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Unsteady Motion with Heat and Mass Transfer for a Non-Newtonian Viscoelastic Fluid Past an Impulsively Infinite Vertical Porous Surface in the Presence of Heat Generation and Chemical Reaction

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ABSTRACT

An analytical solutions of the momentum, heat and concentration equations are obtained. These equations governing the unsteady motion of an incompressible viscoelastic fluid past an infinite porous vertical plate with heat source and chemical reaction. The Lightill methods as well as perturbation technique are used to solve the governing equations and the solutions are obtained as a functions of the physical parameters of the problem. The effects of the porosity parameter *V* of the porous plate, elasticity parameter of the fluid *K*, Grashof numbers for heat and mass *Gr* and *Gc* respectively, heat source parameter *L*, Prandtl number *Pr*, chemical reaction parameter *S* and Schmidt number *Sc*, on these solutions. as well as the skin friction τ, Nusselt number and Sherwood number are discussed numerically and illustrated graphically through a set of figures.

1. INTRODUCTION

The phenomenon of non-Newtonian fluid flow with heat and mass transfer involves interesting challenges in the chemical industry separation processes, in transpiration cooling, and in pollutant dispersion along aquifers. The boundary layer concept for such fluids is of special importance because of its applications to many practical problem, among which we cite the possibility of reducing frictional drag on the hulls of ships and submarines, and also, in polymer production. Most of the previous researchers of the fluid flow on vertical plate [1-10] are based on the assumption that the fluid is Newtonian.

Eldabe *et al*., [11-14] studied some problems of non-Newtonian fluids which flowing over an infinite solid surfaces with different external forces. For these fluids there are several constitutive equations, which do not obey the Newtonian law.

The purpose of the present paper is to analyze the effects of the elasticity of the fluid with heat sink and chemical reactions on the unsteady flow of non-Newtonian fluid which obeying the Rivlin Erickcen model with heat and mass transfer past an infinite porous vertical plate. The governing partial differential equations which governing this motion have been solved analytically by using perturbation technique. The solutions of these equations are obtained as a functions of the physical parameters of the problem

2. FIELD EQUATIONS

For non-Newtonian fluid the constitutive equations for the stress tensor τ_{ij} are assumed in the forms [15]

$$\tau_{ij} = 2\mu d_{ij} - 2k_0 e_{ij} + 4\psi d_i^{\alpha} d_{\alpha j}$$

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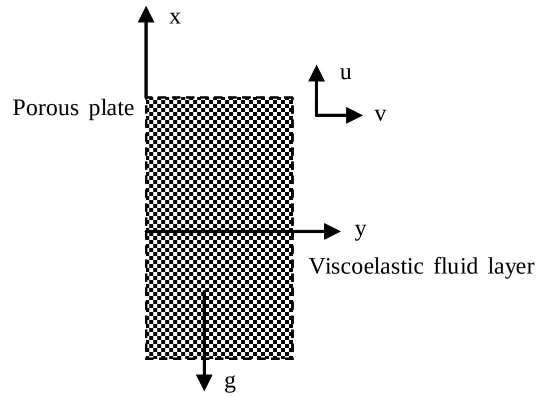

Figure 1: Sketch of the Problem

where

$$d_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$$

$$e_{ij} = \frac{1}{2}(a_{i,j} + a_{j,i} + 2v^{m}_{,i} v_{m,j}),$$

v_i is the velocity vector, and the acceleration vector is given by

$$a_i = \frac{\partial v_i}{\partial t} + v^m v_{i,m} .$$

A comma followed by an index implies covariant differential.

The material constants μ, k_0 and ψ are, respectively, the viscosity, elastic-viscosity, and the cross-viscosity coefficients of the fluid.

The basic equations of the unsteady motion of the incompressible viscoelastic fluid with heat generation and chemical reaction can be written as follows:

The continuity equation

$$\nabla . \underline{V} = 0$$

The momentum equation

The heat equation with heat generation

$$\frac{\partial \underline{V}}{\partial t} + (\underline{V} . \nabla)\underline{V} = -\frac{1}{\rho}\nabla P + \frac{1}{\rho}\nabla . \tau + \underline{F} .$$

The concentration equation with chemical reaction

$$\rho c_p \left(\frac{\partial T}{\partial t} + (\underline{V} . \nabla) T \right) = k\nabla^2 T - \ell (T - T_\infty) .$$

3. FORMULATION OF THE PROBLEM

The unsteady motion of a viscoelastic incompressible fluid past an infinite porous vertical plate is investigated. The Cartesian coordinates (x, y) is considered, where x-axis is taken along the plate in the vertically upward direction and y-axis is normal to the plate (sec Fig. 1). The velocity components of the fluid are u and v in the x and y directions respectively. The temperature and concentration fields of the fluid are T and C, respectively. The influence of the density variations with temperature and concentration are considered only in the body force term. Also, in the heat equation, term represents viscous dissipation is neglected as it assumed to be very small in free convection flows. Since the plate is an infinite extent, hence all physical variables are functions of y and t only. Under these assumptions the equations govern this motion are:

The continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

The momentum equation:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{k_0}{\rho}\left(\frac{\partial^3 u}{\partial y^2\partial t}+v\frac{\partial^3 u}{\partial y^3}\right)+g\beta(T-T_\infty)+g\beta'(C-C_\infty). \tag{2}$$

The heat equation with heat generation:

$$\rho c_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\ell(T-T_\infty). \tag{3}$$

The concentration equation with chemical reaction:

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\lambda(C-C_\infty). \tag{4}$$

Where ν is the kinematic viscosity of the fluid, ρ is the density of the fluid, k_0 is the elastic viscosity of the fluid, g is the gravity acceleration, β is the coefficient of thermal expansion, β' is the coefficient of expansion with concentration, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, ℓ is the coefficient of heat generation, D is the coefficient of mass diffusivity and λ is the coefficient of chemical reaction.

The second term in the right hand side of the momentum equation is due to the viscoelasticity of the non Newtonian fluid, also the second term in the right hand sides of both the equations (3) and (4) is due to the heat source and chemical reaction effects respectively.

The appropriate boundary conditions are:

$$u=U_c e^{iwt},\quad \frac{\partial T}{\partial y}=-\frac{U_c}{\nu}(T+T_\infty e^{iwt}),\quad \frac{\partial C}{\partial y}=-\frac{U_c}{\nu}(C+C_\infty e^{iwt})\quad \text{at}\quad y=0$$

$$u\to 0,\quad T\to T_\infty,\quad C\to C_\infty,\quad \text{as}\quad y\to\infty. \tag{5}$$

It is clear from the continuity equation (1) that $\frac{\partial v}{\partial y}=0$, which gives ν is a constant or a function of t, we shall consider it equals to a constant $-v_0$ which represents the suction at the plate due to its porosity.

Now, we shall consider the following non-dimensional quantities:

$$\left.\begin{aligned}
&u^* = \frac{u}{U_c}, \quad t^* = t\frac{U_c^2}{\nu}, \quad y^* = \frac{yUc}{\nu}, \quad \omega^* = \frac{\omega\nu}{U_c^2} \\
&\theta = \frac{T - T_\infty}{T_\infty}, \quad \phi = \frac{C - C_\infty}{C_\infty}, \quad G_r = \frac{\nu g\beta T_\infty}{U_c^3}, \\
&G_c = \frac{\nu g\beta' C_\infty}{U_c^3}, \quad V = \frac{v_0}{U_c}, \quad K = \frac{U_c^2}{\rho\nu^2}k_0, \\
&p_r = \frac{\mu c_p}{k}, \quad L = \frac{\nu}{U_c^2\rho c_p}\ell, \quad S_c = \frac{\nu}{D}, \quad S = \frac{\lambda\nu}{U_c^2}
\end{aligned}\right\} \tag{6}$$

Equations (3) and (4) with boundary conditions (5) after substituting from equations (6) may be written in dimensionless form after dropping the star mark:

$$\frac{\partial u}{\partial t} - V\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - K\left(\frac{\partial^3 u}{\partial y^2 \partial t} - V\frac{\partial^3 u}{\partial t^3}\right) + G_r\theta + G_c\phi\,, \tag{7}$$

$$\frac{\partial\theta}{\partial t} - V\frac{\partial\theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} - L\theta\,, \tag{8}$$

$$\frac{\partial\phi}{\partial t} - V\frac{\partial\phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2} - S\phi\,. \tag{9}$$

With boundary conditions:

$$\left.\begin{aligned}
&u = e^{i\omega t}, \quad \frac{\partial\theta}{\partial y} = -(1 + \theta + e^{i\omega t}), \quad \frac{\partial\phi}{\partial y} = -(1 + \phi + e^{i\omega t}) \quad \text{at} \quad y = 0 \\
&u \to 0\;, \quad \theta \to 0\;, \quad \phi \to 0 \qquad \text{as} \quad y \to \infty
\end{aligned}\right\} \tag{10}$$

4. METHOD OF SOLUTIONS

To solve the system of partial differential equations (7-9) subjected to the boundary conditions (10), we shall use the following technique according to Lightill method [17]

$$\left.\begin{aligned}
u &= u_0 + u_1 e^{i\omega t} \\
\theta &= \theta_0 + \theta_1 e^{i\omega t} \\
\phi &= \phi_0 + \phi_1 e^{i\omega t}
\end{aligned}\right\} \tag{11}$$

Substituting from (11) into (7-9) and equating the like terms we have the following system of ordinary differential equations subjected to the boundary conditions

$$KVu_0^{\backslash\backslash\backslash} + u_0^{\backslash\backslash} + Vu_0^{\backslash} + G_r\theta_0 + G_c\phi_0 = 0\,, \tag{12}$$

$$KVu_1^{\backslash\backslash\backslash} + (1 - Ki\omega)u_1^{\backslash\backslash} + Vu_1^{\backslash} - i\omega u_1 + G_r\theta_1 + G_c\phi_1 = 0\,, \tag{13}$$

$$\theta_0'' + P_r V\theta_0' - LP_r\,\theta_0 = 0\,, \tag{14}$$

$$\theta_1'' + P_r V\theta_1' - (L + i\omega)P_r\,\theta_1 = 0\,, \tag{15}$$

$$\phi_0'' + S_c V\phi_0' - SS_c\phi_0 = 0\,, \tag{16}$$

$$\phi_1'' + S_c V\phi_1' - (S + i\omega)\,S_c\phi_1 = 0\,, \tag{17}$$

$$\left.\begin{array}{l} u_0 = 0, u_1 = 1, \quad \theta_0' = -(1+\theta_0), \quad \theta_1' = -(1+\theta_1), \\ \phi_0' = -(1+\phi_0), \quad \phi_1' = -(1+\phi_1) \quad \text{at} \quad y = 0 \\ u_0 \to 0, \quad u_1 \to 0 \quad , \quad \theta_0 \to 0, \quad \theta_1 \to 0 \ , \\ \phi_0 \to 0, \quad \phi_1 \to 0 \quad \text{as} \quad y \to \infty, \end{array}\right\} \tag{18}$$

Where the mark dash denotes to the total differentiation with respect to y.

