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Stochastic Analysis of Two Products Production System with Sales When Both Demands and Funds are Available

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ABSTRACT

In this paper we study a two different products finite engenderment inventory system where the engenderment has the impact of seasonal demand, availability of funds and sales. Season for sales commences at time which is identically tantamount to maximum of time to seasonal demand, time to funds availability. The engenderment of the two different products is done alternately. The engenderment times of the products have general distributions. The time to seasonal demand and funds availability are exponential. The double Laplace Stieltjes Transform of engenderment and sales time and their denotes are obtained. Numerical examples are presented

Mathematics subject classification: 30c45, 30c80.

Keywords: Inventory systems, two different products, season, funds availability

1. INTRODUCTION

In this paper we study inventory systems and their cognate inventory subject to seasonal demands. The authoritative ordinance season commences in an exponential time and the time to availability of funds for the purchase of products has an exponential distribution.The season for sales commences when both the above exponential times are over. Several researchers studied single commodity inventory systems of (s, S) type. We utilize an incipient concept of which has been introduced as Setting the Clock Back to Zero(SCBZ) by Raja Rao8 and studied by K.Usha and Eswariprem, Ramanarayanan10 to Time to Vital Organs Failure of Gestational Diabetic Person. S.Murthy and Ramanarayanan4 studied one injuctively authorizing and two authoritatively mandating levels inventory system with SCBZ property. Arrow, Karlin and Scart1 first analysed such inventory system. Bulk demand models were treated by Ramanarayanan and Jacob.9 Murthy and Ramanarayanan 4, 5, 6,7 considered several (s, S) inventory system. Wu and Liang-yuh ouyang11 studied (Q, R, L) inventory model with defective items. Bhattacharya S.K., R. Biswas, M.M.Ghosh, P. Banerjee2 A study of peril factors of diabetes mellitus. Daniel.J.K., and Ramanarayanan. R 3 An (S, s) inventory system with rest periods to the server. In authentic life inventory models variants of pairs of products are engendered for sales by the companies. For example drug manufacturing companies engender antibiotic and vitamin tablets. Oil refining companies engender fuel oil like petrol and engine oil to reduce friction and auto mobile companies engender engine and body of conveyances. In textiles there is demand for shirting cloth and pant cloth, inner garments and outer garments and so on. The expected engenderment time and expected sale time are obtained. In section 2 Numerical examples are presented.

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2. MODEL DESCRIPTION

2.1. Assumptions

The following are the assumptions of the model

(i) The company engenders two different products A & B and at a time only one type is engendered. The engenderment time of product A is desultory variable with Cdf $G_x(x)$ and that of product B has Cdf $G_y(y)$. A & B are engendered in successive manner. The engenderment time X + Y of a dyad has Cdf $G(x)$.

(ii) The demand season for the products commences in a exponential time U, from starting time zero with parameter λ1.

(iii) The funds for the products become available in an exponential time V from the commencement time zero with parameter λ2.

(iv) Sales time commences when *k* number of pair products is engendered or when the maximum of U, V occurs.

(v) The products are sold in pairs and the selling time of a dyad is arbitrary variable with Cdf R(y) and the Cdf of selling time of product A is $R_A(y)$.

2.2. Analysis

We note that the probability of n number of pairs produced in (*o*, *t*)

$= G_n(t) - G_{n+1}(t)$ where $G_n(t)$ is the Cdf of $\sum_{i=1}^{n}\left(X_i + Y_i\right)$. When the n-th pair ($n < k$) is produced at time $x < t$ during t-x there are two possibilities. After the n-th pair production,

(a) The production for product A is over but for B is not over or

(b) The production for product A is also not over. Their respective probabilities are given below. We note that the probability of n number of pairs and one production of A is over before *t*

$$= \int_0^t g_n\left(x\right)\int_0^{t-x} g_x\left(u\right)\bar{G}_y\left(t-x-u\right)dudx \tag{1}$$

$= P(n + 1)$ number of A products and *n* number of B products are produced during (0, t)]. Here $\overline{G(x)} = 1 - G(x)$

Here $g_n(x)$ is the pdf of $\sum_{i=1}^{n}\left(X_i + Y_i\right)$.

