I J C T A, 9(1), 2016, pp. 115-132

A Novel 3-D Conservative Chaotic System with a Sinusoidal Nonlinearity and its Adaptive Control

Sundarapandian Vaidyanathan[1]

Abstract: First, this paper announces a six-term novel 3-D conservative chaotic system with a sinusoidal nonlinearity. The conservative chaotic systems are characterized by the important property that they are volume-conserving. The phase portraits of the novel conservative chaotic system are displayed and the mathematical properties are discussed. The proposed novel conservative chaotic system has infinitely many equilibrium points along the x_1 axis, which are all unstable. The Lyapunov exponents of the novel conservative chaotic system are obtained as $L_1 = 1.1444$, $L_2 = 0$ and $L_3 = -1.1444$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel conservative chaotic system is obtained as $L_1 = 1.1444$. Also, the Kaplan-Yorke dimension of the novel conservative chaotic system is derived as $D_{KY} = 3$. Thus, the novel conservative system exhibits high level of complexity and it is suitable for applications like cryptosystems, secure communications, etc. Next, an adaptive controller is designed to globally stabilize the novel conservative chaotic system with unknown parameters. Moreover, an adaptive controller is also designed to achieve global and exponential synchronization of the identical novel conservative chaotic systems with unknown parameters. The main adaptive results for stabilization and synchronization are established using Lyapunov stability theory.

Keywords: Chaos, chaotic systems, conservative systems, chaos control, chaos synchronization.

1. INTRODUCTION

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1]. Chaos theory has applications in several areas in Science and Engineering.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler found a 3-D chaotic system [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

Chaos theory has applications in several fields of science and engineering such as lasers [39], oscillators [40], chemical reactions [41-50], biology [51-66], ecology [67], artificial neural networks [68-69], robotics [70-71], fuzzy logic [72], electrical circuits [73-76], cryptosystems [77-78], memristors [79-81], etc.

In the chaos literature, there is an active interest in the discovery of conservative chaotic systems [82],

[1] Research and Development Centre, Vel Tech University, Avadi, Chennai, India, *Email: sundarvtu@gmail.com*

which have the special property that the volume of the flow is conserved. If the sum of the Lyapunov exponents of a chaotic system is zero, then the system is called a conservative chaotic system. Classical examples of conservative chaotic systems are Nosé-Hoover system [83], Hénon-Heiles system [84], etc. Vaidyanathan-Pakiriswamy system [29] is a recent example of a 3-D conservative chaotic system.

In this paper, we announce a novel 3-D conservative chaotic system with a sinusoidal nonlinearity. We discuss the qualitative properties of the novel conservative chaotic system and display the phase portraits of the novel conservative chaotic system. The proposed novel conservative chaotic system has infinitely many equilibrium points along the x_1 axis, which are all unstable. The Lyapunov exponents of the novel conservative chaotic system are obtained as $L_1 = 1.1444$, $L_2 = 0$ and $L_3 = -1.1444$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel conservative chaotic system is obtained as $L_1 = 1.1444$. Also, the Kaplan-Yorke dimension of the novel conservative chaotic system is derived as $D_{KY} = 3$. Thus, the novel conservative system exhibits high level of complexity and it is suitable for applications like cryptosystems, secure communications, etc.

Next, this paper derives an adaptive control law that stabilizes the novel conservative chaotic system with unknown system parameters. This paper also derives an adaptive control law that achieves global chaos synchronization of identical conservative chaotic systems with unknown parameters.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system, and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In the chaos literature, a variety of techniques have been proposed to solve the problem of chaos synchronization such as PC method [85], active control method [86-92], adaptive control method [93-122], backstepping control method [123-132], fuzzy control method [133], sliding mode control method [134-143], etc.

This paper is organized as follows. In Section 2, we describe the novel conservative chaotic system with a sinusoidal nonlinearity. In Section 3, we describe the qualitative properties of the novel conservative chaotic system. In Section 4, we detail the adaptive control design for the global chaos stabilization of the novel conservative chaotic system with unknown parameters. In Section 5, we detail the adaptive control design for the global and exponential synchronization of the identical novel conservative chaotic systems. In Section 6, we give a summary of the main results obtained in this research work.

2. A NOVEL 3-D CONSERVATIVE CHAOTIC SYSTEM

In this section, we describe a six-term novel conservative chaotic system with a sinusoidal nonlinearity, which is modeled by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a \sin x_1 + x_2 - b x_3 \\ \dot{x}_3 = x_2 - x_3 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b are constant, positive, parameters of the system.