Since the momentum equation with heat and concentration equations are coupled, then we shall firstly solve the equations of heat and concentration equations (14-17) to obtain θ_0, θ_1, ϕ_0 and ϕ_1 subjected to the corresponding boundary conditions (18); then we get:

$$\theta_0 = -\frac{1}{1+\lambda_1}e^{\lambda_1 y}\,, \tag{19}$$

$$\theta_1 = \frac{-1}{1+\lambda_2}e^{\lambda_2 y}\,, \tag{20}$$

$$\phi_0 = -\frac{1}{1+\lambda_3}e^{\lambda_3 y}\,, \tag{21}$$

$$\phi_1 = \frac{-1}{1+\lambda_4}e^{\lambda_4 y}\,. \tag{22}$$

Now, substituting from Eqs. (19-22) in Eqs. (12) and (13) we get:

$$KVu_0''' + u_0'' + Vu_0' = \frac{G_r}{1+\lambda_1}e^{\lambda_1 y} + \frac{G_c}{1+\lambda_3}e^{\lambda_3 y}\,, \tag{23}$$

and

$$KVu_1''' + (1 - Ki\omega)u_1'' + Vu_1' - i\omega u_1 = \frac{+G_r}{1+\lambda_2}e^{\lambda_2 y} + \frac{G_c}{1+\lambda_4}e^{\lambda_4 y}\,. \tag{24}$$

To solve equations (23) and (24) subjected to the boundary condition (18) in the case of small elasticity parameter of the fluid K, we shall assume that

$$u_0 = u_{00} + Ku_{01}, \qquad u_1 = u_{10} + Ku_{11} \tag{25}$$

Equations (23) and (24) and the boundary conditions (18) after substituting from (25) and equating the likes terms give the following system of equations.

$$u_{00}'' + Vu_{00}' = \frac{G_r}{1+\lambda_1}e^{\lambda_1 y} + \frac{G_c}{1+\lambda_3}e^{\lambda_3 y}\,, \tag{26}$$

$$u_{01}'' + Vu_{01}' = -Vu_{00}''', \tag{27}$$

$$u_{10}'' + Vu_{10}' - i\omega u_{10} = \frac{G_r}{1+\lambda_2} e^{\lambda_2 y} + \frac{G_c}{1+\lambda_4} e^{\lambda_4 y}, \tag{28}$$

$$u_{11}'' + Vu_{11}' - i\omega u_{11} = -Vu_{10}''' + i\omega u_{10}''. \tag{29}$$

With the conditions:

$$\left.\begin{array}{lllll} u_{00} = 0, & u_{01} = 0, & u_{10} = 1, & u_{11} = 0, & \text{at} \quad y = 0 \\ u_{00} \to 0 & u_{01} \to 0 & u_{10} \to 0 & u_{11} \to 0, & y \to \infty \end{array}\right\} \tag{30}$$

The solutions of these equations are:

$$u_{00} = -\lambda_7 e^{-Vy} + \lambda_5 e^{\lambda_1 y} + \lambda_6 e^{\lambda_3 y}, \tag{31}$$

$$u_{01} = -(\lambda_{10} + V^3 \lambda_7 y) e^{-Vy} - \lambda_8 e^{\lambda_1 y} - \lambda_9 e^{\lambda_3 y}, \tag{32}$$

$$u_{10} = \lambda_{14} e^{\lambda_{11} y} + \lambda_{12} e^{\lambda_2 y} + \lambda_{13} e^{\lambda_4 y}, \tag{33}$$

$$u_{11} = (-\lambda_{22} + y\lambda_{19}) e^{\lambda_{11} y} + \lambda_{20} e^{\lambda_2 y} + \lambda_{21} e^{\lambda_4 y}. \tag{34}$$

We can write the complete formula of the velocity, temperature and the concentration as follows:

$$\begin{aligned} u(y,t) = &\{(\lambda_{23} + y\lambda_{24}) e^{-Vy} + \lambda_{25} e^{\lambda_1 y} + \lambda_{26} e^{\lambda_3 y}\} \\ &+ e^{i\omega t}\{(\lambda_{27} + y\lambda_{28}) e^{\lambda_{11} y} + \lambda_{29} e^{\lambda_2 y} + \lambda_{30} e^{\lambda_4 y}\}, \end{aligned} \tag{35}$$

$$\theta(y,t) = -\lambda_{31} e^{\lambda_1 y} - \lambda_{32} e^{\lambda_2 y} e^{i\omega t}, \tag{36}$$

and

$$\phi(y,t) = -\lambda_{33} e^{\lambda_3 y} - \lambda_{34} e^{\lambda_4 y} e^{i\omega t}. \tag{37}$$

The skin-friction can be written as:

$$\tau^* = \left\{\mu \frac{\partial u}{\partial y} - k_0 \left(\frac{\partial^2 u}{\partial t \partial y} - V_0 \frac{\partial^2 u}{\partial y^2}\right)\right\}_{y=0}. \tag{38}$$

And in virtue of equation (6), equation (38) can be written in dimensionless form as:

$$\tau = \left\{\frac{\partial u}{\partial y} - K \left(\frac{\partial^2 u}{\partial y \partial t} - V \frac{\partial^2 u}{\partial y^2}\right)\right\}_{y=0}. \tag{39}$$

By using equation (35), the skin-friction can be written in the following form

$$\tau = \lambda_{41} + \lambda_{48} e^{i\omega t}. \tag{40}$$

The rate of heat transfer is given by

$$N_u^* = -\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{41}$$

and in virtue of equation (6), equation (41) takes the following non-dimensional form of the Nusselt number.

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}. \tag{42}$$

By using the expression of $\theta(y, t)$ from equation (36), the Nusselt number can be written as:

$$N_u = \lambda_{49} + \lambda_{50} e^{i\omega t}. \tag{43}$$

Also, the rate of mass transfer S_h^* is given as

$$S_h^* = -\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{44}$$

Which can be written in dimensionless form after using the equation (6) , hence the Sherwood number may be written as follows :

$$S_h = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}. \tag{45}$$

And after using the expression of $\phi(y, t)$ from equation (37), the Sherwood number takes the following form:

$$S_h = \lambda_{51} + \lambda_{52} e^{i\omega t}. \tag{46}$$

The constants $\lambda_1 \to \lambda_{52}$ appearing in the text are appeared before the reference section of the text through appendix.

5. APPENDIX

$$\lambda_1 = -\left(\frac{P_r V + \sqrt{P_r^2 V^2 + 4LP_r}}{2}\right), \qquad \lambda_2 = -\left(\frac{P_r V + \sqrt{P_r^2 V^2 + 4P_r(L + i\omega)}}{2}\right),$$

$$\lambda_3 = -\left(\frac{S_c V + \sqrt{S_c^2 V^2 + 4SS_c}}{2}\right), \qquad \lambda_4 = -\left(\frac{S_c V + \sqrt{S_c^2 V^2 + 4S_c(S + i\omega)}}{2}\right),$$

$$\lambda_5 = \frac{G_r}{\lambda_1(\lambda_1 + 1)(\lambda_1 + V)}, \qquad \lambda_6 = \frac{G_c}{\lambda_3(\lambda_3 + 1)(\lambda_3 + V)},$$

$$\lambda_7 = \lambda_5 + \lambda_6, \qquad \lambda_8 = \frac{V\lambda_5\lambda_1^2}{\lambda_1 + V}, \qquad \lambda_9 = \frac{V\lambda_6\lambda_3^2}{\lambda_3 + V},$$

$$\lambda_{10} = \lambda_8 + \lambda_9, \qquad \lambda_{11} = -\left(\frac{V + \sqrt{V^2 + 4i\omega}}{2}\right), \qquad \lambda_{12} = \frac{G_r}{(1+\lambda_2)(\lambda_2^2 + V\lambda_2 - i\omega)},$$

$$\lambda_{13} = \frac{G_c}{(1+\lambda_4)(\lambda_4^2 + V\lambda_4 - i\omega)}, \qquad \lambda_{14} = 1 - \lambda_{12} - \lambda_{13}, \qquad \lambda_{15} = \lambda_{11}^2\lambda_{14}(i\omega - V\lambda_{11}),$$

$$\lambda_{16} = \lambda_2^2\lambda_{12}(i\omega - V\lambda_2), \qquad \lambda_{17} = \lambda_4^2\lambda_{13}(i\omega - V\lambda_4), \qquad \lambda_{18} = \left(\frac{-V + \sqrt{V^2 + 4i\omega}}{2}\right),$$

$$\lambda_{19} = \frac{\lambda_{15}}{(\lambda_{11} - \lambda_{18})}, \qquad \lambda_{20} = \frac{\lambda_{16}}{\lambda_2^2 + V\lambda_2 - i\omega}, \qquad \lambda_{21} = \frac{\lambda_{17}}{\lambda_4^2 + V\lambda_4 - i\omega},$$