The probability of n number of pairs in (0, *t*) and the production of A is not completed before *t* is

$$= \int_0^t g_n(x)\overline{G_X}(t-x)dx, \tag{2}$$

Since the selling time starts when *k* pairs are produced or at time *Z* = Max (*U*, *V*), where *U* and *V* are random variables with parameters λ_1 – & λ_2 respectively, the time to start sales is given by

T = Min (Time to produce *k* pairs, *Z*)

Let the probability density function of season starting time be $h(z)$, with distribution function $H(z)$.

Then
$$H(Z) = 1 - e^{-(\lambda_1 Z)} - e^{-\lambda_2 Z} + e^{-(\lambda_1+\lambda_2)Z}$$

$$h(z) = \lambda_1 e^{-(\lambda_1 Z)} +_{\lambda 2} e^{-\lambda_2 Z} - (\lambda_1 + \lambda_2)(e^{-(\lambda_1+\lambda_2)Z})$$

The pdf of T is

$$\left\{\sum_{i=0}^{K-1}\int_0^t g_i(x)\int_0^{t-x} g_x(u)\overline{G_y}(t-x-u)\,du\,dx+\sum_{i=0}^{K-1}\int_0^t g_i(x)\overline{G_X}(t-x)\,dx\right\}$$

The first term of the right side of (3) is the part of the pdf that the time to produce k number of pairs is t and the season has not started up to time t. The first part of the second term is the part of the pdf that the season starts at time t, the time to produce i number of A & B pairs is x, the $(i+1)^{th}$ A product is produced at time $x+u$ and the production of the $(i+1)^{th}$ B product is not over during t-x-u for $0 \le i \le k-1$. The second part of the second term is the part of the pdf that the season starts at time t, the time to produce i number of A & B pairs is x and the $(i+1)^{th}$ A product is not produced during t-x.

This gives the joint pdf of time to start sales T and total sales time of products R as follows.

$$f_{T,R}(x,y)=g_k(x)\left(e^{-(\lambda_2 x)}+e^{-\lambda_2 x}-e^{-(\lambda_1+\lambda_2)x}\right)r_k(y)$$

$$+\left\{\lambda_1 e^{-(\lambda_1 x)}+\lambda_2 e^{-\lambda_2 x}-(\lambda_1+\lambda_2)\left(e^{-(\lambda_1+\lambda_2)x}\right)\right\}$$

$$\left\{\sum_{i=0}^{k-1}\int_0^x g_i(u)\int_0^{x-u} g_x(v)\overline{G_y}(x-u-v)\,dv\,du\int_0^y r_i(w)r_a(y-w)\,dw+\sum_{i=0}^{k-1}\int_0^x g_i(x)\overline{G_x}(x-u)\,du\,r_i(y)\right\} \quad (4)$$

The double Laplace transform of the pdf is given by

$$f_{T,R}^{\Rightarrow}(\varepsilon,\eta)=\int_0^w\int_0^w e^{-\varepsilon x}e^{-\eta y} f_{T,R}(x,y)\,dx\,dy$$

$$=\int_0^w\int_0^w e^{-\varepsilon x-\eta y} g_k(x)\left(e^{-(\lambda_2 x)}+e^{-\lambda_2 x}-e^{-(\lambda_2+\lambda_2)x}\right)r_k(y)\,dx\,dy$$

$$+\int_0^\infty\int_0^\infty e^{-\varepsilon x-\eta y}\left\{\lambda_1 e^{-(\lambda_2 x)}+\lambda_2 e^{-\lambda_2 X}-(\lambda_1+\lambda_2)\left(e^{-(\lambda_2+\lambda_2)x}\right)\right\}$$

$$\left\{\sum_{i=0}^{k-1}\int_0^x g_i(u)\int_0^{x-u} g_x(v)\overline{G_y}(x-u-v)\,dv\,du\int_0^y r_i(w)r_A(y-w)\,dw+\sum_{i=0}^{k-1}\int_0^x g_i(u)\overline{G_x}(x-u)\,du\,r_i(y)\right\}dx\,dy$$