The system (1) exhibits conservative chaotic behaviour for the values

$$a = 600, \quad b = 2 \tag{2}$$

For numerical simulations, we take the initial conditions of the state as

$$x_1(0) = 0, \quad x_2(0) = 1, \quad x_3(0) = 0 \tag{3}$$

The Lyapunov exponents of the conservative chaotic system (1) for the parameter values (2) and the initial conditions (3) are numerically calculated as

$$L_1 = 1.1444, \quad L_2 = 0, \quad L_3 = -1.1444 \tag{4}$$

We note that the sum of the Lyapunov exponents of the conservative system (1) is zero.

The Kaplan-Yorke dimension of the conservative chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2 + \frac{1.1444 + 0}{1.1444} = 2 + 1 = 3 \tag{5}$$

Figure 1 shows the 3-D phase portrait of the conservative chaotic system (1). Figures 2-4 show the 2-D projection of the conservative chaotic system (1) on the (x_1, x_2), (x_2, x_3),and (x_1, x_3) planes, respectively.

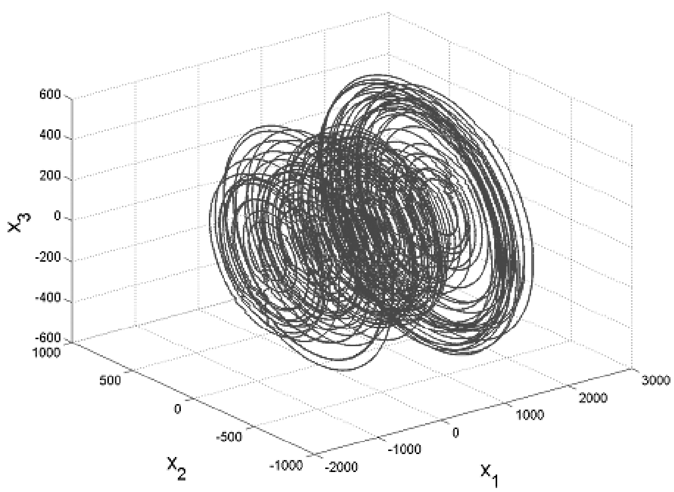

Figure 1: Phase portrait of the conservative chaotic system

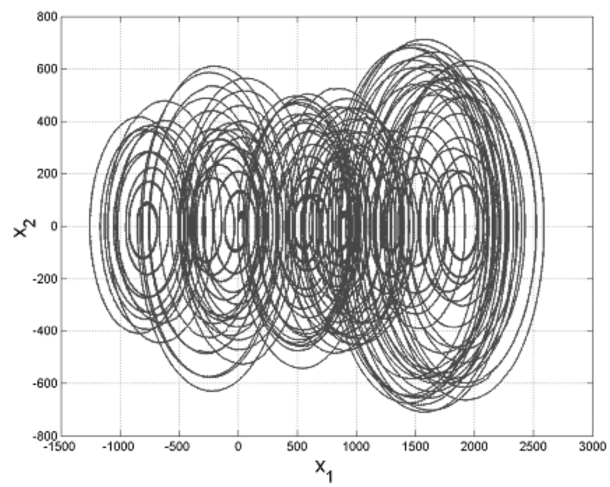

Figure 2: 2-D projection of the conservative chaotic system on the (x_1, x_2) plane

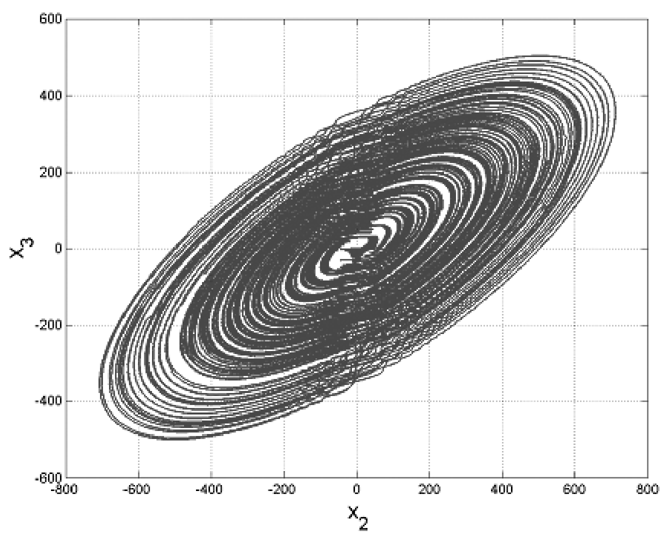

Figure 3: 2-D projection of the conservative chaotic system on the (x_2, x_3)plane