$$\lambda_{22} = \lambda_{20} + \lambda_{21}, \qquad \lambda_{23} = -\lambda_7 + K\lambda_{10}, \qquad \lambda_{24} = KV^3\lambda_7,$$

$$\lambda_{25} = \lambda_5 - \lambda_8 K, \qquad \lambda_{26} = \lambda_6 - \lambda_9 K, \qquad \lambda_{27} = \lambda_{14} - \lambda_{22} K,$$

$$\lambda_{28} = \lambda_{19} K, \qquad \lambda_{29} = \lambda_{12} + K\lambda_{20}, \qquad \lambda_{30} = \lambda_{13} + K\lambda_{21},$$

$$\lambda_{31} = \frac{1}{1+\lambda_1}, \qquad \lambda_{32} = \frac{1}{1+\lambda_2}, \qquad \lambda_{33} = \frac{1}{1+\lambda_3},$$

$$\lambda_{34} = \frac{1}{1+\lambda_4}, \qquad \lambda_{35} = V\lambda_{23}(KV^2 - 1), \qquad \lambda_{36} = \lambda_{24}(1 - 2KV^2),$$

$$\lambda_{37} = \lambda_1\lambda_{25}(1 + KV\lambda_1), \qquad \lambda_{38} = \lambda_3\lambda_{26}(1 + KV\lambda_3), \qquad \lambda_{39} = \lambda_{35} + \lambda_{36},$$

$$\lambda_{40} = \lambda_{37} + \lambda_{38}, \qquad \lambda_{41} = \lambda_{39} + \lambda_{40}, \qquad \lambda_{42} = \lambda_{11}\lambda_{27}(1 - Ki\omega + KV\lambda_{11}),$$

$$\lambda_{43} = \lambda_{28}(1 - Ki\omega + 2KV\lambda_{11}), \qquad \lambda_{44} = \lambda_2\lambda_{29}(1 - Ki\omega + KV\lambda_2), \qquad \lambda_{45} = \lambda_4\lambda_{30}(1 - Ki\omega + KV\lambda_4),$$

$$\lambda_{46} = \lambda_{42} + \lambda_{43}, \qquad \lambda_{47} = \lambda_{44} + \lambda_{45}, \qquad \lambda_{48} = \lambda_{46} + \lambda_{47},$$

$$\lambda_{49} = \lambda_1\lambda_{31}, \qquad \lambda_{50} = \lambda_2\lambda_{32} \qquad \lambda_{51} = \lambda_3\lambda_{33},$$

$$\lambda_{52} = \lambda_4\lambda_{34}.$$

6. RESULTS AND DISCUSSION

In this study we have bean modulated the motion with heat and mass transfer of the non-Newtonian fluid past an infinite vertical porous plate mathematically, the equation of continuity, momentum, heat and mass are used to describe this phenomena. These equations are solved analytically by using Lightill method to separate the solutions into two parts, and a perturbation method for small value of the elasticity parameters. The solution which are, velocity, temperature, concentration as well as skin-friction, and the rate of heat and mass transfer are obtained as a functions of the physical parameters of the problem. The effects of these parameters on these solutions are calculated numerically and illustrated graphically through a set of Figs. (2-50).

Since the motion along a vertical plate, the momentum equation exclude the temperature and concentration this due to the density variation with temperature and concentration in the body force term, hence the momentum and heat and mass equations are coupled and the velocity depends on the temperature and the concentration of the fluid.

Figure 2, illustrate the dependence of velocity u on the Prandtl number P_r. It is clear that the velocity decreasing with the increasing of Prandtl number. The velocity u, decreases with increases of the porosity parameter of the plate V, this shown through Fig. (3). From Fig. (4) its seen that the velocity u decreases with increasing the Schmidt number S_c. The velocity u is plotted for different values of the Grashof heat and mass numbers G_r and G_c its clear that the velocity u increases with the increasing of both G_r and G_c,this shown through the Figs. (5) and (9).

The effect of the heat source parameter L on the velocity u is given through the figure (6), it is seen that the velocity u decreases with increases L, while the velocity u increases with increasing the elasticity parameter K , this shown from Fig. (7). Figure (8) gives the relation between the chemical reaction parameters S and the velocity u, it is clear that the velocity decreases with increasing of S. Figures. (10-17) represent the effects of the above parameters on the velocity when plotted against time. It's found that the velocity changes periodically with change of time. Figures (18-21) illustrate the effects of the porosity parameter V, heat source parameter L, prandtl number P_r and the angular velocity w on the temperature when plotted against distance y. It is observed that the temperature decreases by increasing these parameters. Figures (22-25) represent the effects of the above parameters on the temperature when plotted against time. It's found that the temperature changes periodically with the change of time. The effects of the different parameters on the concentration are indicated graphically through Figs. (26-33). In Figs. (26-29) the concentration is plotted against distance y. It's clear that the concentration is decreases by increasing the Schmit number S_c ,chemical reaction parameter S, porosity parameter V and the angular velocity w. In Figs. (30-33) the concentration is plotted against time and it's observed from these figures that the concentration changes periodically with the change of time. Figures (34-42) illustrate the effect of the Prandtl number P_r, porosity parameter V, Schmidt number, chemical reaction parameter S, the Grashof heat and mass numbers G_r and G_c, the heat source parameter L, the elasticity parameter K and the angular velocity parameter W on the skin friction when plotted against time and it's observed from these figures that the skin friction changes periodically with the change of time. Also we found that from Fig. (41) the effect of K change the skin friction from increasing to decreasing periodically with the change of time. Figures (43-46) are graphed to illustrate the effects of Prandtl number P_r, the porosity parameter V, the heat source parameter L and the angular velocity w on Nusselt number when plotted against time, It's found that the Nusselt number changes periodically with change of time. Also, the change of Sherwood number dependes on the porosity parameter, Schmidt number, chemical reaction parameter and the angular velocity are illustrate through the Figs. (46-50) when plotted against time. It is found that the Sherwood number changes periodically with change of time.

7. CONCLUSION

Rivlin-Ericksen model is used to describes the fluid motion with heat and mass transfer past an infinite porous vertical plate. The resulting partial differential equations which control the motion are solved analytically by using the perturbation technique with lightill method. The velocity, temperature, concenteration, skin-friction, Nusselt number and Sherwood number distributions are obtained. The effects of the problem's parameters such as non-Newtonian parameter K, the porosity parameter V of the porous plate, Grashof numbers for heat and mass respectively, heat source parameter L, Prandtl number, chemical reaction parameter, and Schmidt number on these distributions are discussed and illustrated by a set of graphs. The results of this problem are of great interest for many scientific and engineering applications, among which we cite the possibility of reducing frictional drag on the hulls of ships and submarines, and also, in polymer production.

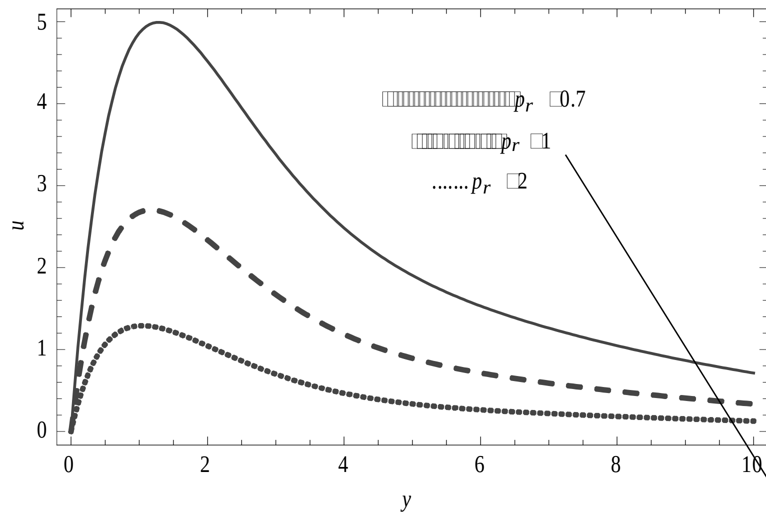

Figure 2: The Velocity Distribution is Plotted Against *y* for Different Values of P_r, with $W = 2$, $t = \frac{\pi}{4}$, $V = 0.2$, $L = 3$, $G_r = 10$, $G_c = 5$, $S = 3$, $S_c = 2$, $k = 0.01$

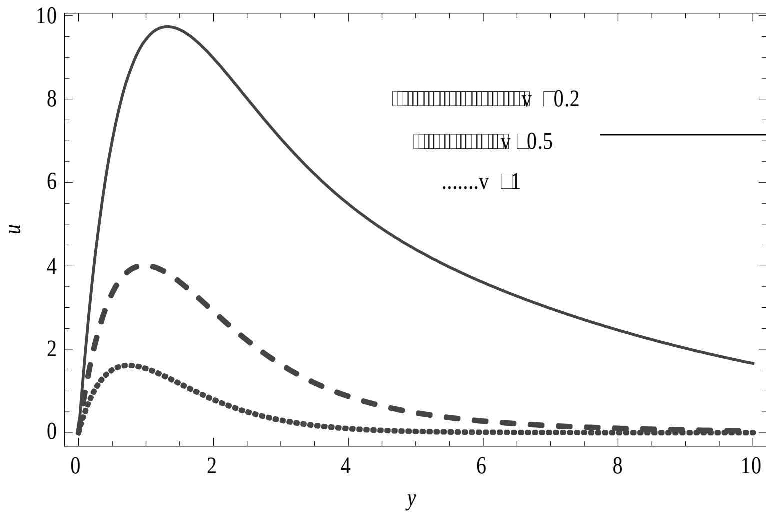

Figure 3: The Velocity Distribution is Plotted Against *y* for Different Values of *V*, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $L = 3$, $G_r = 10$, $G_c = 5$, $S = 3$, $S_c = 2$, $k = 0.01$