We get

$$f_{T,R}^{\Rightarrow}(\varepsilon,\eta)=g^{\Rightarrow k}(\varepsilon+\lambda_1)r^{\Rightarrow k}(\eta)+g^{\Rightarrow k}(\varepsilon+\lambda_2)r^{\Rightarrow k}(\eta)+g^{\Rightarrow k}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow k}(\eta)+\lambda_1$$

$$\sum_{i=0}^{k-1} g^{\Rightarrow i}(\varepsilon+\lambda_1)g_X^{\Rightarrow}(\varepsilon+\lambda_1)\overline{G_y^{\Rightarrow}}(\varepsilon+\lambda_1)r^{\Rightarrow i}(\eta)r_A^{\Rightarrow}(\eta)$$

$$+\lambda_2\sum_{i=0}^{k-1} g^{\Rightarrow i}(\varepsilon+\lambda_2)g_X^{\Rightarrow}(\varepsilon+\lambda_2)\overline{G_y^{\Rightarrow}}(\varepsilon+\lambda_2)r^{\Rightarrow i}(\eta)r_A^{\Rightarrow}(\eta)$$

$$-(\lambda_1+\lambda_2)\sum_{i=0}^{k-1} g^{\Rightarrow i}(\varepsilon+\lambda_1+\lambda_2)g_X^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)\overline{G_y^{\Rightarrow}}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow i}(\eta)r_A^{\Rightarrow}(\eta)+\lambda_1$$

$$\sum_{i=0}^{k-1} g^{\Rightarrow i}(\varepsilon+\lambda_1)\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_1)r^{\Rightarrow i}(\eta)$$

$$+\lambda_2\sum_{i=0}^{k-1} g^{\Rightarrow i}(\varepsilon+\lambda_2)\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_2)r^{\Rightarrow i}(\eta)-(\lambda_1+\lambda_2)\sum_{i=0}^{k-1} g^{\Rightarrow i}(\varepsilon+\lambda_1+\lambda_2)\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow i}(\eta)$$

$$f_{T,R}^{\Rightarrow}(\varepsilon,\eta)=g^{\Rightarrow k}(\varepsilon+\lambda_1)r^{\Rightarrow k}(\eta)+g^{\Rightarrow k}(\varepsilon+\lambda_2)r^{\Rightarrow k}(\eta)+g^{\Rightarrow k}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow k}(\eta)$$

$$+\lambda_1\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_1)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_1)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_1)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_1)r_A^{\Rightarrow}(\eta)\right]+\lambda_2$$

$$\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_2)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_2)r_A^{\Rightarrow}(\eta)\right]$$

$$-(\lambda_1+\lambda_2)\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_1+\lambda_2)r_A^{\Rightarrow}(\eta)\right]$$

$$+\lambda_1\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_1)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_1)r^{\Rightarrow}(\eta)}\right]\left[\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_1)\right]+\lambda_2\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_2)\right]$$

$$-(\lambda_1+\lambda_2)\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_1+\lambda_2)\right]$$

$$f_{T,R}^{\Rightarrow}(\varepsilon,\eta)=g^{\Rightarrow k}(\varepsilon+\lambda_1)r^{\Rightarrow k}(\eta)+g^{\Rightarrow k}(\varepsilon+\lambda_2)r^{\Rightarrow k}(\eta)+$$

$$g^{\Rightarrow k}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow k}(\eta)+\lambda_1\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_1)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_1)r_A^{\Rightarrow}(\eta)+\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_1)\right]$$

$$+\lambda_2\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_2)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_2)r_A^{\Rightarrow}(\eta)+\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_2)\right]$$

$$g^{\Rightarrow k}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow k}(\eta)+\lambda_1\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_1)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_1)r_A^{\Rightarrow}(\eta)+\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_1)\right]$$

$$+\lambda_2\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_2)r^{\Rightarrow}(\eta)}\right]\left[g_X^{\Rightarrow}(\varepsilon+\lambda_2)\overline{G_Y^{\Rightarrow}}(\varepsilon+\lambda_2)r_A^{\Rightarrow}(\eta)+\overline{G_X^{\Rightarrow}}(\varepsilon+\lambda_2)\right]$$

$$-(\lambda_1+\lambda_2)\left[\frac{1-\left(g^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow}(\eta)\right)^k}{1-g^{\Rightarrow}(\varepsilon+\lambda_1+\lambda_2)r^{\Rightarrow}(\eta)}\right]$$

$$\left[g_X^{\Rightarrow}\left(\varepsilon+\lambda_1+\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\varepsilon+\lambda_1+\lambda_2\right)r_A^{\Rightarrow}\left(\eta\right)+\overline{G_X^{\Rightarrow}}\left(\varepsilon+\lambda_1+\lambda_2\right)\right] \quad (5)$$

