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Security Constrained Optimal Power Flow Using Benders Cut Principle

D. Kishore Kumar[1] and Mr. G. Sasikumar[2]

[1]M Tech (Power Electronics), VNR VJIET, Hyderabad

[2]Associate Professor, EEE Dept, VNR VJIET, Hyderabad

Email:-kishorwithk@gmail.com; sasikumar_g@vnrvjiet.in;

ABSTRACT

This paper presents an efficient approach to find Security Constrained Optimal Power Flow (SCOPF) determining the dispatch schedule of power generators with minimum cost. Satisfying system constraints, transmission constraints along with mandates to ensure optimal power flow has led to SCOPF. Revenue paid for electricity will be reduced considerably while the generation cost is reduced. Power world simulator employs linear programming method for finding optimal power flow. The proposed method has been tested and examined on ieee-14 bus system using Benders cut principle. Results shows this method is advantageous than other conventional methods for solving OPF with security constraints for the same case.

Keywords— Optimal Power Flow, Security Constrained Optimal Power Flow (SCOPF), Benders cut principle.

1. INTRODUCTION

In power systems, operation and planning needs operators to make decisions with respect to different objectives. Several tools have been developed to assist operators in this aspect. Optimal Power Flow (OPF) is one of them which help the operators in running the system optimally under specific constraints. Operating power systems at minimum cost without maintaining the security is no longer the sufficient criterion for dispatching electric power and therefore suitable control measures should be made. This has led to Security Constraint Optimal Power Flow.

As the power industry moves into a more competitive environment, use of Optimal Power Flow will become increasingly more important in maximizing the capability of the existing transmission system asset. A typical OPF solution adjusting the appropriate control variables, so that a specific objective in operating a power system network is optimized (maximizing or minimizing) with respect to the power system constraints, dictated by the electrical network and with security constraints is solved in this paper. Security constrained optimal power flow takes into account outages of certain transmission lines or equipment. A SCOPF solution is secure for all credible contingencies or can be made secure by corrective means. Due to the computational complexity of the problem, more work has been devoted to obtaining faster solutions requiring less storage. Several classical methods have been used in solving OPF for decades and here one of it, linear programming is used to find the optimal solution for this problem as it is the fast method among the classical methods for solving SCOPF. Generator / Transformer / Line / Load / Static or Synchronous compensator failure / Apparatus failure is termed as Security Constrained Optimal Power flow (SCOPF).

The recent Blackouts lead to the importance of the system which is capable to withstand any contingencies, or to have system which can work on the specified limits when a contingency occurs, without effecting the overall operation of the system. SCOPF problem is the perfect incorporation of the contradictory doctrines of maximum economy, safer operation and augmented security.

2. PROBLEM FORMULATION

The optimal power flow is constrained optimization problem requiring the minimization of :

$$F = (x,u)$$

Subject to constraints

$$Gi\ (x, u) = 0,\ i=0, 1, 2, 3....m \text{ (equality constraints)}$$

$$Hi(x, u) \leq 0,\ i=0, 1, 2, 3....m \text{ (inequality constraints)}$$

$$umin \leq u \leq umax$$

$$xmin \leq x \leq xmax$$

Here f (x,u) is the scalar objective function,

G (x,u) represents nonlinear equality constraints (power flow equations),

H (x,u) is the nonlinear inequality constraints.

u and x represents set of controllable and dependent variables respectively.

The vector x contains dependent variables consisting of bus voltage magnitudes and phase angles, as well as the Mvar output of Generators designated for bus voltage control and fixed parameters such as the reference bus angle, non- controlled Generator MW and Mvar outputs, non-controlled MW and Mvar loads, fixed bus voltages, line parameters, etc.

A. Objective function

In the solution of SCOPF, the main objective is to minimize total operating costs of the system with maintaining the system security. In OPF, when the load is light, the cheapest generators are always the ones chosen to run first. As the load increases, more and more expensive generators will then be brought in.

