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Report: Delta function

B.G. Sidharth*
Abhishek Das**

It is known that one can write the innite sum of delta functions as

$$\sum_{n=-\infty}^{+\infty} \delta(x-nt) = \cdots + \delta(x-2t) + \delta(x-t) + \delta(x) + \delta(x+t) + \delta(x+2t) + \cdots \quad (1)$$

Now, by the property of delta function we know that at $x=0, \delta(x)=\infty$ and anywhere else $\delta(x)=0$. Thus, resorting to this basic property we have from equation (1) for all non-zero values of *x*

$$\sum_{n=-\infty}^{+\infty} \delta(x-nt) = \cdots + \delta(x-2t) + \delta(x-t) + \delta(x+t) + \delta(x+2t) + \cdots \quad (2)$$

Essentially, for all non-zero values of x we have pairs such as:

$$\delta(x-2t) \; and \; \delta(x+2t)$$

$$\delta(x-t) \; and \; \delta(x+t)$$

$$\delta(x-lt) \; and \; \delta(x+lt)$$

and so on, where l is some arbitrary value of n. These pair terms can be interpreted as elements of a double Wiener process, where, time fluctuates between *'−lt'* and *'+lt'*. As it seems, this epitomizes a backward-forward nature of time. This indicates that at some position *x* every instant of times, say *'lt',* has a backward-forward nature. Now, it is known that the innite sum of the delta function can also be written as

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*birlasc@gmail.com
**parbihtih3@gmail.com

$$\sum_{n=-\infty}^{+\infty} \delta(x - nt) = \frac{1}{t} \sum_{n=-\infty}^{+\infty} e^{\frac{2n\pi xi}{t}} \tag{3}$$

From equation (3) we can draw an interesting conclusion. For t = 0 we will have the sum as innite and for $t = \infty$ we shall have the sum as zero. This means that the sum in the relation (3) essentially corresponds to all the values of time *t*. As if, all the values of time are incorporated in the sum $\sum_{n=-\infty}^{+\infty} \delta(x - nt)$. One can argue that the birth of a new universe and it's end is included in this innite sum of the delta function. Now, let us consider the instants of time nested along some axis that shows the evolution of time in both the backward and forward directions. So, it is obvious from our considerations that if we take the collection of all open subsets of S (containing all instants of time), say T, then they will dene a topological space: *(S,T)*. Now, considering any two disjoint closed subsets, one can nd a continuous function that separates the disjoint closed subsets, say *f*. Thus, according to Urysohn's lemma [1, 2] we will have a normal space. This means that we will also have a metric dened for the topological space. It is worth mentioning that this metric of the topological space has all the information regarding the arrow of time. The fluctuations and the backward-forward nature of time are also incorporated here in the sense that relation (3) provides epitomizes all instants of time. Also, since the topological space (S; T) is metrizable, the separation or distance between two points of time in the axis will be given by the given metric along with many other elaborate details on the evolution of time.

References

[1] B. G. Sidharth, The Thermodynamic Universe, p - 164, World Scientic, 2008.

[2] Pavel Urysohn, Sur une classe d'equations integrales non lineaires, Mat. Sb. 31, 256- 255, 1923.