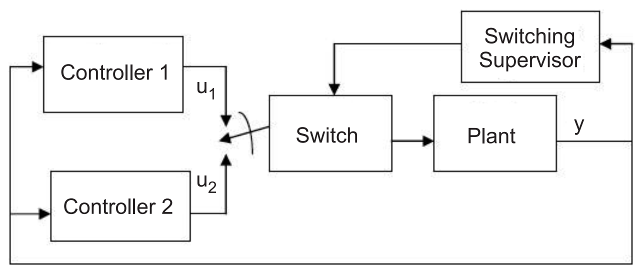

Figure 3: General implementation of switched linear systems

The switching signal σ(*t*) indicates

$$\dot{x}(t) = A_1 = (A - BK_1)x(t) \tag{9}$$
$$= A_2 = (A - BK_2)x(t)$$

The switching strategy σ(*t*) shown in Figure 3 (i.e. the rule of when to switch to which system parameters) determines the behavior and performance of the overall system. Typically, switched-linear system results when an open-loop plant parameter A is controlled by switching between two or more controllers in a state-variable feedback. The controller gain vectors shown in Figure 3 can be obtained by any of the standard control theory tools. Since controller gains K_1 and K_2 presented in this research are derived from, LQR by tuning the weighting matrices. For, the sake of completeness LQR control method are explained briefly.

B. Linear Quadratic Regulator Algorithm

Optimal control theory, an extension of the calculus of variations, is a mathematical optimization method for deriving control policies. The method is largely due to the work of Lev Pontryagin and his collaborators in the Soviet Union and Richard Bellman in the United States. Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. A control problem includes a cost functional that is a function of state and control variables. An optimal control is a set of differential equations describing the paths of the control variables that minimize the cost functional. The optimal control can be derived using Pontryagin's maximum principle (a necessary condition), or by solving the Hamilton-Jacobi-Bellman equation (a sufficient condition).

A special case of optimal control problem which is of particular importance arises when the objective function is a quadratic function of *x* and *u*, and the dynamic equations are linear. The resulting feedback law in this case is known as the linear quadratic regulator (LQR). Consider a linear system characterized by Eqn. (1) where (A, B) is stabilizable. Then the cost index that determines the matrix K of the LQR vector is (Yathisha L et. al., 2013; S Patil Kulkarni et. al., 2013)

$$J(X, U, Q, R) = \int_0^{\infty} (X^T QX + U^T RU)\, dt,\ Q \geq 0, R \geq 0 \tag{10}$$

where Q and R are the positive-definite Hermitian or real symmetric matrix. Note that the second term on the right side account for the expenditure of the energy on the control efforts. The matrix Q and R determine the relative importance of the error and the expenditure of this energy. From the above equations we get

$$J = \int_0^{\infty} (X^T QX + X^T K^T RKX)\, dt \tag{11}$$

$$J = \int_0^{\infty} X^T (QX + K^T RK)\, X\, dt \tag{12}$$

where (A, $Q^{1/2}$) is detectable and (A − BK) is stable. The linear quadratic regulation problem is to find a control U = –Kx law such that and J is minimized, the solution is given by

$$K = -R^{-1} B^T P \tag{13}$$

and hence the control law is,

$$u(t) = -Kx(t) = -R^{-1} B^T Px(t) \tag{14}$$

In which P must satisfy reduced Riccati equation:

$$PA + A^T P - PBR^{-1} + B^T P + Q = 0 \tag{15}$$

The selection of Q and R is weakly connected to the performance specifications, and a certain amount of trial and error is required with an interactive computer simulation before a satisfactory design results. The LQR function allows you to choose two parameters, R and Q, which will balance the relative importance of the input and state in the cost function that you are trying to optimize. Essentially, the LQR method allows for the control of all outputs. In this case, it is pretty easy to do. The controller can be tuned by changing the nonzero elements in the Q matrix to get a desirable response the Matlab function lqr can be used to derive optimal control gains for a continuous controller.