Thus, the operating cost plays a very important role in the solution of OPF. The amount of fuel or input to a generator is usually expressed in Btu/hr (British Thermal units per hour) and its output in MW (Mega Watts). Figure 1 shows a typical input-output curve of a generator (slack bus generator) also commonly known as the heat-rate curve.

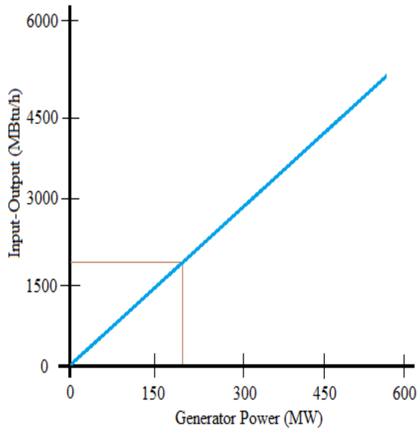

Fig 1: Typical Input-output Curve of a Generator

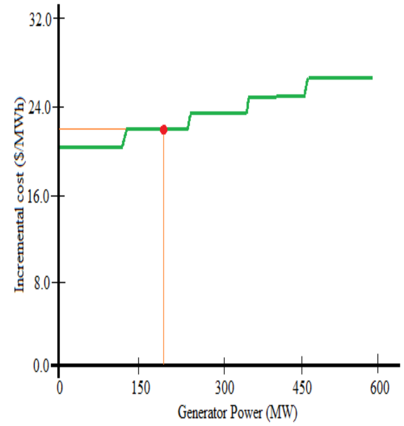

Fig 2: Typical incremental cost curve

It specifies the relationship between how much heat must be input to the generator and its resulting MW output. In all practical cases, the cost of generator i can be represented as

$$C_i = (a_i + b_i P_i + c_i P_i^2) * \text{fuel cost}$$

Where P_i is the real power output of generator i, and a_i, b_i, c_i are the cost coefficients.

B. Control Variables

The control variables in an optimal power flow problem are the quantities whose value can be adjusted directly to help minimize the objective function and satisfy the constraints. The control variables can be given as:

1. Real power and reactive power generation.
2. Phase-shifter angles
3. Net interchange
4. Load MW and Mvar (load shedding)
5. DC transmission line flows
6. Control voltage settings
7. LTC transformer tap settings

C. Dependent Variables

These variables are the optimal power flow variables that are not controlled. These include all type of variables that are free, within limits, to assume value to solve the problem. The main dependent variables are complex bus voltage angles and magnitude.

D. Constraints

Constraints are the operating limits to the problem. In a conventional power flow, equipment limits are normally supplied by the user for monitoring purposes, such as printing out of any violations of circuit flow limits. Only the power flow algorithm enforces a small set of limits, such as, tap limits and generator Mvar limits. In contrast, an OPF enforces all equipment limits input by the user. This may easily lead to problem infeasibility if the limits are too restrictive or inconsistent. Careless input of limits should therefore be avoided.

A commercially available OPF normally offers a facility to relax limits in case of unfeasibility. Once a solution is obtained for the relaxed problem, the OPF will provide means to investigate how the original limits had caused convergence difficulties. Such a mechanism may provide valuable information concerning the power system being modelled. For instance, a region which requires relaxation of voltage limits may have implications of requiring new reactive compensation sources.

Some OPF programs require users to give them guidance as to which limits can be relaxed and in what sequence. This flexibility in fact places much burden on the users who need to appreciate how an OPF algorithm performs before the preferred strategy for constraint relaxation can be formalized as input to the program.

E. Equality constraint

The equality constraints of the OPF reflect the physics of the power system as well as the desired voltage set points throughout the system. The physics of the power system are enforced through the power flow equations which require that the net injection of real and reactive power at each bus sum to zero. This can be achieved by active and reactive power analysis.

$$P\,i = P\,Load + P\,Loss$$

$$Q\,i = Q\,Load + Q\,Loss$$

Where P i & Q i are the active and reactive power outputs

P Load & Q Load is the active and reactive load power

P Loss & Q Loss is the active and reactive power loss

The power flow equations of the network can be given as:

$G(V, \delta) = 0$

Where $P_i(V, \delta) - P_{i\,net}$

$G(V, \delta) = \{Q_i(V, \delta) - Q_{i\,net}\}$

$P_m(V, \delta) - P_{m\,net}$

Pi & Qi are the calculated real and reactive power at PQ bus.