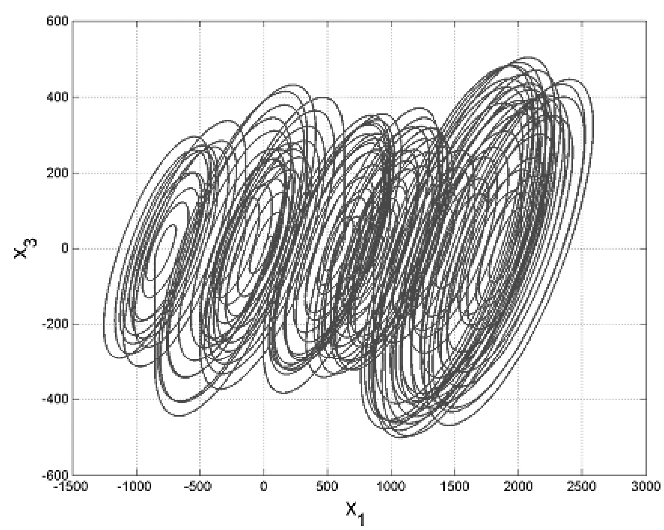

Figure 4: 2-D projection of the conservative chaotic system on the (x_1, x_3)plane

3. PROPERTIES OF THE CONSERVATIVE CHAOTIC SYSTEM

In this section, we discuss the qualitative properties of the conservative chaotic system (1) introduced in Section 2. We suppose that the parameter values of the system (1) are as in the chaotic case, i.e. $a = 600$and $b = 2$.

3.1. Volume Conservation of The Flow

In vector notation, we may express the system (1) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{6}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = x_2 \\ f_2(x_1, x_2, x_3) = a \sin x_1 + x_2 - b x_3 \\ f_3(x_1, x_2, x_3) = x_2 - x_3 \end{cases} \tag{7}$$

Let Ω be any region in R^3 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector field f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\dot{V} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3 \tag{8}$$

The divergence of the novel chaotic system (1) is easily found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = 0 + 1 - 1 = 0 \tag{9}$$

Substituting (9) into (8), we obtain the first order ODE

$$\dot{V} = 0 \tag{10}$$

Integrating (10), we obtain the unique solution as

$$V(t) = V(0) \text{ for all } t \geq 0 \tag{11}$$

This shows that the 3-D novel chaotic system (1) is volume-conserving. Hence, the system (1) is a conservative chaotic system.

3.2. Symmetry

It is easy to see that the system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) = (-x_1, -x_2, -x_3) \tag{12}$$

Thus, the system (1) exhibits *point reflection symmetry* about the origin in R^3.

3.3. Equilibrium Points

The equilibrium points of the system (1) are obtained by solving the system of equations

$$\begin{cases} x_2 = 0 \\ a\sin x_1 + x_2 - bx_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \tag{13}$$

Solving the system (13), we obtain an infinite set of equilibrium points given by

$$E_\alpha = \begin{bmatrix} \alpha\pi \\ 0 \\ 0 \end{bmatrix}, \text{ where } \alpha = 0, \ \pm 1, \ \pm 2, \ \pm 3, \ldots \tag{14}$$

The Jacobian of the system (1) at any point $x \in R^3$ is given by

$$J(x) = \begin{bmatrix} 0 & 1 & 0 \\ a\cos x_1 & 1 & -b \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 600\cos x_1 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \tag{15}$$

Case A: When α is an even integer

In this case, the Jacobian matrix of the system (1) at E_α is obtained as

$$J_\alpha = J(E_\alpha) = \begin{bmatrix} 0 & 1 & 0 \\ 600 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \tag{16}$$

The matrix J_α has the eigenvalues

$$\lambda_1 = -1.0034, \ \ \lambda_2 = -23.9574, \ \ \lambda_3 = 24.9607 \tag{17}$$

This shows that the equilibrium point E_α is a saddle point, when α is an even integer.

Case B: When α is an odd integer

In this case, the Jacobian matrix of the system (1) at E_α is obtained as

$$J_\alpha = J(E_\alpha) = \begin{bmatrix} 0 & 1 & 0 \\ -600 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \tag{18}$$

The matrix J_α has the eigenvalues

$$\lambda_1 = -0.9967, \ \ \lambda_{2,3} = 0.4983 \pm 24.5305i \tag{19}$$

This shows that the equilibrium point E_α is a saddle-focus point, when α is an odd integer.