The Laplace transform of T is

$$f_{T,R}^{\Rightarrow}\left(\varepsilon,0\right)=g^{\Rightarrow k}\left(\varepsilon+\lambda_1\right)+g^{\Rightarrow k}\left(\varepsilon+\lambda_1\right)+g^{\Rightarrow k}\left(\varepsilon+\lambda_1+\lambda_2\right)+$$

$$\lambda_1\left[\frac{1-\left\{g^{\Rightarrow}\left(\varepsilon+\lambda_2\right)\right\}^k}{1-g^{\Rightarrow}\left(\varepsilon+\lambda_2\right)}\right]\left[g_X^{\Rightarrow}\left(\varepsilon+\lambda_1\right)\overline{G_Y^{\Rightarrow}}\left(\varepsilon+\lambda_1\right)+\overline{G_X^{\Rightarrow}}\left(\varepsilon+\lambda_1\right)\right]+$$

$$\lambda_2\left[\frac{1-\left\{g^{\Rightarrow}\left(\varepsilon+\lambda_2\right)\right\}^k}{1-g^{\Rightarrow}\left(\varepsilon+\lambda_2\right)}\right]\left[g_X^{\Rightarrow}\left(\varepsilon+\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\varepsilon+\lambda_2\right)r_A^{\Rightarrow}\left(\eta\right)+\overline{G_X^{\Rightarrow}}\left(\varepsilon+\lambda_2\right)\right]-$$

$$\left(\lambda_1+\lambda_2\right)\left[\frac{1-\left\{g^{\Rightarrow}\left(\varepsilon+\lambda_1+\lambda_2\right)\right\}^k}{1-g^{\Rightarrow}\left(\varepsilon+\lambda_1+\lambda_2\right)}\right]\left[g_X^{\Rightarrow}\left(\varepsilon+\lambda_1+\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\varepsilon+\lambda_1+\lambda_2\right)+\overline{G_X^{\Rightarrow}}\left(\varepsilon+\lambda_1+\lambda_2\right)\right] \quad (6)$$

On differentiation of equation (6) at ε = 0 we get, $\frac{\partial}{\partial\varepsilon}f_{T,R}^{\Rightarrow}\left(0,0\right)=-E\left(T\right)$ and we obtain

$$E\left(T\right)=-kg^{\Rightarrow k-1}\left(\lambda_1\right)g^{\Rightarrow}\left(\lambda_1\right)-kg^{\Rightarrow k-1}\left(\lambda_2\right)g^{\Rightarrow}\left(\lambda_2\right)-kg^{\Rightarrow k-1}\left(\lambda_1+\lambda_2\right)g^{\Rightarrow}\left(\lambda_1+\lambda_2\right)+\lambda_1$$

$$\left[\frac{kg^{\Rightarrow k-1}\left(\lambda_2\right)g^{\Rightarrow}\left(\lambda_2\right)}{1-g^{\Rightarrow}\left(\lambda_2\right)}\right]\left[\left[g_X^{\Rightarrow}\left(\lambda_1\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_1\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_1\right)\right]\right]-\lambda_1\frac{1-g^{\Rightarrow k}\left(\lambda_2\right)}{\left(1-g^{\Rightarrow}\left(\lambda_2\right)\right)^2}g^{\Rightarrow}\left(\lambda_1\right)$$

$$\left[g_X^{\Rightarrow}\left(\lambda_1\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_1\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_1\right)\right]-\lambda_1\frac{1-g^{\Rightarrow k}\left(\lambda_2\right)}{\left(1-g^{\Rightarrow}\left(\lambda_2\right)\right)}$$

$$\left[g_X^{\Rightarrow}\left(\lambda_1\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_1\right)+g_X^{\Rightarrow}\left(\lambda_1\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_1\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_1\right)\right]+$$

$$\lambda_2\left[\frac{kg^{\Rightarrow k-1}\left(\lambda_2\right)g^{\Rightarrow}\left(\lambda_2\right)}{1-g^{\Rightarrow}\left(\lambda_2\right)}\right]\left[\left[g_X^{\Rightarrow}\left(\lambda_2\right)G_Y^{\Rightarrow}\left(\lambda_2\right)+G_X^{\Rightarrow}\left(\lambda_2\right)\right]\right]-$$