C. Optimal Switching Strategy

The proposed research concentrates on developing a switching criterion, which would improve the output performance of the system over the performance obtained by using standard state feedback. Most conventional control strategies attempt to provide a trade off between the control cost and the performance of the controlled variables. For example, the Linear Quadratic Regulator (LQR) attains this trade off by minimizing a quadratic cost given by Eqn. (16),

$$J = \int_0^{\infty} (\|U\|^2 + \|Y\|^2)\, dt \tag{16}$$

However, it does not necessarily give a good performance. It gives a performance that is acceptable to some extent. In order to obtain optimal performance of the controlled variable, the index given by Eqn. (17), should be minimized.

$$J = \int_0^{\infty} (\|Y\|^2)\, dt \tag{17}$$

Since the problem of minimization of the above cost, Eqn. (17), does not have an optimal solution, we do not make any such attempt here. We try to find a control strategy that would give lesser value of the performance index (J) measured using Eqn. (17), compared to the equivalent cost obtained when using state feedback control strategy.

$$\alpha(\xi, \tau) = \prec \xi, e^{A_1^T \tau}(T(\tau) - T_0)e^{A_1 \tau}\xi \succ > 0 \tag{18}$$

The above Eqn. (18) (L Aravena et. al., 2006; Lalitha Devarakonda et. al., 2006) shows that if the alternate feedback is applied for τ seconds, the performance of the system is improved. From the above theorem it was difficult to establish a time of switching. One alternative is to maximize the function α, which might be difficult to implement. Here we can note that since $T(0) = T_0$, the function $\alpha(\xi, \tau)$ is zero for $\tau = 0$. Hence if the derivative of α at $\tau > 0$ is positive, then we can say that there will be a time interval of length $\tau > 0$ where the function α will be positive.

Theorem: For the switched linear system Eqn. (8), If $\sigma(t) = 1$ and if for initial state ζ, the following condition holds,

$$\left.\frac{d\alpha(\zeta, t_1)}{dt_1}\right|_{t_1 = 0} = -<\zeta, (A_2^T\tau_0 + \tau_0 A_2 + C^T C) > 0 \tag{19}$$

then for small values of t_1, the function α will be positive & the alternate control is beneficial.

Proof:

$$\alpha(\zeta, t_1) = <\zeta, e^{A_2^T t_1}(\tau(t_1) - \tau_0)e^{A_2 t_1} >$$

$$\alpha(\zeta, t_1) = \zeta^T e^{A_2^T t_1}(\tau(t_1) - \tau_0)e^{A_2 t_1}\zeta$$

$$\alpha(\zeta, t_1) = \zeta^T e^{A_2^T t_1}\tau(t_1)e^{A_2 t_1}\zeta - \zeta^T e^{A_2^T t_1}\tau_0 e^{A_2 t_1}\zeta$$

$$\left.\frac{d\alpha(\zeta, t_1)}{dt_1}\right|_{t_1 = 0} = \zeta^T e^{A_2^T t_1}\tau(t_1)e^{A_2 t_1}\zeta - \zeta^T e^{A_2^T t_1}\tau_0 e^{A_2 t_1}\zeta \tag{19}$$

$$\left.\frac{d\alpha(\zeta, t_1)}{dt_1}\right|_{t_1 = 0} = \zeta^T A_2^T\tau_0\zeta + \zeta^T A_2\tau_0\zeta - [\zeta^T A_2^T\tau_0\zeta + \zeta^T A_2\tau_0\zeta] \tag{20}$$

$$= \zeta^T[A_2^T\tau_0 + \tau_0 + A_2]\zeta - \zeta^T[A_2^T\tau_0 + \tau_0 + A_2]\zeta \tag{21}$$

$$= \zeta^T[-Q]\zeta - \zeta^T A_2^T\tau_0\zeta \tag{22}$$

$$\left.\frac{d\alpha(\zeta, t_1)}{dt_1}\right|_{t_1 = 0} = -<\zeta, (A_2^T\tau_0 + \tau_0 A_2 + C^T C) > 0 \tag{23}$$

Above result can be used to design the following switching algorithm.