3.4. Lyapunov Exponents And Kaplan-yorke Dimension

We take the parameter values of the novel system (1) as in the chaotic case, *i.e.* $a = 600$ and $b = 2$.

We choose the initial values of the state as $x_1(0) = 0$, $x_2(0) = 1$ and $x_3(0) = 0$.

Then we obtain the Lyapunov exponents of the system (1) as

$$L_1 = 1.1444, \quad L_2 = 0, \quad L_3 = -1.1444. \tag{20}$$

Figure 5 shows the Lyapunov exponents of the system (1) as determined by MATLAB.

We note that the sum of the Lyapunov exponents of the system (1) is zero.

Also, the Maximal Lyapunov Exponent of the system (1) is $L_1 = 1.1444$.

The Kaplan-Yorke dimension of the conservative chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2 + \frac{1.1444 + 0}{1.1444} = 2 + 1 = 3 \tag{21}$$

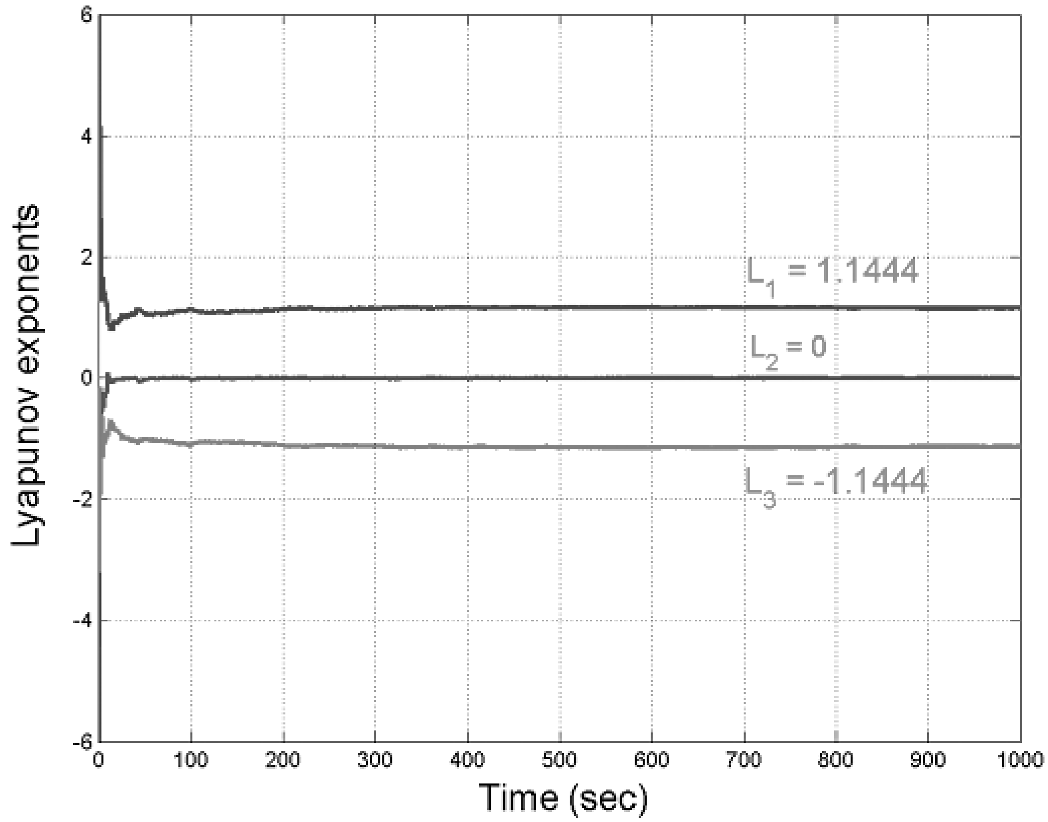

Figure 5: Lyapunov exponents of the conservative chaotic system

4. ADAPTIVE CONTROL DESIGN FOR THE STABILIZATION OF THE CONSERVATIVE CHAOTIC SYSTEM

In this section, we use adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the novel 3-D conservative chaotic system with unknown parameters.

Thus, we consider the novel 3-D conservative system given by

$$\begin{cases} \dot{x}_1 = x_2 + u_1 \\ \dot{x}_2 = a \sin x_1 + x_2 - b x_3 + u_2 \\ \dot{x}_3 = x_2 - x_3 + u_3 \end{cases} \tag{22}$$

In (22), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$ and $\hat{b}(t)$ for the unknown parameters a and b, respectively.