<u>A.Q: ???</u>

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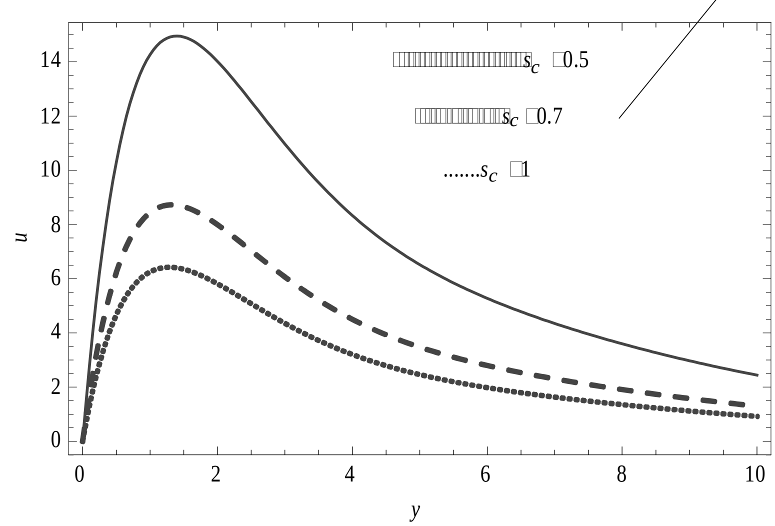

Figure 4: The Velocity Distribution is Plotted Against *y* for Different Values of S_c, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $L = 3$, $G_r = 10$, $G_c = 5$, $S = 3$, $V = 0.2$, $k = 0.01$

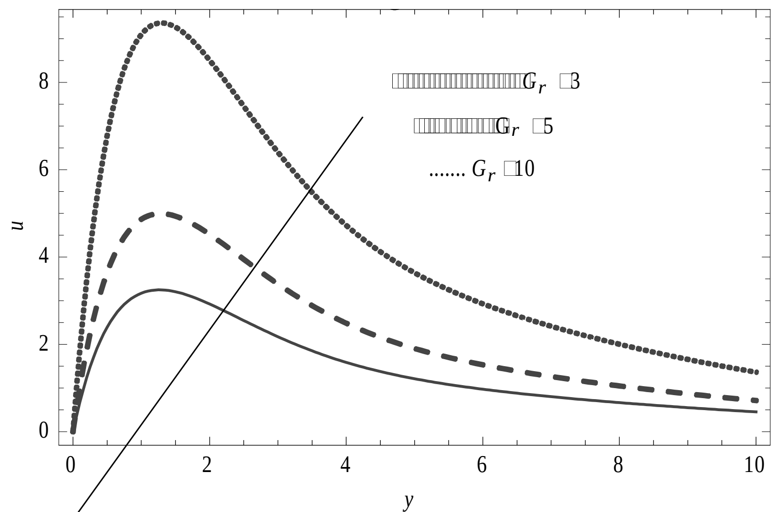

Figure 5: The Velocity Distribution is Plotted Against *y* for Different Values of G_r, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $L = 0.5$, $S_c = 2$, $G_c = 5$, $S = 3$, $V = 0.2$, $k = 0.01$

A.Q: ???

Please Check the Figures. There are **Mathematica4 Font** missing. Please sent all the Figures with (Arial font & Symbols where necessary)

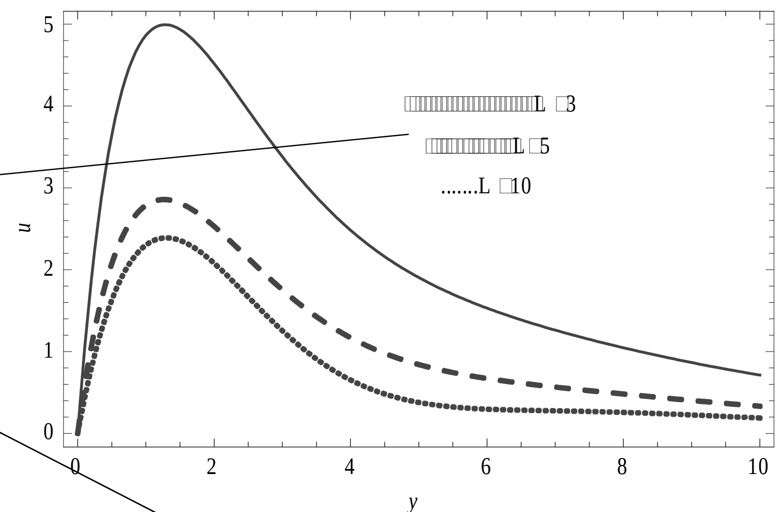

Figure 6: The Velocity Distribution is Plotted Against *y* for Different Values of *L*, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $G_r = 10$, $S_c = 2$, $G_c = 5$, $S = 3$, $V = 0.2$, $k = 0.01$

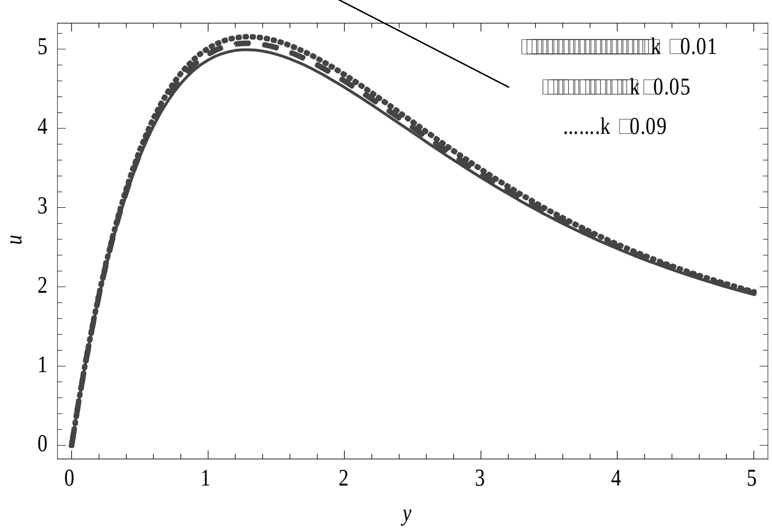

Figure 7: The Velocity Distribution is Plotted Against y for Different Values of *K*, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $G_r = 10$, $S_c = 2$, $G_c = 5$, $S = 3$, $V = 0.2$, $L = 3$

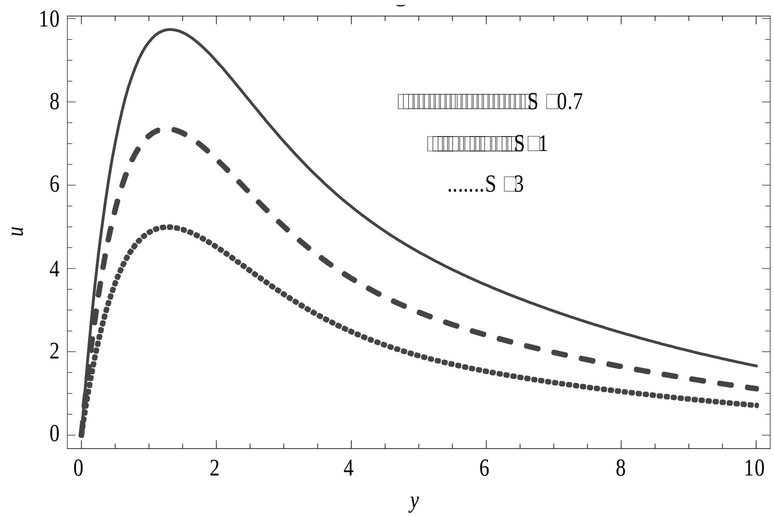

Figure 8: The Velocity Distribution is Plotted Against y for Different Values of S, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $G_r = 10$, $S_c = 2$, $K = 0.01$, $V = 0.2$, $L = 3$

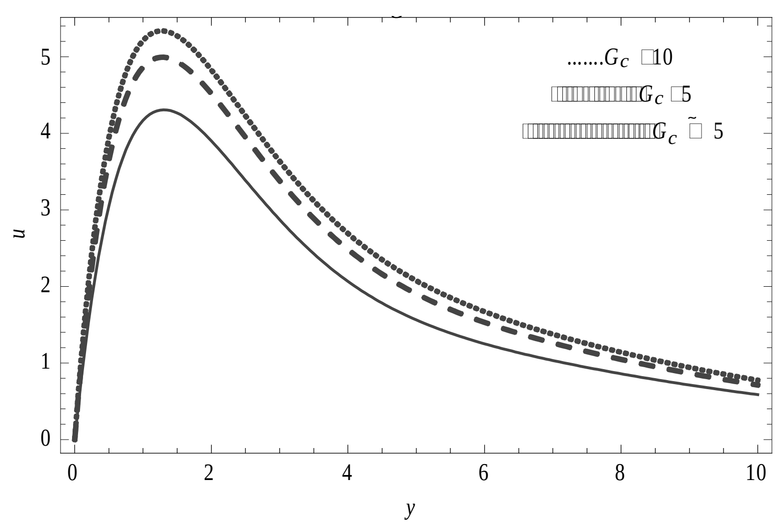

Figure 9: The Velocity Distribution is Plotted Against y for Different Values of G_c, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $S = 3$, $S_c = 2$, $G_c = 5$, $K = 0.01$, $V = 0.2$, $L = 3$