D. Switching Algorithm Design

Design of a performance based switching control law is equivalent to finding a matrix S and vectors K_1 and K_2 such that (L Aravena et. al., 2006; Lalitha Devarakonda et. al., 2006) & (Yathisha L et. al., 2015; S Patil Kulkarni et. al., 2015)

1. Intialize, master controller as $A_1 = A - BK_1$ & alternate controller as $A_2 = A - BK_2$.
2. Determine P by solving the Lyapunov equation,

$$A_1^T P + PA_1 = -C^T C$$

3. Define the switching matrix,

$$S = -(A_2^T P + PA_2 + C^T C)$$

4. The switching function is $S(x) = < x, Sx >$ if, $S(x) > 0$, Use alternate control else, master control

E. Stability of the Switching Strategy

In this subsection asymptotic stability of the proposed optimal switching control is explained.

If the systems with both A_1 and A_2 is observable and sliding does not exist, the feedback control system using the switching strategy defined in section 3. D is asymptotically stable.

Proof: Firstly, we assume that the switching occurs ideally; i.e., when $S(X) \leq 0$ the master feedback, is in place and when $S(X) > 0$ the alternative feedback K_2 is used without any delays. Recall that

$$S(X) = - \prec X, (A_2^T T_0 + T_0 A_2 + C^T C)X \succ \tag{24}$$

Let,

$$A_C = \gamma A_2 + (1 - \gamma)A_1 \tag{25}$$

for $\gamma = 0, 1$. When $S(X) > 0$ we use alternate feedback, hence $\gamma = 1$ When $S(X) < 0$, we use master feedback and hence $\gamma = 0$. In the absence of sliding, $\dot{X} = A_C X$ at any instant of time. Consider the positive definite function

$$V(X) = \prec X, T_0 X \succ \tag{26}$$

For Lyapunov stability consider

$$\begin{aligned} \dot{V} &= \frac{d \prec X, T_0 X \succ}{dt} \\ &= \prec X, (A_C^T T_0 + T_0 A_C)X \succ \\ &= \prec X, A_C^T T_0 X \succ + \prec X, T_0 A_C X \succ \\ &= -\gamma S(X) - (1 - \gamma) \prec X, C^T C X \succ \end{aligned} \tag{27}$$

If the system is observable, it is well known that the function $-(X, C^T CX)$ is non positive and cannot be zero over any time interval. When $S < 0$, $\gamma = 0$, the term $\gamma S(X)$ vanishes from Eqn. (27). When $S > 0$, $\gamma = 1$, and $\dot{V}$ is negative. It is clear we can find a single lyapunov function $V(X)$ such that $\dot{V}$ is negative for any $X \in R^n$. Hence, it states that the system is asymptotically stable, provided there is no sliding.

4. SIMULATION RESULTS

The experimental set-up to test the proposed algorithm consists of linearised Phillps-Heffron model installed with UPFC described by A and B matrices below. The master and alternate controllers of K_1 and K_2 for all the four input matrices B are obtained by using LQR technique respectively by choosing the wighing matrices as ($Q = I$; $R = 1$) for K_1 and ($Q = I$; $R = 0.01$) for K_2. The matrix C is vector with zeros along with 1 in any one position depending on the state variables.

The proposed optimal switching strategy S between two vectors of K_1 and K_2 for the four alternatives UPFC based damping controllers m_E, δ_E, m_B and δ_B of control matrix B are also given below.

In order to investigate the performance of the proposed switching controller and the system behavior under nominal loading condition the performance index is tabulated with the present individual controllers as well as the existing optimization techniques.