We consider the adaptive control law defined by

$$\begin{cases} u_1 = -x_2 - k_1 x_1 \\ u_2 = -\hat{a}(t)\sin x_1 - x_2 + \hat{b}(t)x_3 - k_2 x_2 \\ u_3 = -x_2 + x_3 - k_3 x_3 \end{cases} \tag{23}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (23) into (22), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = -k_1 x_1 \\ \dot{x}_2 = [a - \hat{a}(t)]\sin x_1 - [b - \hat{b}(t)]x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \end{cases} \tag{24}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{25}$$

Using (25), we can simplify the plant dynamics (24) as

$$\begin{cases} \dot{x}_1 = -k_1 x_1 \\ \dot{x}_2 = e_a \sin x_1 - e_b x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \end{cases} \tag{26}$$

Differentiating (25) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{27}$$

We use adaptive control theory to find an update law for the parameter estimates.

We consider the quadratic candidate Lyapunov function defined by

$$V(\mathbf{x}, e_a, e_b) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2\right) \tag{28}$$

Clearly, V is a positive definite function on R^5.

Differentiating V along the trajectories of (26) and (27), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_2 \sin x_1 - \dot{\hat{a}}\right] + e_b\left[-x_2 x_3 - \dot{\hat{b}}\right] \tag{29}$$

In view of (29), we take the parameter update law as follows:

$$\begin{cases} \dot{\hat{a}} = x_2 \sin x_1 \\ \dot{\hat{b}} = -x_2 x_3 \end{cases} \tag{30}$$

Theorem 1. The novel 3-D conservative chaotic system (22) with unknown system parameters is globally and exponentially stabilized for all initial conditions $x(0)\in R^3$ by the adaptive control law (23) and the parameter update law (30), where k_1, k_2, k_3 are positive gain constants.

Proof. We prove this result by using Lyapunov stability theory [144]. We consider the quadratic Lyapunov function defined by (28), which is positive definite on R^5.

By substituting the parameter update law (30) into (29), we obtain the time derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{31}$$

From (31), it is clear $\dot{V}$ that is a negative semi-definite function on R^5.

Thus, we can conclude that the state vector $x(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\left[x_1(t) \quad x_2(t) \quad x_3(t) \quad e_a(t) \quad e_b(t) \right]^T \in L_\infty$$

We define $k = \min\{k_1, k_2, k_3\}$. Thus, it follows from (31) that

$$\dot{V} \le -k \left\| \boldsymbol{x}(t) \right\|^2 \tag{32}$$

Thus, we have

$$k \left\| \boldsymbol{x}(t) \right\|^2 \le -\dot{V} \tag{33}$$

Integrating the inequality (33) from 0 to t, we get

$$k \int_0^t \left\| \boldsymbol{x}(\tau) \right\|^2 d\tau \le V(0) - V(t) \tag{34}$$

From (34), it follows that $x\in L_2$. Using (26), we can conclude that $\dot{x} \in L_\infty$.

Using Barbalat's lemma [144], we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0)\in R^3$. This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (22) and (30), when the adaptive control law (23) is applied.

The parameter values of the novel conservative chaotic system are taken as in the chaotic case, i.e. $a = 600$ and $b = 2$. We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

Furthermore, as initial conditions of the novel conservative chaotic system (22), we take

$$x_1(0) = 7.4,\ x_2(0) = -10.9,\ x_3(0) = 12.3 \tag{35}$$

Also, as initial conditions of the parameter estimates $\hat{a}(t)$ and $\hat{b}(t)$ we take

$$\hat{a}(0) = 16.3, \quad \hat{b}(0) = 9.4 \tag{36}$$

Figure 6 shows the exponential convergence of the controlled state trajectories of the 3-D novel conservative chaotic system (22).

5. ADAPTIVE CONTROL DESIGN FOR THE SYNCHRONIZATION OF THE IDENTICAL CONSERVATIVE CHAOTIC SYSTEMS

In this section, we use adaptive control method to derive an adaptive feedback control law for globally synchronizing identical 3-D novel conservative chaotic systems with unknown parameters.