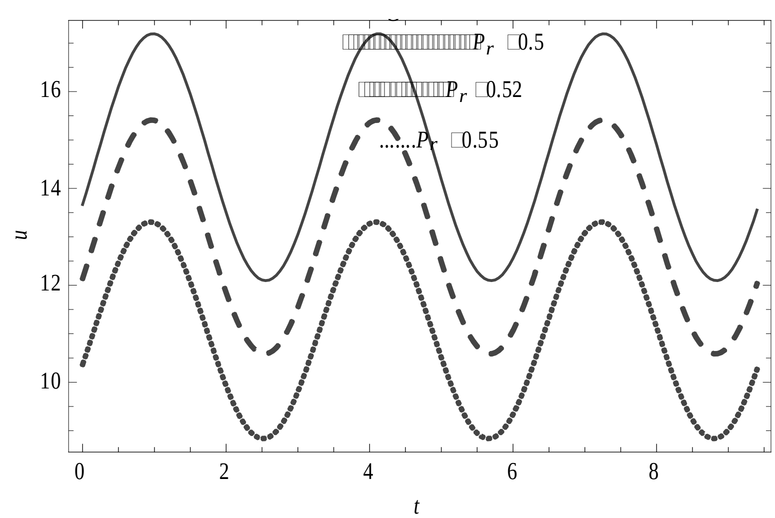

Figure 10: The Velocity Distribution is Plotted Against t for Different Values of P_r, with $W = 2$, $t = \frac{\pi}{4}$, $V = 0.2$, $L = 3$, $G_r = 10$, $G_c = 5$, $S = 3$, $S_c = 2$, $k = 0.01$

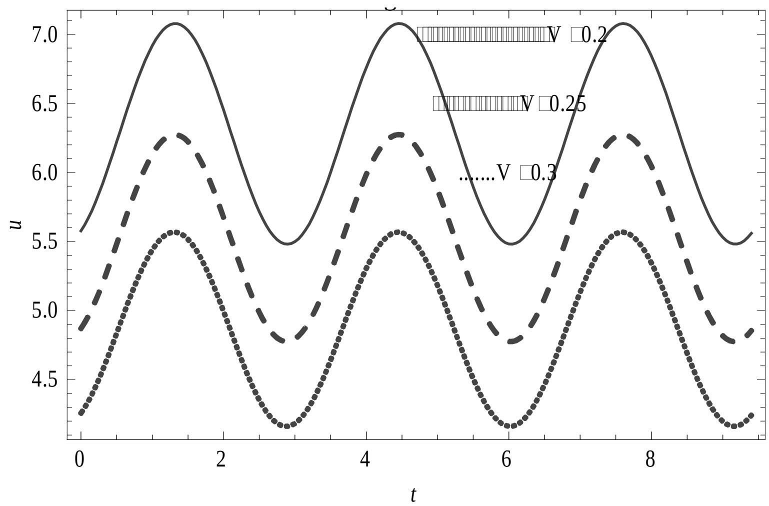

Figure 11: The Velocity Distribution is Plotted Against *t* for Different Values of *V*, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $L = 3$, $G_r = 10$, $G_c = 5$, $S = 3$, $S_c = 2$, $k = 0.01$

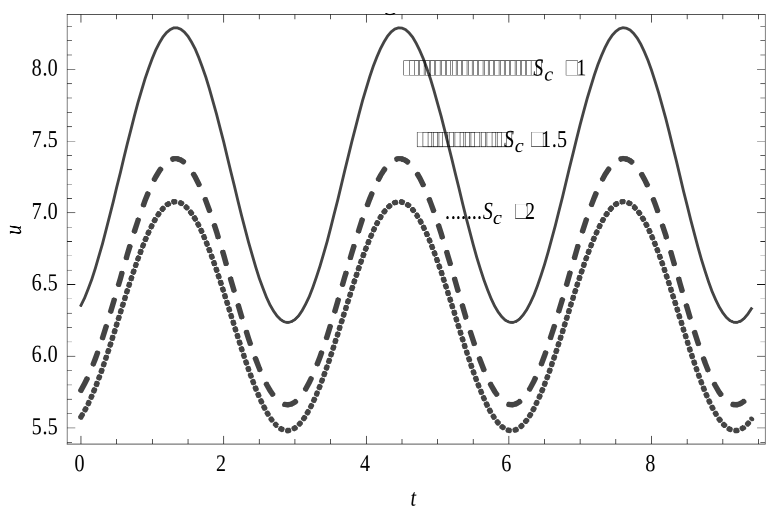

Figure 12: The Velocity Distribution is Plotted Against *t* for Different Values of S_c, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $L = 3$, $G_r = 10$, $G_c = 5$, $S = 3$, $V = 0.2$, $k = 0.01$

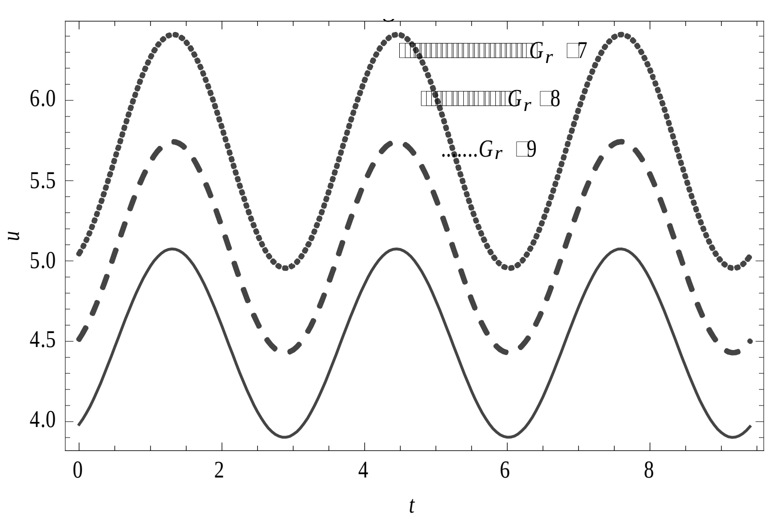

Figure 13: The Velocity Distribution is Plotted Against *t* for Different Values of G_r, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $L = 3$, $S_c = 2$, $G_c = 5$, $S = 3$, $V = 0.2$, $k = 0.01$

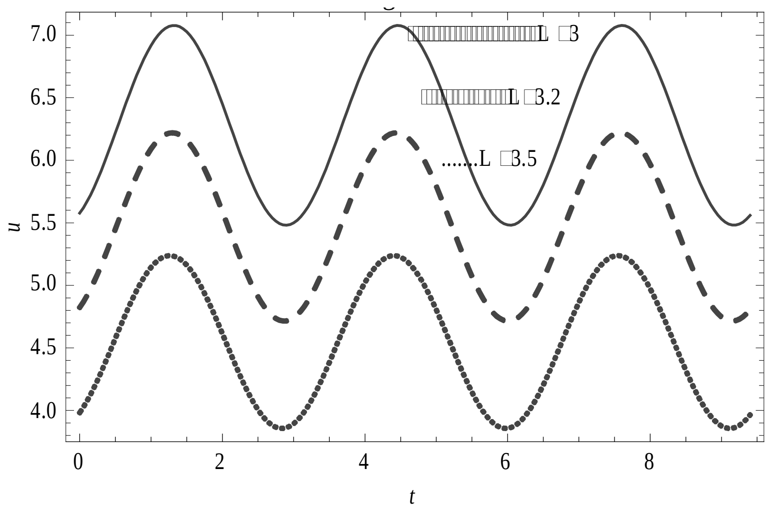

Figure 14: The Velocity Distribution is Plotted Against t for Different Values of L, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $G_r = 10$, $S_c = 2$, $G_c = 5$, $S = 3$, $V = 0.2$, $k = 0.01$

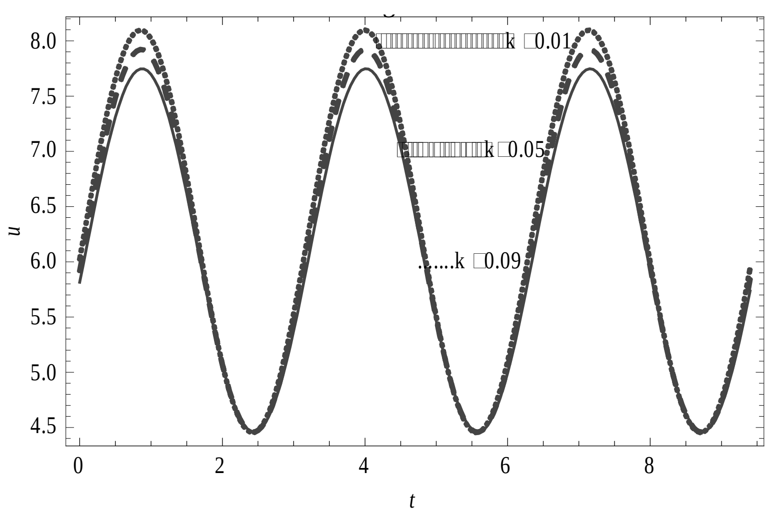

Figure 15: The Velocity Distribution is Plotted Against t for Different Values of K, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $G_r = 10$, $S_c = 2$, $G_c = 5$, $S = 3$, $V = 0.2$, $L = 3$.