In the present study, the performance index J, is expressed as:

$$J = \int_0^{t_s} |M_p^2|\, dt \tag{28}$$

$$A = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.07076 & 0 & -0.0214 & 0 \\ -0.08322 & 0 & -0.4873 & 0.1982 \\ 1513 & 0 & -3516 & -100 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.0474 & -0.1492 & -0.023 & -6.612\times 1.0e-03 \\ -0.2305 & 7.53\times 1.0e-03 & -0.056 & 8.386\times 1.0e-03 \\ 4591 & -311 & 1096 & -189 \end{bmatrix}$$

$$Q = I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R = [1] (Master control) & R = [0.01] (Alternate control)

The master and alternate controllers along with the switching matrices for modulating index and phase angle of series & shunt inverters are:

A. Modulating Index of Series Inverter

$$K_1 = [1.1660 \quad -48.5192 \quad 0.6684 \quad 0.9781]$$

$$K_2 = [8.3838 \quad -445.7982 \quad 11.3497 \quad 9.9743]$$

$$S = \begin{bmatrix} 0.0008 & -0.0460 & 0.0011 & 0.0005 \\ -0.0460 & 2.5498 & -0.0597 & -0.0289 \\ 0.0011 & -0.0596 & 0.0014 & 0.0006 \\ 0.0005 & -0.0289 & 0.0006 & -0.0000 \end{bmatrix}$$

B. Phase Angle of Series Inverter

$$K_1 = [-2.9311 \quad -101.3626 \quad 5.9240 \quad -0.6932]$$

$$K_2 = [-2.7049 \quad -257.2307 \quad -2.1846 \quad -9.5648]$$

$$S = \begin{bmatrix} 0.0000 & -0.0079 & -0.0004 & -0.0004 \\ -0.0079 & 1.4343 & 0.0539 & 0.0406 \\ -0.0004 & 0.0539 & 0.0017 & 0.0009 \\ -0.0004 & 0.0406 & 0.0009 & -0.0000 \end{bmatrix}$$

C. Modulating Index of Shunt Inverter

$$K_1 = [2.1271 \quad -149.8195 \quad 3.8598 \quad 0.9110]$$

$$K_2 = [8.6342 \quad -665.4444 \quad 27.2748 \quad 9.8972]$$

$$S = \begin{bmatrix} 0.0002 & -0.0252 & 0.0008 & 0.0002 \\ -0.0252 & 2.4708 & -0.0820 & -0.0215 \\ 0.0008 & -0.0820 & 0.0023 & 0.0005 \\ 0.0002 & -0.0215 & 0.0005 & -0.0000 \end{bmatrix}$$

D. Phase Angle of Shunt Inverter

$$K_1 = [-1.8283 \quad -289.4468 \quad 3.1935 \quad -0.5944]$$

$$K_2 = [-0.3962 \quad -603.5448 \quad 6.7290 \quad -9.4647]$$

$$S = \begin{bmatrix} 0.0005 & 0.0005 & -0.0004 & -0.0000 \\ 0.0005 & 1.6167 & 0.0020 & -0.0001 \\ -0.0004 & 0.0020 & 0.0006 & -0.0000 \\ -0.0000 & -0.0001 & -0.0000 & 0.0000 \end{bmatrix}$$

The digital simulation results of all the four UPFC control inputs modulating index and phase angle of series & shunt inverters for the locally available state variable rotor speed deviation are shown below.