F. Inequality Constraints

In a power system components and devices have operating limits & these limits are created for security constraints. Thus the required objective function can be minimized by maintaining the network components within the security limits.

$P_{gi\,min} \leq P_{gi} \leq P_{gi\,max}$

$Q_{gi\,min} \leq Q_{gi} \leq Q_{gi\,max}$

$\Sigma P_{gi} - P_D - P_{Loss} = 0$

Where Pgi is the amount of generation in MW at generator i

Qgi is the amount of generation in Mvar at generator i

The inequality constraints on voltage magnitude V of each PQ bus

$V_{i\,min} \leq V_i \leq V_{i\,max}$

Where Vi min & Vi max are the minimum and maximum values of voltages at bus i

The inequality constraints on phase angle δ of voltages at all buses i

$\delta_{i\,min} \leq \delta_i \leq \delta_{i\,max}$

Where δi min & δi max are the minimum and maximum values of phase angle at bus i.

3. SECURITY CONSTRAINED OPF

Security-constrained OPF (SCOPF) problems are a special class of OPF problems. It iterates between a base case OPF problem and a set of predefined contingency system states. To ensure the security of system, a so-called "N-1 criteria" is applied, i.e. there should be no violations after the outage of any single element in the system. This leads to the implementation of preventive mode of SCOPF. Figure 3 below compares the optimal power flow (OPF) with the security constrained optimal power flow (SCOPF).

Notice that there are c contingencies to be addressed in the SCOPF, and that there are a complete new set of constraints for each of these c contingencies. Observe:

I. Each set of contingency-related equality constraints is exactly like the original set of equality constraints.

II. Each set of contingency-related inequality constraints is exactly like the original set of inequality except it corresponds to the system with an element removed and, for branch flow constraints and for voltage magnitudes, the limits will be different.

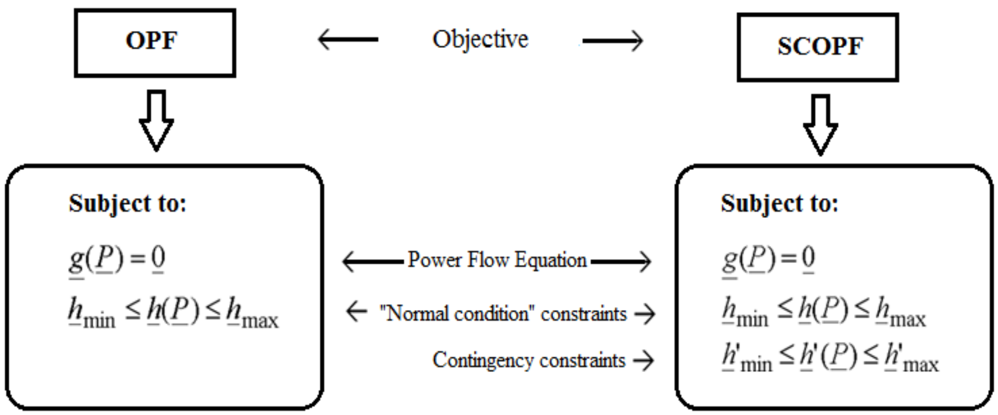

Fig 3: Comparison of OPF and SCOPF

Also notice that the constraints are a function of x_k, the voltage magnitudes and angles under the pre-contingency (k=0) and contingency conditions (k>1, 2,…, c), and u0, the controls which were set under the pre-contingency conditions (k=0).