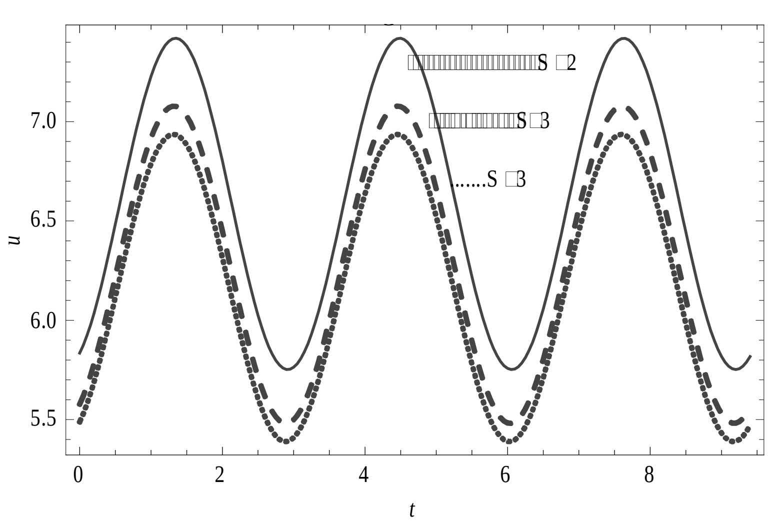

Figure 16: The Velocity Distribution is Plotted Against t for Different Values of S, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $G_r = 10$, $S_c = 2$, $K = 0.01$, $V = 0.2$, $L = 0.5$

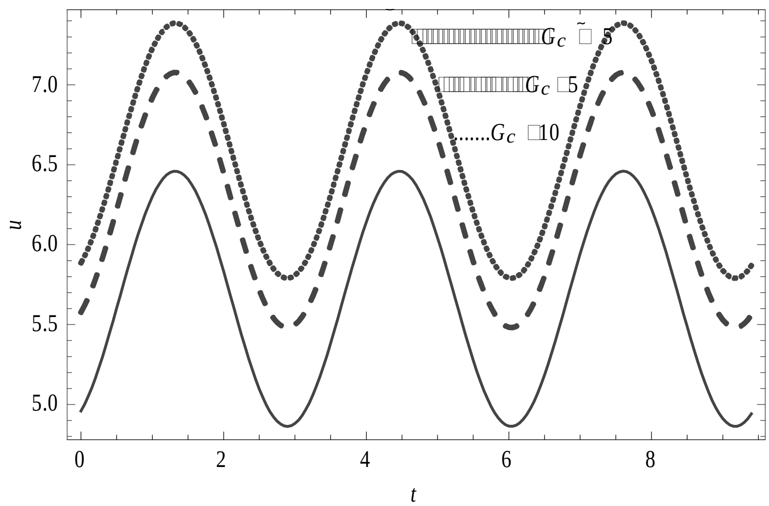

Figure 17: The Velocity Distribution is Plotted Against *t* for Different Values of G_c, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $S = 0.2$, $S_c = 0.3$, $G_c = 5$, $K = 0.01$, $V = 0.2$, $L = 0.5$

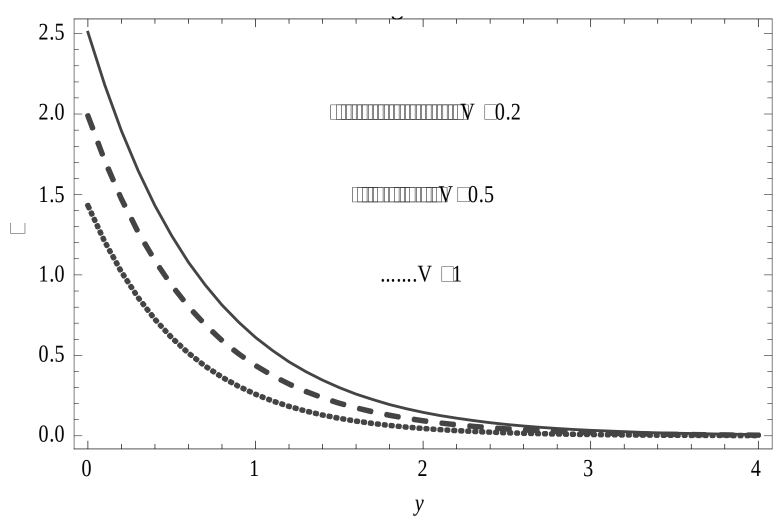

Figure 18: The Temperature Distribution is Plotted Against *y* for Several Values of *V*, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $S = 3$, $S_c = 2$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $L = 0.5$

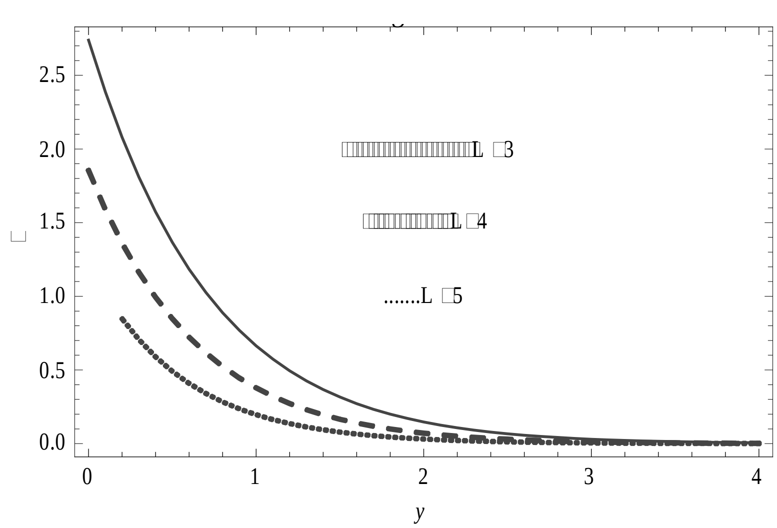

Figure 19: The Temperature Distribution is Plotted Against *y* for Several Values of *L*, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $S = 3$, $S_c = 2$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $V = 0.2$

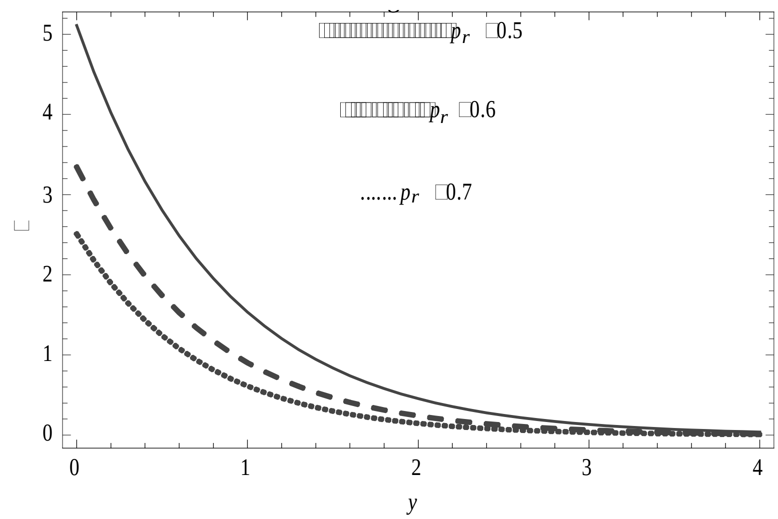

Figure 20: The Temperature Distribution is Plotted Against *y* for Several Values of P_r, with $W = 2, t = \frac{\pi}{4}, L = 3, S = 3, S_c = 2, G_c = 5, K = 0.01, G_r = 10, V = 0.2$

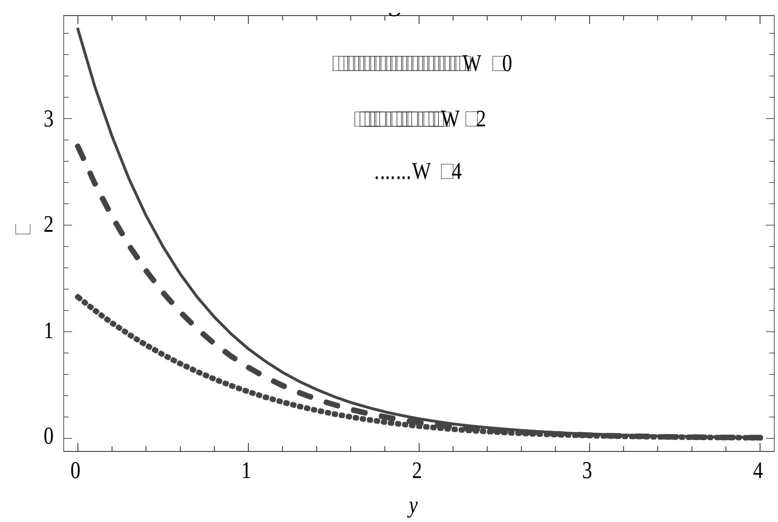

Figure 21: The Temperature Distribution is Plotted Against *y* for Several Values of *W*, with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, S = 3, S_c = 2, G_c = 5, K = 0.01, G_r = 10, V = 0.2$

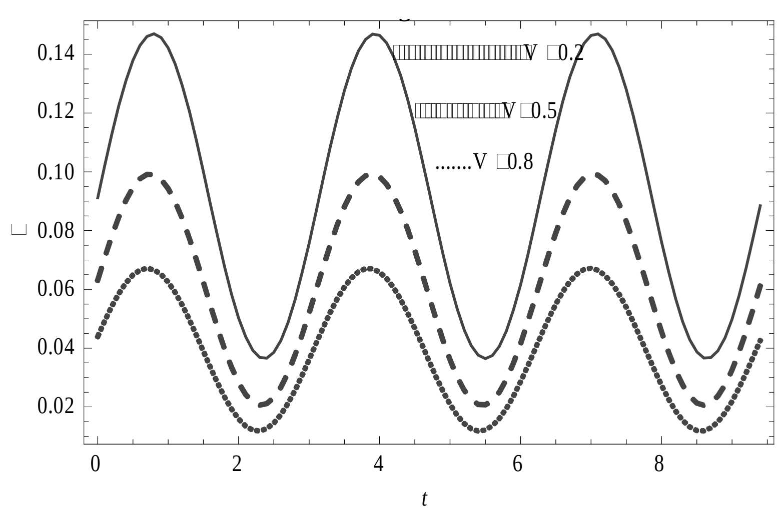

Figure 22: The Temperature Distribution is Plotted Against *t* for Several Values of *V*, with $W = 2, t = \frac{\pi}{4}, P_r = 0.7, S = 3, S_c = 2, G_c = 5, K = 0.01, G_r = 10, L = 3$

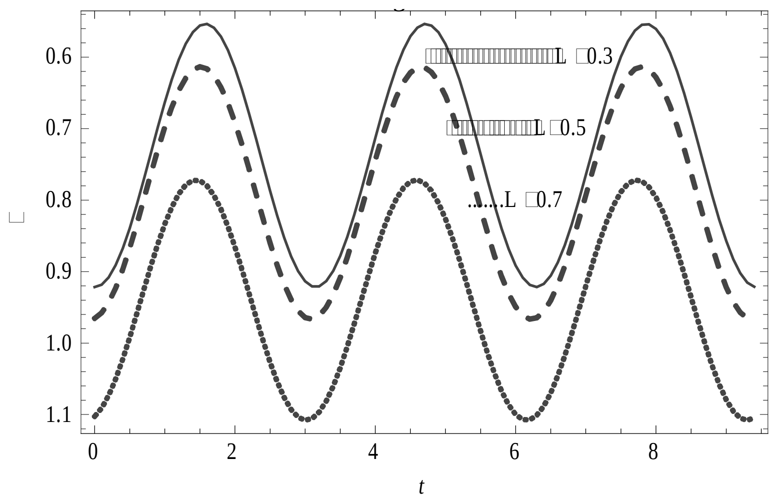

Figure 23: The Temperature Distribution is Plotted Against t for Several Values of L, with $W = 2$, $t = \frac{\pi}{4}$, $P_r = 0.7$, $S = 3$, $S_c = 2$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $V = 0.2$