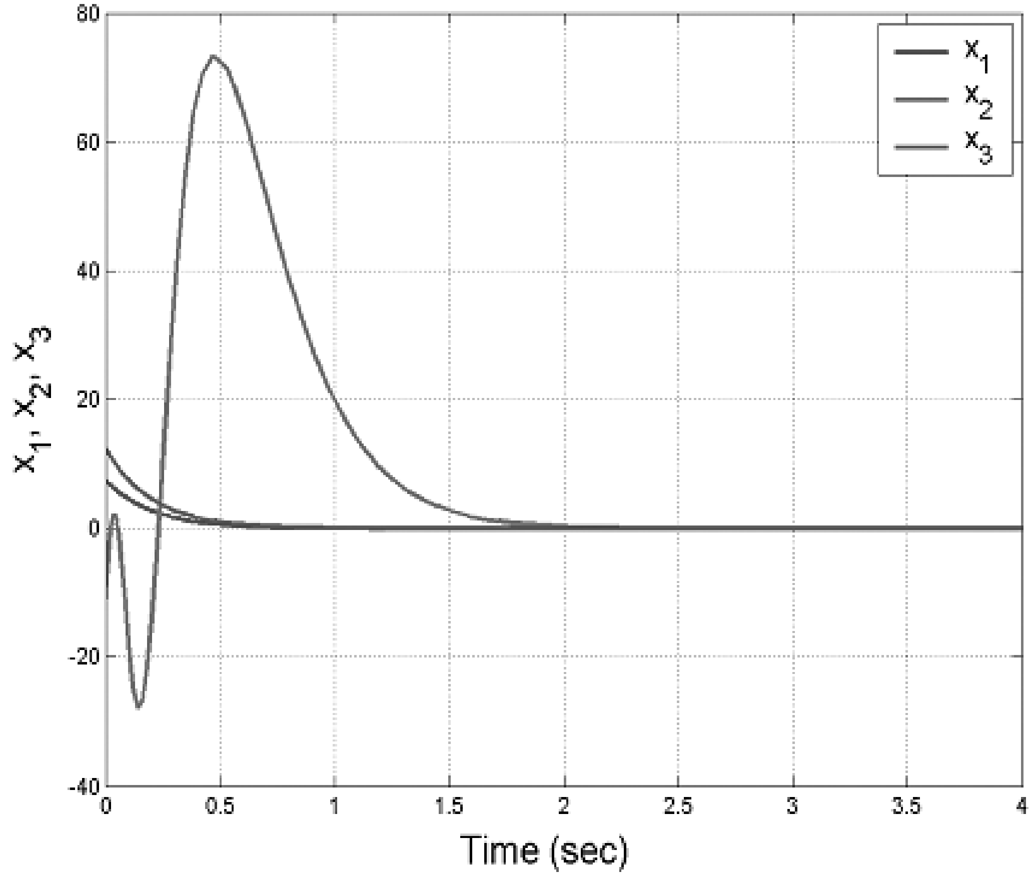

Figure 6: Time-history of the controlled state trajectories of the conservative chaotic system

As the master system, we consider the 3-D novel conservative chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a \sin x_1 + x_2 - bx_3 \\ \dot{x}_3 = x_2 - x_3 \end{cases} \tag{37}$$

where x_1, x_2, x_3 are the states and a, b are unknown system parameters.

As the slave system, we consider the controlled 3-D novel conservative chaotic system given by

$$\begin{cases} \dot{y}_1 = y_2 + u_1 \\ \dot{y}_2 = a \sin y_1 + y_2 - by_3 + u_2 \\ \dot{y}_3 = y_2 - y_3 + u_3 \end{cases} \tag{38}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$ and $\hat{b}(t)$ for the unknown system parameters a and b, respectively.

The synchronization error between the novel conservative chaotic systems (37) and (38) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{39}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_2 + u_1 \\ \dot{e}_2 = a(\sin y_1 - \sin x_1) + e_2 - be_3 + u_2 \\ \dot{e}_3 = e_2 - e_3 + u_3 \end{cases} \tag{40}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 = -e_2 - k_1 e_1 \\ u_2 = -\hat{a}(t)(\sin y_1 - \sin x_1) - e_2 + \hat{b}(t)e_3 - k_2 e_2 \\ u_3 = -e_2 + e_3 - k_3 e_3 \end{cases} \tag{41}$$

where k_1, k_2, k_3 are positive constants and $\hat{a}(t)$, $\hat{b}(t)$ are estimates of the unknown parameters a, b respectively.