The solution strategy first solves the OPF (master problem) and then takes contingency 1 and re-solves the OPF, then contingency 2 and resolves the OPF, and so on (these are the sub problems). For any contingency-OPFs which require a redispatch, relative to the k=0 OPF, an appropriate constraint is generated, at the end of the cycle, these constraints are gathered and applied to the k=0 OPF. Then the k=0 OPF is resolved, and the cycle starts again. Experience has it that such an approach usually requires only 2-3 cycles.

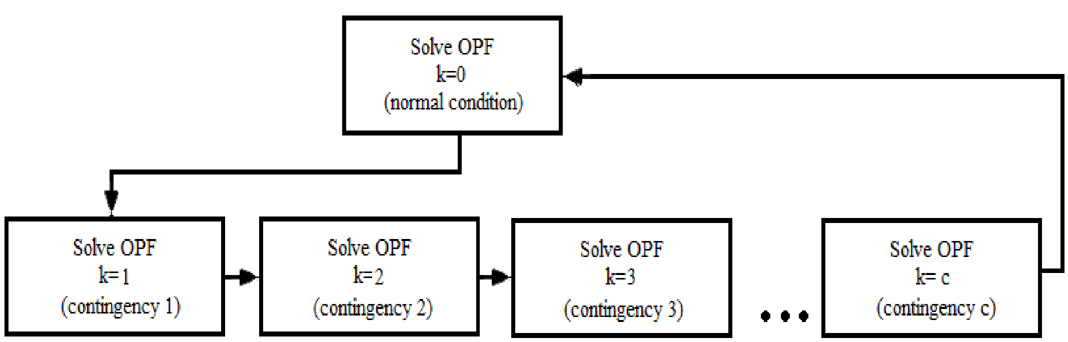

Fig 4: Decomposition solution strategy

4. BENDERS CUT PRINCIPLE

J.F. Benders introduced the Benders decomposition (BD) algorithm for solving large-scale, mixed-integer linear programming problems (MILP), which partition the problem into a programming problem (which may be linear or non-linear, and continuous or integer) and a linear programming problem. The procedure followed in this paper includes the steps illustrated in the flowchart of Figure 5.

Problems for which Benders cut principle can be applied are those that have the following structure:

$$\text{Min} \quad z = c(x) + d\,(y) \qquad \ldots (4.1)$$

$$\text{s.t.} \quad A(x) \geq b$$

$$E(x) + F(y) \geq h$$

In Benders decomposition, the second part problem is required to be a linear programming problem, which is convex and dual theory can be applied to.

Although in all the objective function and constraints can be completely nonlinear functions, a so called "P property" is preferred for better performance, which means that the decision variables are partitioned explicitly in the objective function and constraints.

Outline of the Methodology

We repeat problem (4.1) here again and put it into the BD form, which means that all the objective and constraints are linear. For simple explanation, the coupler is put into explicitly linear form. This form is called the standard form in this dissertation.

$$\text{Min} \quad z = c(x) + d(y)$$

$$\text{s.t.} \quad A(x) \geq b \qquad \ldots (4.2)$$

$$E \cdot x + F(y) \geq h \qquad \ldots (4.3)$$

This problem can be decomposed into three sub problems: master problem, feasibility sub problem, and optimality sub problem.

- **Master Problem**

Decide on a feasible $x*$ considering only constraint (4.2) via what is referred to as the master Problem:

$$\text{Min} \quad z = c(x) + \alpha\,(x)$$

$$\text{s.t.} \quad A(x) \geq b$$

where $\alpha(x)$ is a piecewise function of the optimality subproblem optimal value as a function of the master problem decision variable x. z is a lower bound of the whole problem and will be updated iteratively by the optimality subproblem.