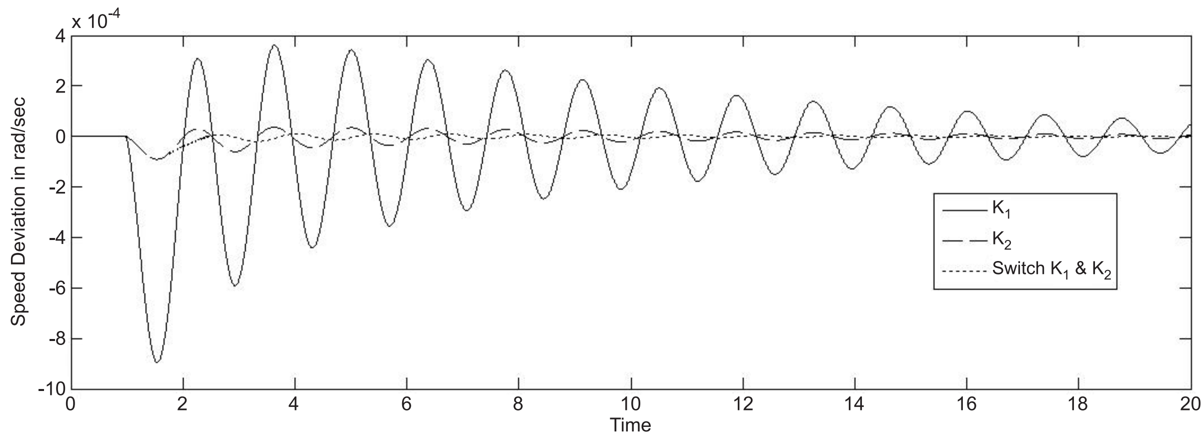

Figure 4: Speed Deviation response for m_E

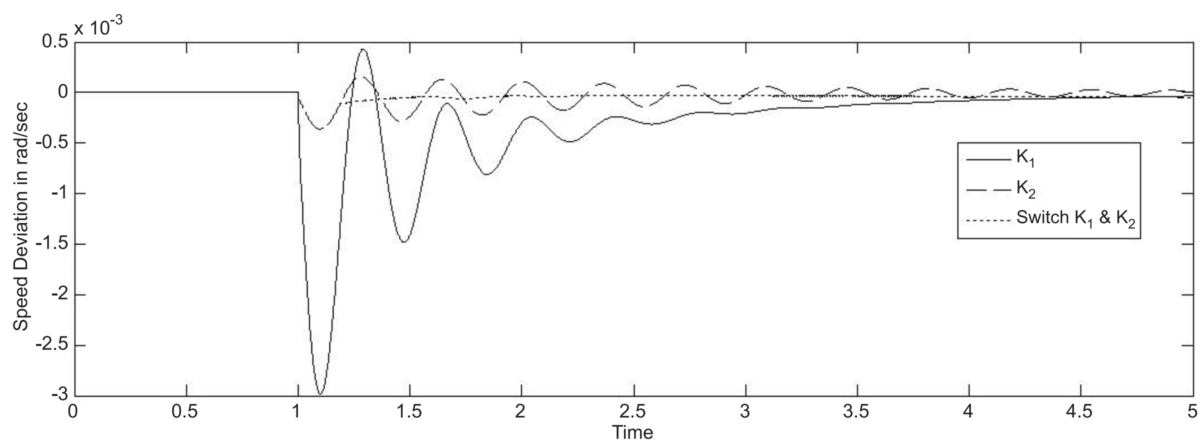

Figure 5: Speed Deviation response for δ_E

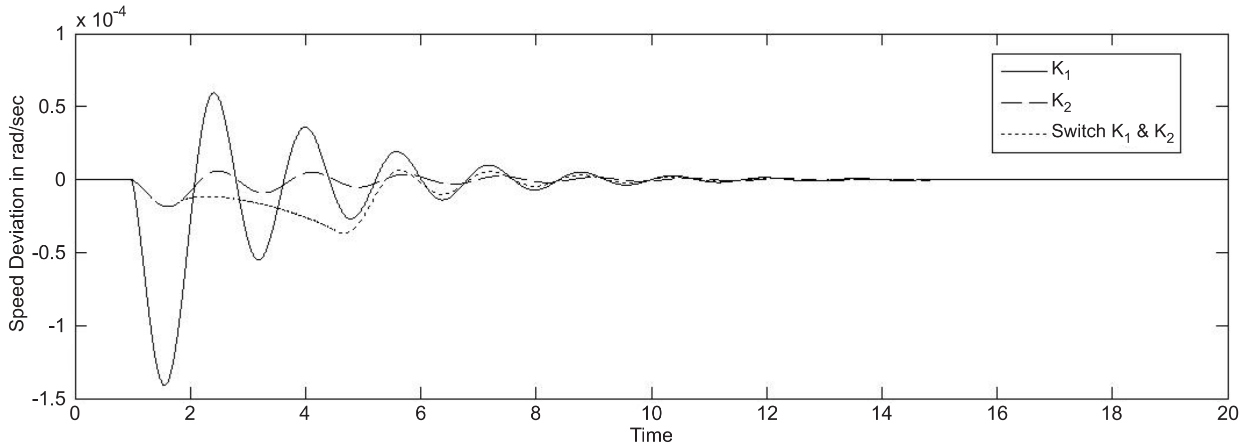

Figure 6. Speed Deviation response for m_B

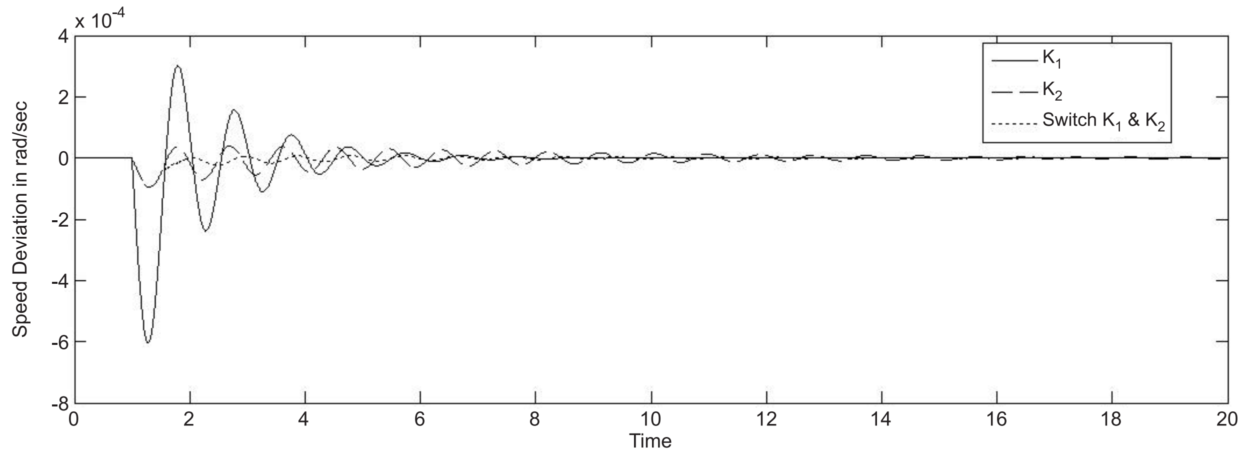

Figure 7. Speed Deviation response for δ_B

Table 1
Comparison of performance index J with individual controllers of LQR

$J = \int_0^{t_s} \lvert M_p^2 \rvert dt$	m_E	δ_E	m_B	δ_B
K_1	$2.025 \times 1.0e-05$	$4.05 \times 1.0e-05$	$2.352 \times 1.0e-05$	$3.6 \times 1.0e-06$
K_2	$2.2 \times 1.0e-07$	$7.2 \times 1.0e-07$	$4.8 \times 1.0e-07$	$1.6 \times 1.0e-07$
Switch K_1 & K_2	$1.6 \times 1.0e-07$	$1.8 \times 1.0e-07$	$1.08 \times 1.0e-06$	$1.1 \times 1.0e-07$

Table 2
Comparison of performance index J with Oter optimization techniques