Substituting (41) into (40), we can simplify the error dynamics (40) as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = [a - \hat{a}(t)](\sin y_1 - \sin x_1) - [b - \hat{b}(t)]e_3 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{42}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \end{cases} \tag{43}$$

Substituting (43) into (42), the error dynamics is simplified as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = e_a(\sin y_1 - \sin x_1) - e_b e_3 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{44}$$

Differentiating (43) with respect to t, we obtain

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \end{cases} \tag{45}$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right) \tag{46}$$

Differentiating V along the trajectories of (44) and (45), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_2(\sin y_1 - \sin x_1) - \dot{\hat{a}}\right] + e_b\left[-e_2 e_3 - \dot{\hat{b}}\right] \tag{47}$$

In view of (47), we take the parameter update law

$$\begin{cases} \dot{\hat{a}} = e_2\left(\sin y_1 - \sin x_1\right) \\ \dot{\hat{b}} = -e_2 e_3 \end{cases} \tag{48}$$

Next, we state and prove the main result of this section.

Theorem 2. The novel 3-D conservative chaotic systems (37) and (38) with unknown system parameters are globally and exponentially synchronized for all initial conditions $x(0)$, $y(0) \in R^3$ by the adaptive control law (41) and the parameter update law (48), where k_1, k_2, k_3 are positive constants.

Proof. We prove this result by applying Lyapunov stability theory [144].

We consider the quadratic Lyapunov function defined by (46), which is positive definite in R^5.

By substituting the parameter update law (48) into (47), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{49}$$

From (49), it is clear that $\dot{V}$ is a negative semi-definite function on R^5.

Thus, we can conclude that the synchronization error vector $e(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\begin{bmatrix} e_1(t) & e_2(t) & e_3(t) & e_a(t) & e_b(t) \end{bmatrix}^T \in L_\infty \tag{50}$$

We define $k = \min\{k_1, k_2, k_3\}$. Then it follows from (49) that

$$\dot{V} \le -k\left\|\boldsymbol{e}(t)\right\|^2 \tag{51}$$

Thus, we have

$$k\left\|\boldsymbol{e}(t)\right\|^2 \le -\dot{V} \tag{52}$$

Integrating the inequality (52) from 0 to t, we get

$$\int_0^t k\left\|\boldsymbol{e}(\tau)\right\|^2 d\tau \le V(0) - V(t) \tag{53}$$

From (53), it follows that $e \in L_2$. Using (44), we can conclude that $\dot{e} \in L_\infty$.

Using Barbalat's lemma [144], we conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$. This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ is used to solve the systems (37), (38) and (48), when the adaptive control law (41) is applied.

We take the parameter values of the conservative systems (37) and (38) as in the chaotic case, *i.e.* $a = 600$ and $b = 2$. We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

As initial conditions of the master system (37), we take

$$x_1(0) = 6.2, \quad x_2(0) = -3.9, \quad x_3(0) = 8.5 \tag{54}$$

As initial conditions of the slave system (38), we take

$$y_1(0) = 4.7, \quad y_2(0) = 7.3, \quad y_3(0) = 2.1 \tag{55}$$

As initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 9.1, \quad \hat{b}(0) = 3.8 \tag{56}$$

Figures 7-9 depict the synchronization of the novel conservative chaotic systems (37) and (38).

Figure 10 depicts the time-history of the complete synchronization errors e_1, e_2, e_3