- **Feasibility Subproblem**

In order to check whether (4.3) is satisfied based on x^* given in the master problem, a slack vector is introduced and the corresponding subproblem is formulated as:

$$\begin{aligned} \text{Min} \quad & v = 1^T \cdot s \\ \text{s.t.} \quad F(y) + s \quad & \geq h - E \cdot x^* \end{aligned} \qquad \text{... (4.4)}$$

Here, 1^T is the vector of ones, and $v > 0$ means that violations occur in the subproblem. In order to eliminate the violations, the feasibility cut (4.5) is added to the master problem:

$$v + \lambda E (x^* - x) \quad \leq 0 \qquad \text{… (4.5)}$$

where λ is the Lagrangian multiplier vector for inequality constraints (4.4). This problem is called feasibility check subproblem or feasibility problem in short.

- **Optimality Subproblem**

Decide on a feasible $y*$ considering constraint (4.3) given $x*$ from the master problem.

$$\begin{aligned} \omega = \text{Min} \quad & d(y) \\ \text{s.t.} \quad F(y) \quad & \geq h - E \cdot x^* \end{aligned} \qquad \text{… (4.6)}$$

where ω is the value of $\alpha(x)$ at x^*.

If the solution is not optimal, the optimality cut (4.7) is added to the master problem:

$$\omega + \pi E (x^* - x) \leq \alpha \qquad \text{… (4.7)}$$

where π is the Lagrangian multiplier vector of inequality constraints (4.6).

The algorithm to solve SCOPF by Benders cut principle is shown below.

Benders Decomposition algorithm

1. Solve the regular OPF to get $(x^1{}_0, u^1{}_0)$:

Min $f(u_0, x_0)$ s.t. $\{h_0(x_0, u_0) = 0, g_0(x_0, u_0) \leq 0\}$

2. For k = 1, 2, …
   a) For c=1,2,…C:

      Solve a problem of form (22) to get the optimal value $\Sigma_i \propto i$,where u^k_0 is in place of □.

- If $\Sigma_i \propto i > 0$, then add the cut into the master problem
- If $\Sigma_i \propto i = 0$ for all $c \in C$, terminate the algorithm.

b) Solve the master problem to get (x_0^{k+1}, u_0^{k+1}).

This subproblem is called the optimality check subproblem or optimality problem because it is used to check the optimality of the master problem according to the Benders Rule. In the algorithm, we first solve the master problem to obtain a lower bound of the objective value.

We then fix all the first-stage decisions and solve each scenario subproblem to get an upper bound. If the lower bound and the upper bound are within a tolerance, then the algorithm stops.