$J = \int_0^{t_s} \lvert M_p^2 \rvert dt$	m_E	δ_E	m_B	δ_B
Switching Approach	$1.6 \times 1.0e-07$	$1.8 \times 1.0e-07$	$1.08 \times 1.0e-06$	$1.1 \times 1.0e-07$
RCGA	$8.67 \times 1.0e-06$	$6.86 \times 1.0e-06$	$4.32 \times 1.0e-06$	$5.04 \times 1.0e-06$
PSO		$9.8 \times 1.0e-04$	$1.805 \times 1.0e-03$	
IPSO		$4.32 \times 1.0e-04$	$1.44 \times 1.0e-03$	
θ PSO		$1.2375 \times 1.0e-03$		
ANN	$2.5 \times 1.0e-06$	$1.9687 \times 1.0e-06$	$1.715 \times 1.0e-06$	$1.4062 \times 1.0e-06$
ICA		$1.6 \times 1.0e-05$	$1.2 \times 1.0e-05$	

5. DISCUSSIONS

The dynamic response curves for the state space variable speed deviation ($\Delta\omega$), after the fault at 1 sec, for all the four control inputs of UPFC m_E, δ_E, m_B, and δ_B are plotted as shown in the Fig's 4-7 with the legend K_1, K_2 and Switch K_1 and K_2 for the proposed state feedback optimal switching control.