$$\lambda_2\frac{1-g^{\Rightarrow k}\left(\lambda_2\right)}{\left(1-g^{\Rightarrow}\left(\lambda_2\right)\right)^2}g^{\Rightarrow}\left(\lambda_2\right)\left[g_X^{\Rightarrow}\left(\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_2\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_2\right)\right]-$$

$$\lambda_2\frac{1-g^{\Rightarrow k}\left(\lambda_2\right)}{\left(1-g^{\Rightarrow}\left(\beta\right)\right)}\left[g_X^{\Rightarrow}\left(\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_2\right)+g_X^{\Rightarrow}\left(\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_2\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_2\right)\right]-\left(\lambda_1+\lambda_2\right)$$

$$\left[\frac{kg^{\Rightarrow k-1}\left(\lambda_2+\lambda_2\right)g^{\Rightarrow}\left(\lambda_2+\lambda_2\right)}{1-g^{\Rightarrow}\left(\lambda_2+\lambda_2\right)}\right]\left[\left[g_X^{\Rightarrow}\left(\lambda_1+\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_1+\lambda_2\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_1+\lambda_2\right)\right]\right]+\left(\lambda_1+\lambda_2\right)$$

$$\frac{1-g^{\Rightarrow k}(\lambda_2+\lambda_2)}{\left(1-g^{\Rightarrow}\left(\lambda_2+\lambda_2\right)\right)^2}g^{\Rightarrow}\left(\lambda_1+\lambda_2\right)\left[g_X^{\Rightarrow}\left(\lambda_1+\lambda_2\right)\overline{G_Y^{\Rightarrow}}\left(\lambda_1+\lambda_2\right)+\overline{G_X^{\Rightarrow}}\left(\lambda_1+\lambda_2\right)\right]+\left(\lambda_1+\lambda_2\right)$$

$$\frac{1-g^{\Rightarrow k}(\lambda_2+\lambda_2)}{\left(1-g^{\Rightarrow}(\lambda_2+\lambda_2)\right)}\left[\frac{g_X^{\Rightarrow}(\lambda_1+\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)+g_X^{\Rightarrow}(\lambda_1+\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)}{\overline{G_X^{\Rightarrow}}(\lambda_1+\lambda_2)}\right] \tag{7}$$

Similarly we note $\frac{\partial}{\partial\eta} f_{T,R}^{\Rightarrow}(0,0)=-E(R)$ and we obtain

$$E(R)=kg^{\Rightarrow k}(\lambda_1-)E(R_1)+kg^{\Rightarrow k}(\lambda_2)E(R_1)+kg^{\Rightarrow k}(\lambda_1+\lambda_2)$$

$$E(R_1)-k\lambda_1 E(R_1)\frac{g^{\Rightarrow k}(\lambda_2)}{1-g^{\Rightarrow}(\lambda_2)}\left[g_X^{\Rightarrow}(\lambda_1)\overline{G_Y^{\Rightarrow}}(\lambda_1)+\overline{G_X^{\Rightarrow}}(\lambda_1)\right]+$$

$$\lambda_1 E(R_1)\frac{1-g^{\Rightarrow k}(\lambda_2)}{\left(1-g^{\Rightarrow}(\lambda_2)\right)^2}g^{\Rightarrow}(\lambda_1)\left[g_X^{\Rightarrow}(\lambda_1)\overline{G_Y^{\Rightarrow}}(\lambda_1)+\overline{G_X^{\Rightarrow}}(\lambda_1)\right]+$$

$$\lambda_1\frac{1-g^{\Rightarrow k}(\lambda_2)}{1-g^{\Rightarrow}(\lambda_2)}\left[g_X^{\Rightarrow}(\lambda_1)\overline{G_Y^{\Rightarrow}}(\lambda_1)\left(E(RA)\right)\right]-$$

$$\lambda_2 kE(R_1)\frac{g^{\Rightarrow k}(\lambda_2)}{1-g^{\Rightarrow}(\lambda_2)}\left[G_X^{\Rightarrow}(\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_2)+\overline{G_X^{\Rightarrow}}(\lambda_2)\right]+$$

$$\lambda_2 E(R_1)\frac{1-g^{\Rightarrow k}(\lambda_2)}{\left(1-g^{\Rightarrow}(\lambda_2)\right)^2}g^{\Rightarrow}(\lambda_2)\left[g_X^{\Rightarrow}(\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_2)+\overline{G_X^{\Rightarrow}}(\lambda_2)\right]+$$