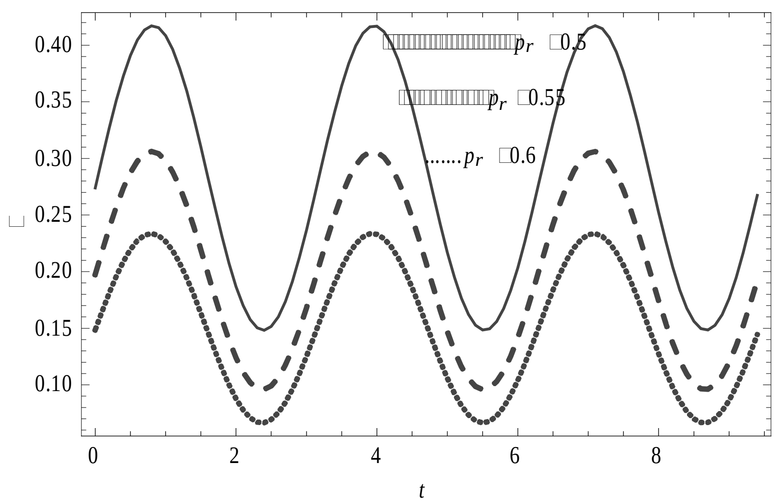

Figure 24: The Temperature Distribution is Plotted Against t for Several Values of P_r, with $W = 2$, $t = \frac{\pi}{4}$, $L = 3$, $S = 3$, $S_c = 2$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $V = 0.2$

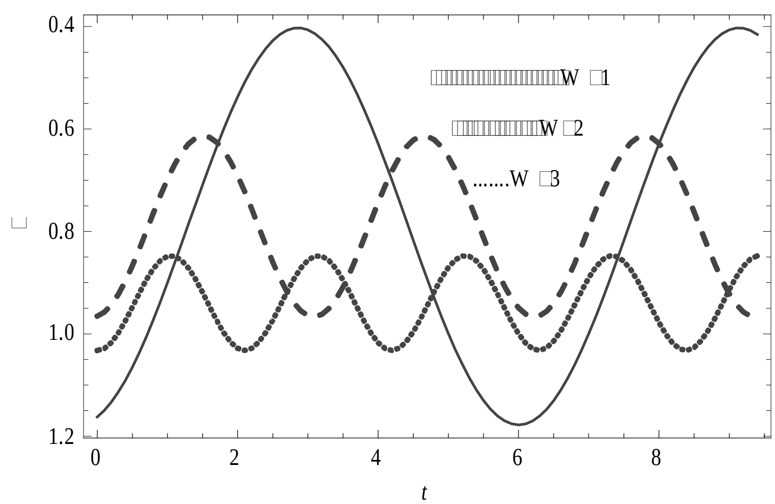

Figure 25: The Temperature Distribution is Plotted Against t for Several Values of W, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $S = 3$, $S_c = 2$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $V = 0.2$

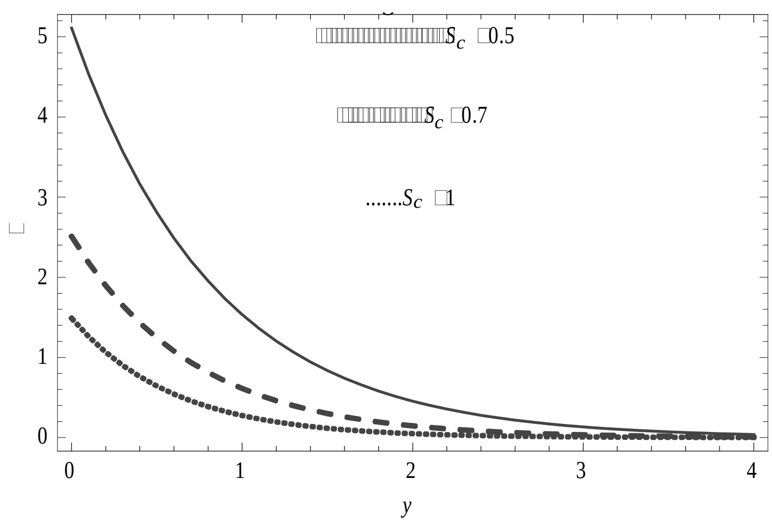

Figure 26: The Concentration Distribution is Plotted Against *y* for Several Values of S_c, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $S = 3$, $W = 2.$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $V = 0.2$

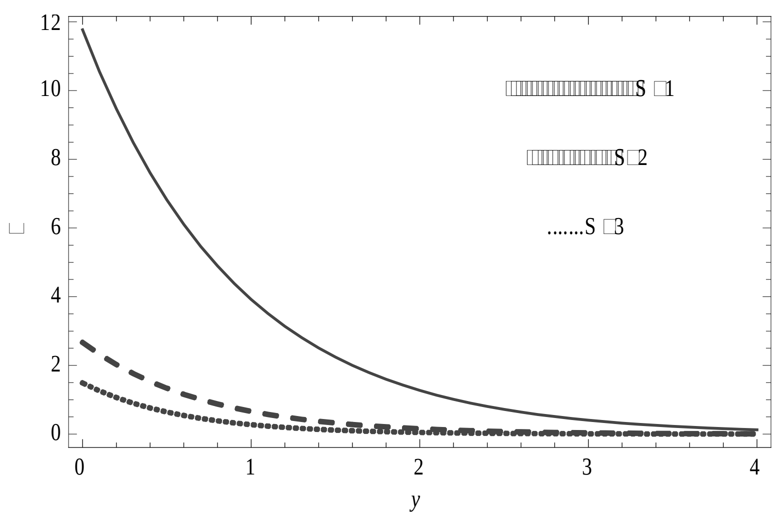

Figure 27: The Concentration Distribution is Plotted Against *y* for Several Values of *S*, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $S_c = 2$, $W = 2.$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $V = 0.2$

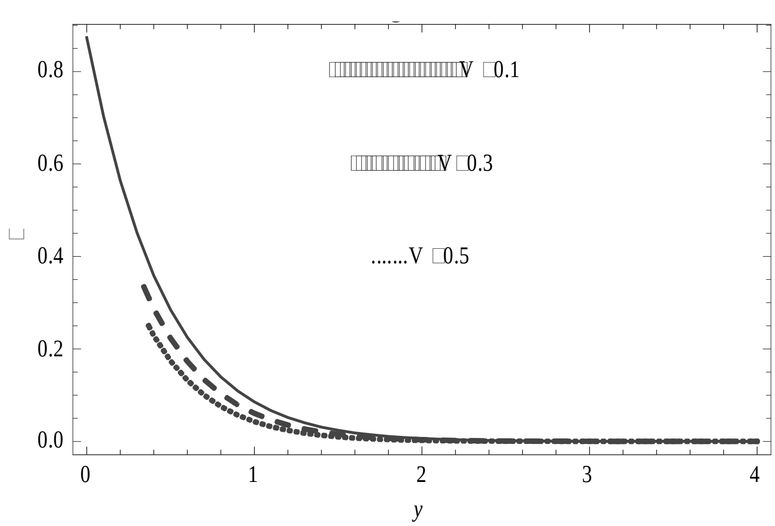

Figure 28: The Concentration Distribution is Plotted Against *y* for Several Values of *V* with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $S_c = 2$, $W = 2.$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $S = 0.2$

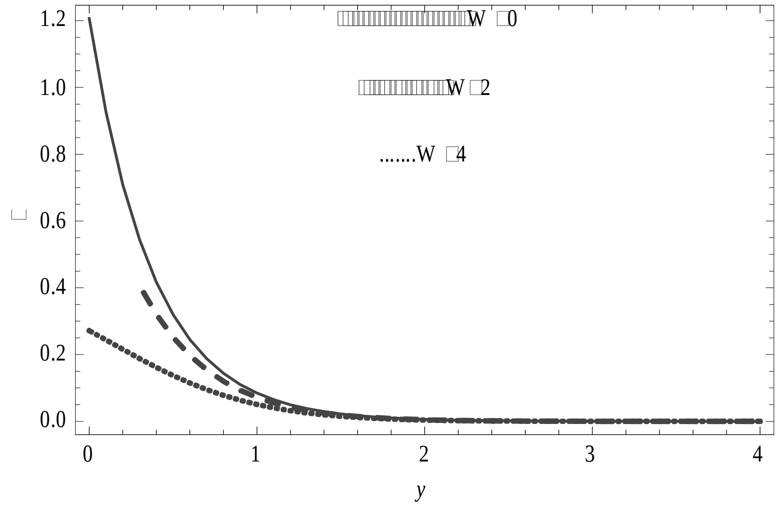

Figure 29: The Concentration Distribution is Plotted Against *y* for Several Values of *W* with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, S_c = 2, W = 2., G_c = 5, K = 0.01, G_r = 10, S = 0.2$