From Fig's 4-7 concludes that the proposed state feedback optimal switching control between two LQR (K_1 & K_2) controllers provides improvements in performance compared to system response with individual controllers K_1 and K_2 with respect to peak overshoots and settling time.

To validate effectiveness of the proposed optimal switching strategy the performance index $J = \int_0^{t_s} |M_p^2| dt$ is compared with individual controllers of LQR in Table 1 and other optimization techniques in Table 2. From Tables 1 & 2, it concludes that the proposed switching approach is a robust controller compared to individual controllers of LQR and other optimization techniques like Real coded genetic algorithm (RCGA), Particle swarm optimization (PSO), Iterative PSO (IPSO), θ-PSO, Artificial neural network (ANN) & Imperlist competative algorithm.

6. CONCLUSION

The optimal switching strategy has been successfully applied to the design of state feedback UPFC based damping controllers. The design problem of the robustly selecting state feedback controller parameters is converted into an optimization problem which is solved by a Linear Quadratic Regulator (LQR) technique with the time do-main based objective function. The two LQR controllers are designed by choosing wiehting matrices as $Q = I$ & $R = 1$ for master controller and $Q = I$ & $R = 0.01$ to alternate controller. Only the local and available state variable $\Delta\omega$ is considered for the proposed work. The effectiveness of the proposed UPFC controllers (m_E, δ_E, m_B, and δ_B) for improving transient stability performance of a power system are demonstrated by a weakly connected example power system subjected to step change. The linear time domain plots results show that the proposed optimal switching control provides improvements in performance of the system compared to individual controllers of LQR.

The system performance characteristics in terms of $J = \int_0^{t_s} |M_p^2| dt$ is tabulated and concludes that the proposed optimal switching strategy provides robust performance compared to individual controllers of LQR and other optimization techniques for all the control inputs of UPFC modulating index of series inverter (m_E), modulating index of shunt inverter (m_B), phase angle of series inverter (δ_E) and phase angle of the shunt inverter (δ_B).

Acknowledgments

The Authors wish to thank Dr. Syed Shakeeb ur Rehman, Principal and Dr. M N Shanmukhaswamy, Head of Electronics & Communication Department of Shree Jayachamarajendra College of Engineering, Mysore for their encouragement during this research work.

References

1. Douglas Leith, Robert Shorten, William Leithead and Oliver Mason (2010) "Issues in the design of switched linear control systems: A benchmark study", *Regular paper, Department of Electronic and Electrical Engineering NUI*, Dublin Ireland, pp. 1-2.

2. Zhi Hong Huang, Cheng Xiang, Hai Lin and Tong Heng Lee (2007) "A Necessary and Sufficient Condition for Stability of Arbitrarily Switched Second-Order LTI System: Marginally Stable Case", *22nd IEEE International Symposium on Intelligent Control Part of IEEE Multi-conference on Systems and Control*, Singapore, 1-3 October 2007, pp. 83-88.

3. Keith R. Santarelli and Munther A. Dahleh (2008) "Comparison of a Switching Controller to two LTI Controllers for a Class of LTI Plants", *American Control Conference, Westin Seattle Hotel, Seattle* , Washington, USA June 11-13, 2008, pp. 4640-4646.

4. L.. Aravena and Lalitha Devarakonda (2006) "performance driven Switching Control", *IEEE Transactions, Industrial Electronics*, E-ISBN: 1-4244- 0497-5, July 2006, PP. 31-36. DOI:10.1109/ISIE.2006.295564.