$$\lambda_2\frac{1-g^{\Rightarrow}(\lambda_2)}{1-g^{\Rightarrow}(\lambda_2)}\left[g_X^{\Rightarrow}(\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_2)\left(E(R_A)\right)\right]+$$

$$k(\lambda_1+\lambda_2)E(R_1)\frac{g^{\Rightarrow k}(\lambda_2+\lambda_2)}{1-g^{\Rightarrow}(\lambda_2+\lambda_2)}\left[g_X^{\Rightarrow}(\lambda_1+\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)+\overline{G_X^{\Rightarrow}}(\lambda_1+\lambda_2)\right]-$$

$$(\lambda_1+\lambda_2)E(R_1)\frac{1-g^{\Rightarrow k}(\lambda_2+\lambda_2)}{\left(1-g^{\Rightarrow}(\lambda_2+\lambda_2)\right)^2}g^{\Rightarrow}(\lambda_1+\lambda_2)$$

$$\left[g_X^{\Rightarrow}(\lambda_1+\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)+G_X^{\Rightarrow}(\lambda_1+\lambda_2)\right]-(\lambda_1+\lambda_2)$$

$$\frac{1-g^{\Rightarrow k}(\lambda_2+\lambda_2)}{1-g^{\Rightarrow}(\lambda_2+\lambda_2)}\left[g_X^{\Rightarrow}(\lambda_1+\lambda_2)\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)\left(E(R_A)\right)\right] \tag{8}$$

We now consider a special case in which X & Y are exponential random variables with parameters a and b respectively which gives

$$g_X^{\Rightarrow}(\lambda_2+\lambda_2)=\frac{a}{a+\lambda_2+\lambda_2}\; g_X^{\Rightarrow}(\lambda_2+\lambda_2)=\frac{-a}{(a+\lambda_2+\lambda_2)^z}$$

$$\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)=\frac{1}{b+\lambda_1+\lambda_2}\;\overline{G_Y^{\Rightarrow}}(\lambda_1+\lambda_2)=\frac{-1}{(b+\lambda_1+\lambda_2)^2}$$

$$\overline{G_X^{\Rightarrow}}(\lambda_1+\lambda_2)=\frac{1}{a+\lambda_1+\lambda_2}\overline{G_X^{\Rightarrow}}(\lambda_1+\lambda_2)=\frac{-1}{(a+\lambda_1+\lambda_2)^2}$$

$$g_x^{\Rightarrow}(\lambda_1)=\frac{a}{a+\lambda_1}\; g_x^{\Rightarrow}(\lambda_1)=\frac{-a}{(a+\lambda_1)^2}$$

$$\overline{G_Y^{\Rightarrow}}(\lambda_1)=\frac{1}{b+\lambda_1}\overline{G_Y^{\Rightarrow}}(\lambda_1)=\frac{-1}{(b+\lambda_1)^2}$$

$$\overline{G_X^{\Rightarrow}}(\lambda_1)=\frac{1}{a+\lambda_2}\overline{G_X^{\Rightarrow}}(\lambda_1)=\frac{-1}{(a+\lambda_2)^2} \tag{9}$$

Using (7), (8) & (9) we find E(T) & E(R), when *X* & *Y* are exponentials with parameters *a* & *b* as follows.

$$E(T)=\left[1-\left(\frac{ab}{(a+\lambda_2)(b+\lambda_2)}\right)^k\right]\left(\frac{1}{\lambda_1}\right)+$$

$$\left[1-\left(\frac{ab}{(a+\lambda_2)(b+\lambda_2)}\right)^k\left(\frac{1}{\lambda_2}\right)\right]+2k\left(\frac{ab}{(a+\lambda_2+\lambda_2)(b+\lambda_2+\lambda_2)}\right)k\left(\frac{a+b+2\lambda_1+2\lambda_2}{(a+\lambda_2+\lambda_2)(b+\lambda_2+\lambda_2)}\right)-$$

$$\left[1-\left(\frac{ab}{(a+\lambda_1+\lambda_2)(b+\lambda_1+\lambda_2)}\right)k\right]\left(\frac{1}{\lambda_1+\lambda_2}\right) \tag{10}$$

and

$$E(R)=\left[1-\left(\frac{ab}{(a+\lambda_2)(b+\lambda_1)}\right)^k\right]\left[\frac{a}{a+b+\lambda_1}\right]\left[\frac{bE(R_2)+(\lambda_2)E(R_A)}{\lambda_2}\right]+$$