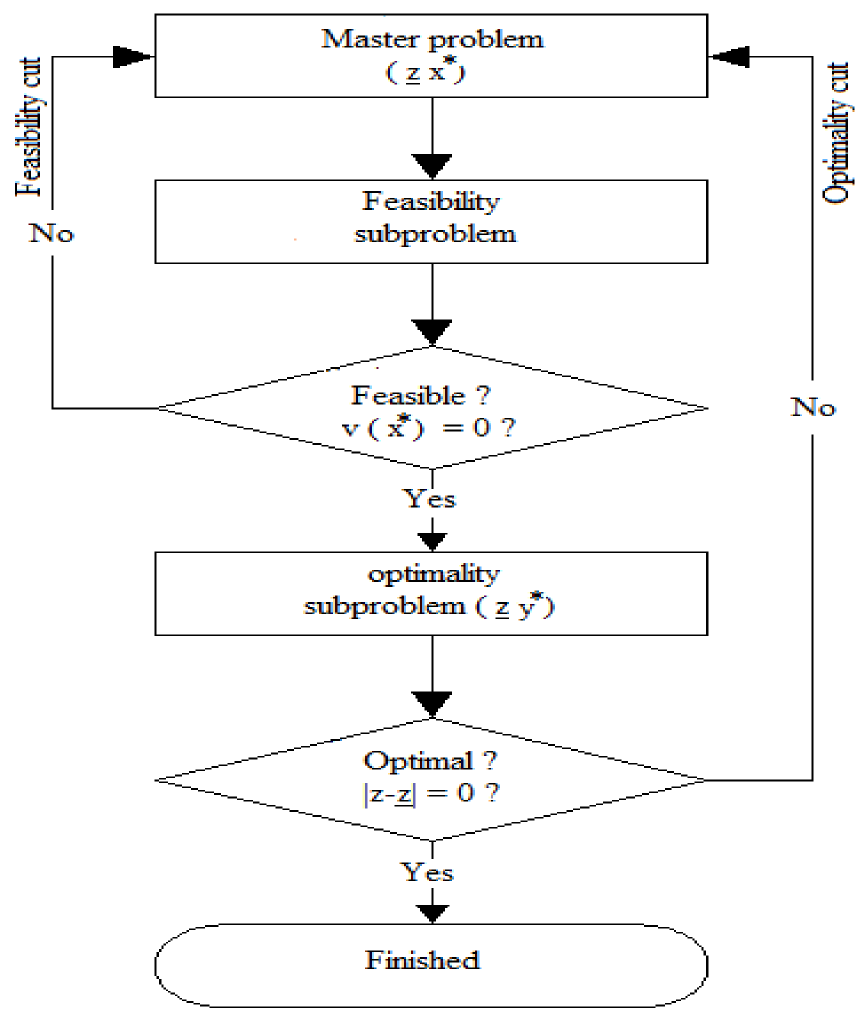

Fig 5: Benders process

4. RESULTS

An IEEE 14-bus power system is considered for SCOPF Simulation. Figure 6 shows a 14 bus IEEE system solved for SCOPF. It can be noted that line from bus 1 to bus 2 and line from bus 1 to bus 5 are loaded to 155 % and 76 % respectively. This is a threat to security of the power system as it could reduce the reliability of the transmission system.