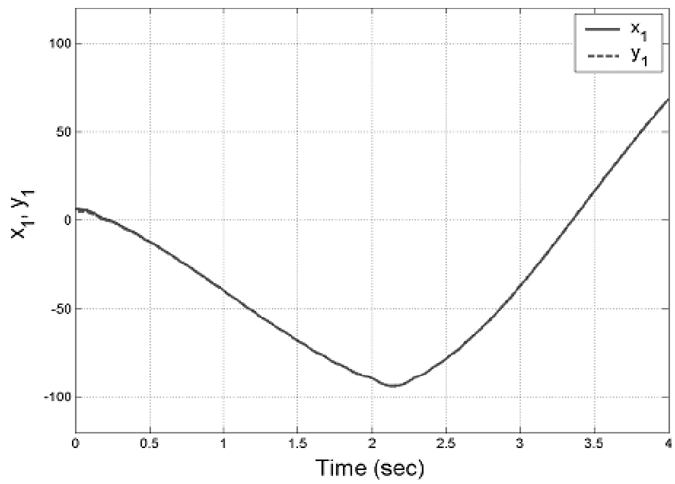

Figure 7: Synchronization of the states x_1 and y_1

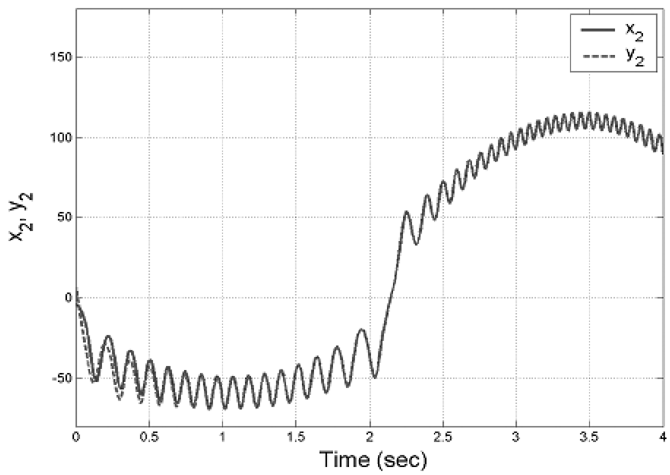

Figure 8: Synchronization of the states x_2 and y_2

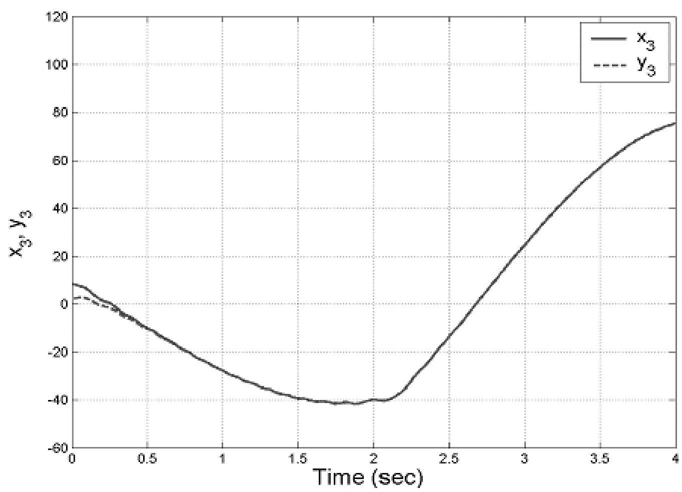

Figure 9. Synchronization of the states x_3 and y_3

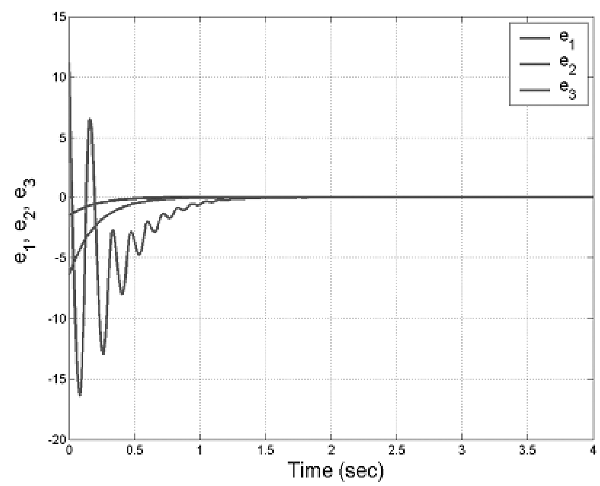

Figure 10. Time-history of the synchronization errors e_1, e_2, e_3

6. CONCLUSIONS

In this paper, we derived new results for a six-term novel 3-D conservative chaotic system with a sinusoidal nonlinearity. We described the dynamics and qualitative properties of the novel conservative chaotic system. We showed that the novel 3-D conservative chaotic system has infinitely many equilibrium points along the axis, which are all unstable. The Lyapunov exponents of the novel conservative chaotic system have been obtained as $L_1 = 1.1444$, $L_2 = 0$ and $L_3 = -1.1444$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel conservative chaotic system is seen as $L_1 = 1.1444$. Also, the Kaplan-Yorke dimension of the novel conservative chaotic system has been derived as $D_{KY} = 3$. Thus, the novel conservative system exhibits high level of complexity and it is suitable for applications like cryptosystems, secure communications, etc. Next, we designed an adaptive controller to globally stabilize the novel conservative chaotic system with unknown parameters. We also designed an adaptive controller to achieve global and exponential synchronization of the identical novel conservative chaotic systems with unknown parameters. The main adaptive results for stabilization and synchronization were established using Lyapunov stability theory. We illustrated all main results with MATLAB simulations.

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