$$\left[1-\left(\frac{ab}{(a+\lambda_2)(b+\lambda_2)}\right)^k\right]\left(\frac{a}{a+b+\lambda_2}\right)\left[\frac{bE(R_2)+\lambda_2E(R_A)}{\lambda_2}\right]+$$

$$2k\left(\frac{ab}{(a+\lambda_2+\lambda_2)(b+\lambda_2+\lambda_2)}\right)^k E(R1)-$$

$$\left[1-\left(\frac{a}{\beta+b+\lambda_2+\lambda_2}\right)\left[\frac{bE(R_2)+(\lambda_2+\lambda_2)E(RA)}{\lambda_2+\lambda_2}\right]\right] \tag{11}$$

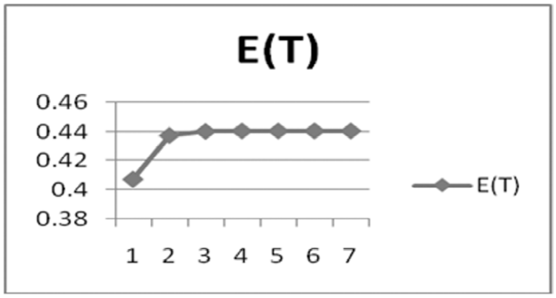
3. NUMERICAL EXAMPLE

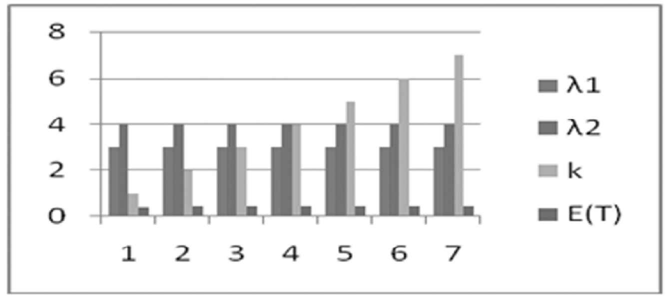
To illustrate the applications of the above result we fix $a = 1$, $b = 2$, $\lambda_1 = 3$, $\lambda_2 = 4$ and vary k we obtain $E(T)$, $E(R)$ in the following table. For $E(R_1) = 20$ and $E(R_A) = 10$

4. CONCLUSION

When k number of products increase, the inter-occurrence time to product E(T) increases and time to sales E(R) are decreases. When the exponential parameter, increases, the inter-occurrence time to product E(T) and time to sales E(R) are decreases.

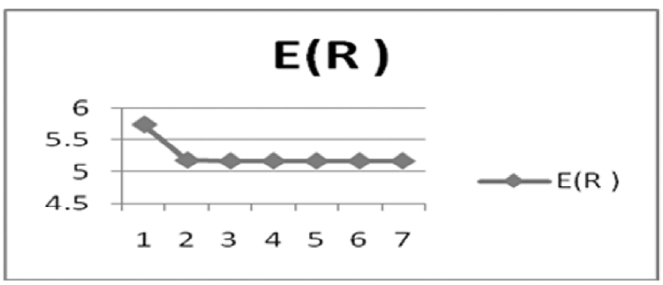
λ_1	λ_2	k	$E(T)$	$E(R)$
3	4	1	0.407561728	5.75
3	4	2	0.437426269	5.18595679
3	4	3	0.440157765	5.172473422
3	4	4	0.44044345	5.174254044
3	4	5	0.440472878	5.174563857
3	4	6	0.440475858	5.174599146

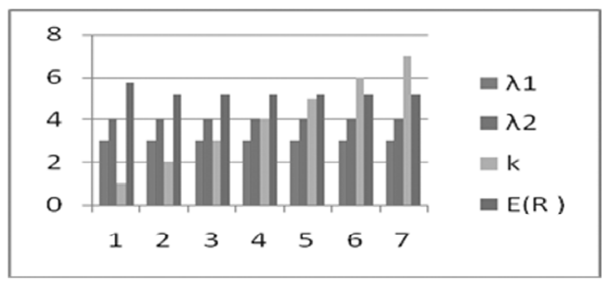




k and E(T)

Figure (1 & 2)





k and E(R)

Figure (3 & 4)

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