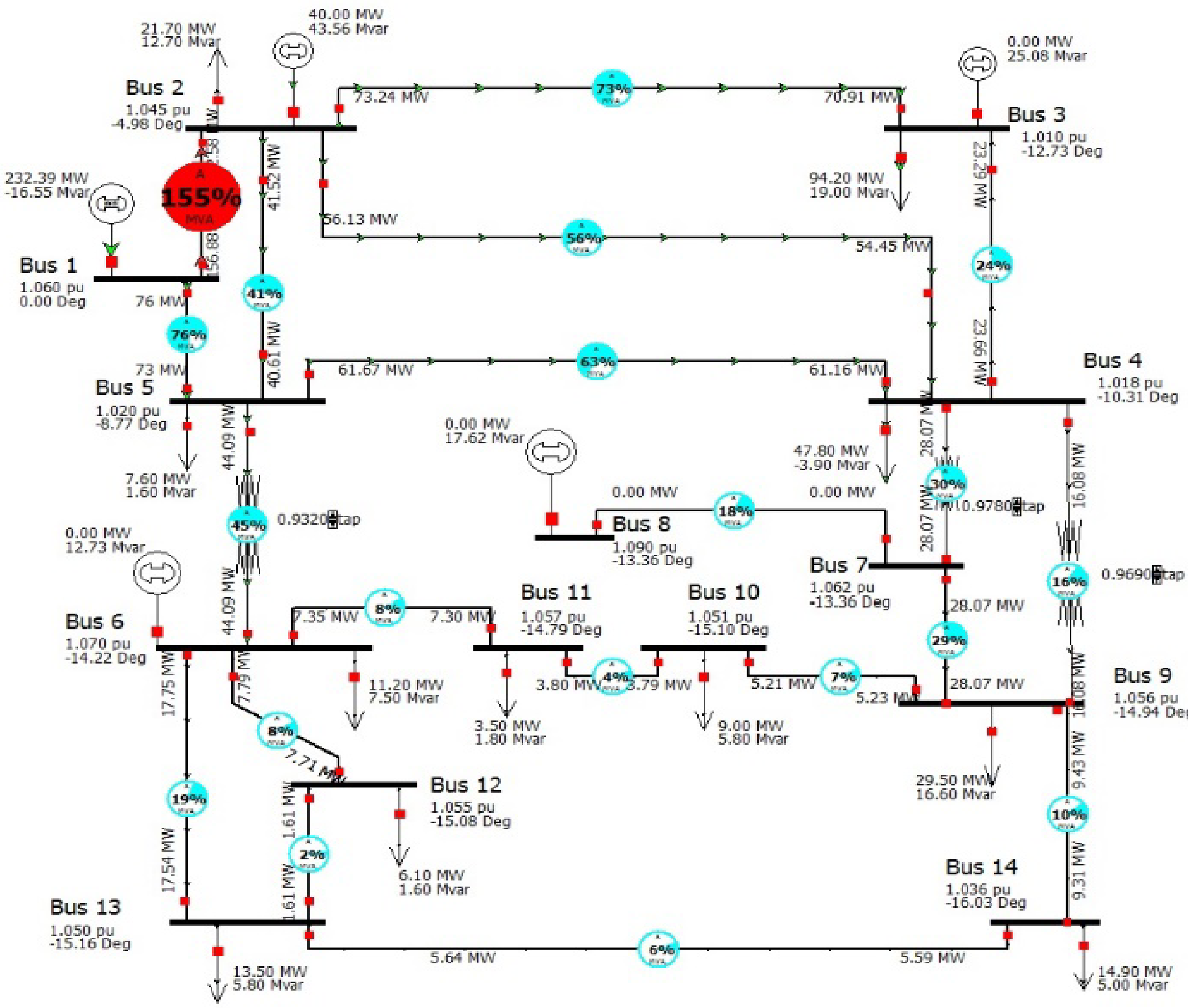

Fig 6: Load flow analysis of a 14 bus system without considering network constraints (conventional OPF).

Thus to ensure security of the system by reducing the burden on the transmission line for the same system, security constrained optimal power flow is solved by using Benders cut principle and the simulation is shown in figure 7.