5. Keith R. Santarelli (2011) "A Switched State Feedback Law for the Stabilization of LTI Systems, *IEEE Transactions*, Volume:56, Issue: 5 ,ISSN :0018-9286, May 2011, pp. 998-1013. DOI:10.1109/TAC.2010.2073590

6. H. Shayeghi, H.A. Shayanfar, S. Jalilzadeh and A. Safari (2009) "Design of output feedback UPFC controller for damping of electromechanical oscillations using PSO, *Energy Conversion and Management, Elsevier journal*, pp. 1-8.

7. Al-Awami AT, Abdel-Magid YL and Abido M A (2007) "A particle-swarm-based approach of power system stability enhancement with unified power flow controller, *Elect Power Energy Syst*,Vol 29, pp. 251-259.

8. Tambey N and Kothari M (2003) "Unified power flow controller based damping controllers for damping low frequency oscillations in a power system, *http:// www.ieindia. org/publish/el/0603¿*.

9. Vilathgamuwa M, Zhu X and Choi S S (2000) "A robust control method to improve the performance of a unified power flow controller", *Elect Power Syst Res*, Vol. 55, pp. 103-111.

10. H. F. Wang(2000)"A Unified Model for the Analysis of FACTS Devices in Damping Power System Oscillations Part III: Unified Power Flow Controller," *IEEE Transactions on Power Delivery*, Vol. 15, No. 3, pp. 978-983, 2000.

11. N. Tambey and M.L. Kothari (2003) "Unified Power Flow Controller(UPFC) Based Damping Controllers for Damping Low Frequency Oscilltions in a power system," *IE(I) Journal - EL*, Vol. 84, pp. 35-41, 2003.

12. Amin Safari and Shayeghi (2009) "Optimal Design of UPFC Based Damping Controller using Iteration PSO," *International Journal of Electrical Power and Energy System Engineering*, pp. 151-156, 2009.

13. R.K. Pandey (2010) "Analysis and Design of Multi - stage LQR UPFC," *IEEE Proceedings*, 2010.

14. M. Sobha, R. Sreerama Kumar and Saly George (2010) "ANFIS Based UPFC Supplemetary Controller for Damping Low Frequency Oscillations in Power Systems," *Regular Paper*, JES 2010.

15. H. Shayeghi , H. A. Shayanfar and A. Safari (2009) "Design of Output Feed-back UPFC Controllers for Damping of Electromechanical Oscillations Using PSO", *ELSEVIER Article of Energy Conversion and Management*, JES 2009.

16. A. K. Baliarsingh, S. Panda, A.K. Mohanty and C. Ardil (2010) "UPFC Supplementary Controller Design Using Real-Coded Genetic Algorithm for Damping Low Frequency Oscillations in Power Systems", *World Academy of Science, Engineering and Technology*, pp. 759-769.

17. Yathisha L and S Patil Kulkarni "Optimum LQR Switching Approach for the Improvement of STATCOM Performance", Springer LNEE, Vol 150, E-ISSN: 1876-1100 , 2013, pp. 259-266, DOI: 10.1007/978-1-4614-3363-7 28.

18. Yathisha L and S Patil Kulkarni (2015) "Application and comparison of switching control algorithms for power system stabilizer", *IEEE Xplore, ICIC Conference*, Pune, 28th to 30th May 2015, pp. 1300-1305.

Appendix A. K-Constants and Variable Values

Synchronous Machine:

$H = 4.0$, $D = 0.0$, $T'_{do} = 5.044$.

Excitation System:

$k_A = 100$, $T_A = 0.01$

k constants for the nominal operating conitions:

$k_1 = 0.5661$, $k_2 = 0.1712$, $k_3 = 2.4583$

$k_4 = 0.4198$, $k_5 = -0.1513$, $k_6 = 0.3516$

$k_{pe} = 0.3795$, $k_{qe} = 1.1628$, $k_{ve} = -0.4591$

$k_{pb} = 0.1864$, $k_{qb} = 0.2855$, $k_{vb} = -0.1096$

$k_{p\delta e} = 1.1936$, $k_{q\delta e} = -0.0380$, $k_{v\delta e} = 0.0311$

$k_{p\delta b} = 0.0529$, $k_{q\delta b} = -0.0423$, $k_{v\delta b} = 0.0189$