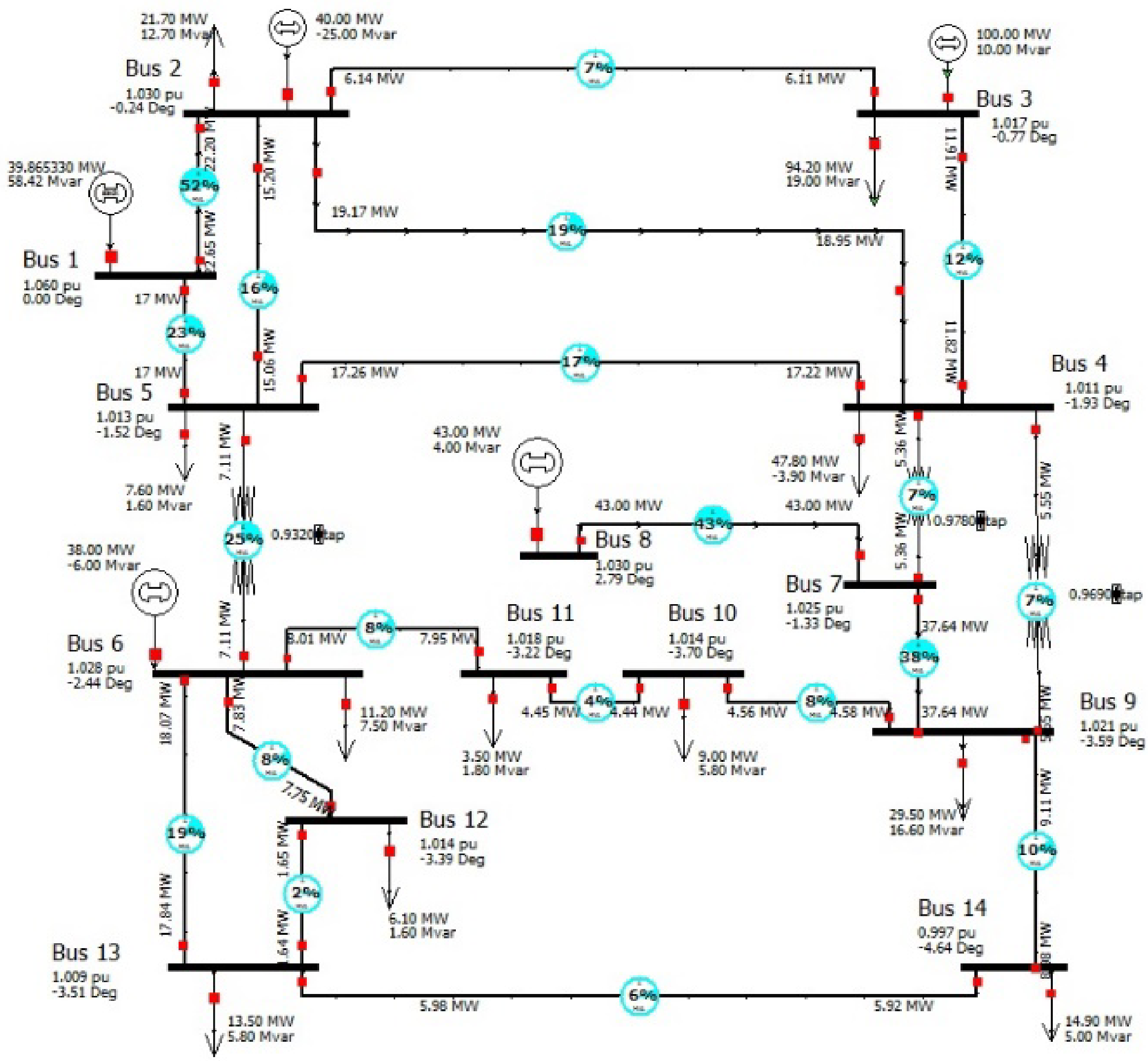

Fig 7: Load flow analysis of a 14 bus system by considering network constraints (Benders cut principle).

The final comparison has been made between the three cases. The three cases are normal optimal power flow, conventional security constrained optimal power flow and security constrained optimal power flow by Benders cut principle.

Table I: Comparison of both loss and Generation cost of three methods.

	Loss (MW)	Generation Cost (Rs/hr)
Normal OPF	13.39238	3581.02
Conventional SCOPF	1.995991	1544.7511
SCOPF by Benders cut principle	1.864838	1542.9069

Table 2 shows the difference in percentage of line loading for the 14-bus system while solving OPF and SCOPF. While solving the base case OPF the total cost for the system to generate a power of 259 MW is 3581.02 Rs/h when SCOPF solved for the same case by using conventional method the total generation cost for generating power of 259 MW is reduced to 1544.7511 Rs/h and finally when SCOPF is solved by using Benders cut principle the total generation cost for generating the power of 259 MW is reduced to 1543.9069.

Table II: Comparison of line loadings between conventional OPF and SCOPF by Benders cut principle

From bus	To bus	Line loading (% MVA) (conventional OPF)	Line loading (% MVA) (SCOPF by Benders cut principle)
1	2	155	52
1	5	76	23
2	3	73	7
2	4	56	19
2	5	41	16
3	4	24	12
4	5	63	17
4	7	30	7
4	9	16	7
5	6	45	25
6	11	8	8
6	12	8	8
6	13	19	19
7	8	18	43
7	9	29	38
9	10	7	8
9	14	10	10
10	11	4	4
12	13	2	2
13	14	6	6

From the results it is observed that the cost of generation obtained by SCOPF of Benders cut principle is lower than that obtained by conventional method. The comparison of costs for the three cases are shown in table 1.

5. CONCLUSION

In this work, an attempt is made to find optimum power solution for IEEE-14 bus system. Different objectives are considered to solve the problem and to minimize the cost of operation. The proposed method is used to consider different cases by varying generation schedule and the cost of operation is compared. A comparison has made between conventional SCOPF and SCOPF by Benders cut principle. Larger power systems can be considered in order to obtain more realistic results. The SCOPF problem solved using power world simulator by using Benders cut principle has ability to display the SCOPF results on system one-line diagram. By solving security constraint OPF the stability and reliability of the system is maintained.

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