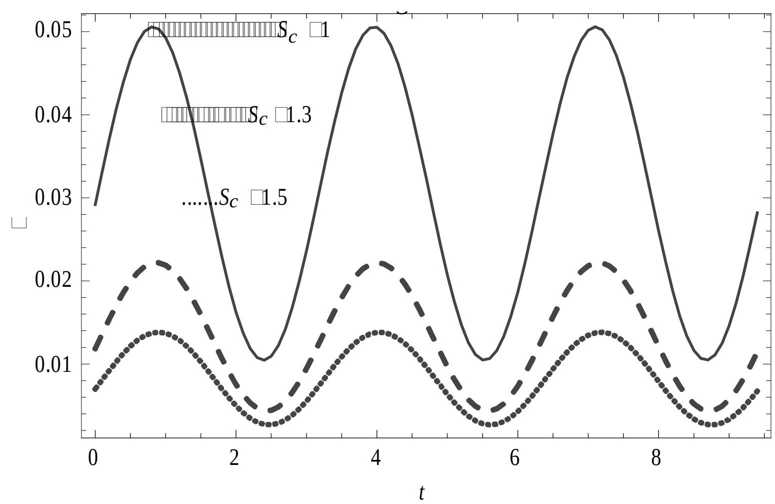

Figure 30: The Concentration Distribution is Plotted Against *t* for Several Values of S_c, with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, S = 3, W = 2., G_c = 5, K = 0.01, G_r = 10, V = 0.2$

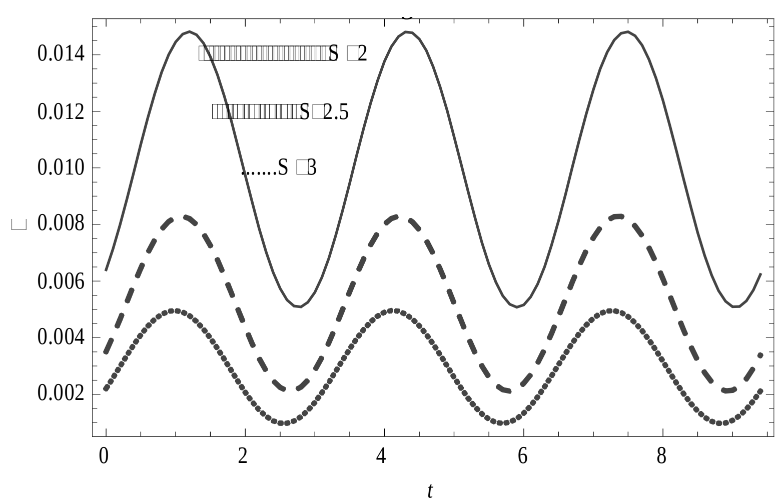

Figure 31: The Concentration Distribution is Plotted Against *t* for Several Values of *S*, with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, S_c = 2, W = 2., G_c = 5, K = 0.01, G_r = 10, V = 0.2$

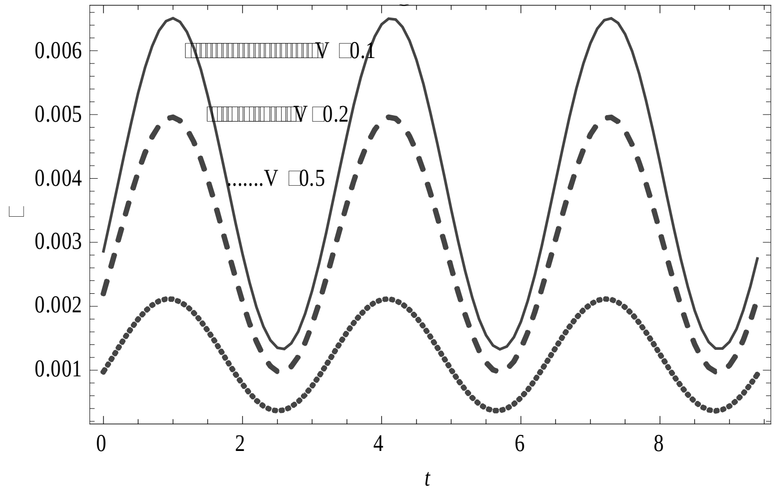

Figure 32: The Concentration Distribution is Plotted Against *t* for Several Values of *V*, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $S_c = 2$, $W = 2.$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $S = 3$

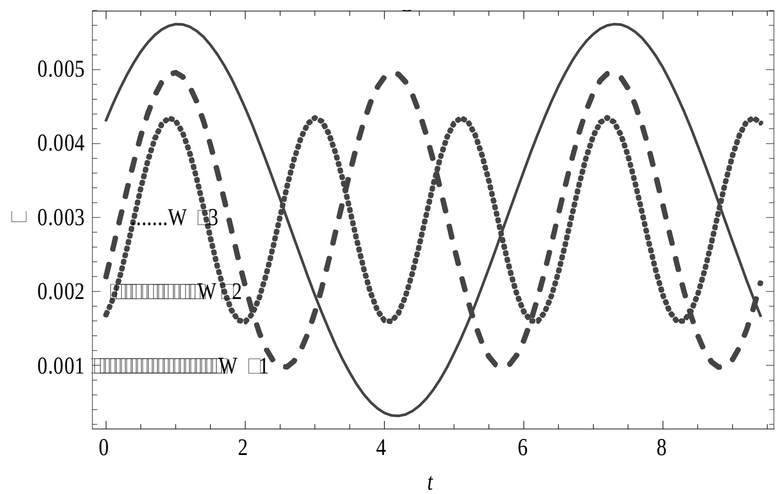

Figure 33: The Concentration distribution is plotted against t for several values of *W*, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $S_c = 2$, $W = 2.$, $G_c = 5$, $K = 0.01$, $G_r = 10$, $S = 3$

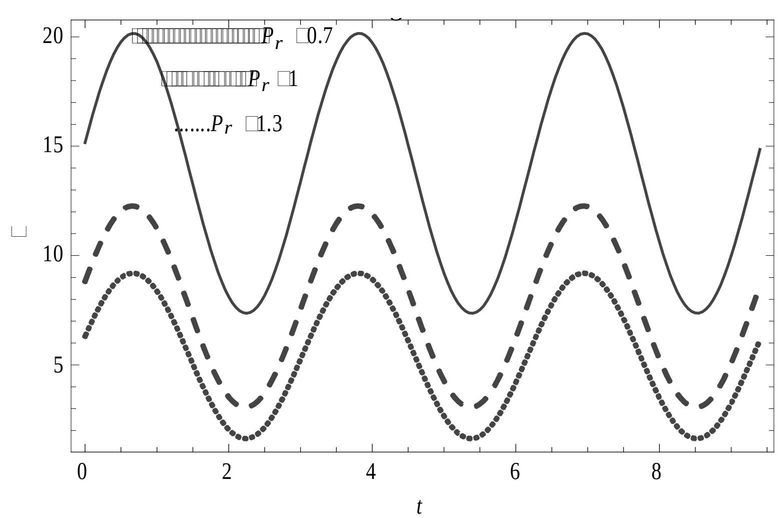

Figure 34: The Skin-Friction Distribution is Plotted Against *t* for Several Values of P_r, with $V = 0.2$, $t = \frac{\pi}{4}$, $L = 3$, $S_c = 2$, $W = 2.$, $S_c = 5$, $K = 0.01$, $G_r = 10$, $S = 3$

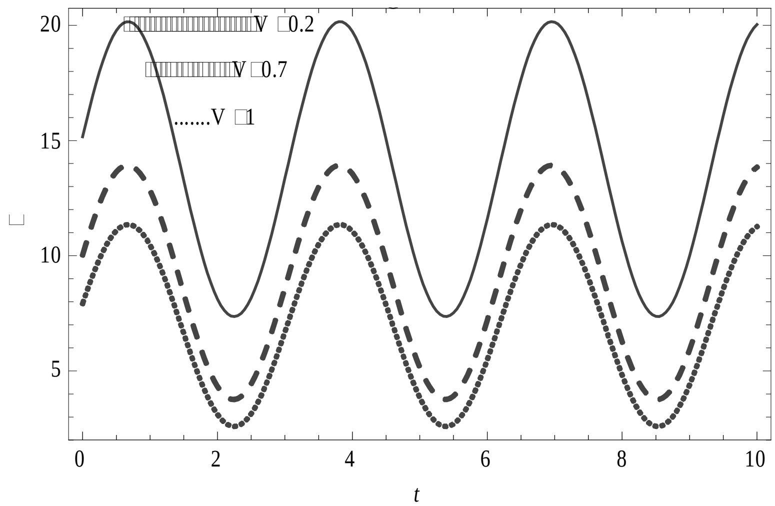

Figure 35: The Skin-Friction Distribution is Plotted Against *t* for Several Values of *V*, with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, S_c = 2, W = 2., G_c = 5, K = 0.01, G_r = 10, S = 3$

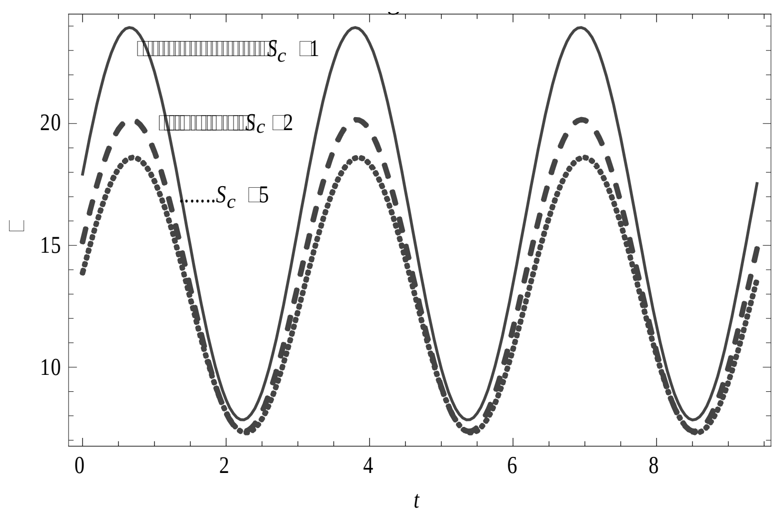

Figure 36: The Skin-Friction Distribution is Plotted Against *t* for Several Values of S_c, with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, V = 0.2, W = 2., G_c = 5, K = 0.01, G_r = 10, S = 3$

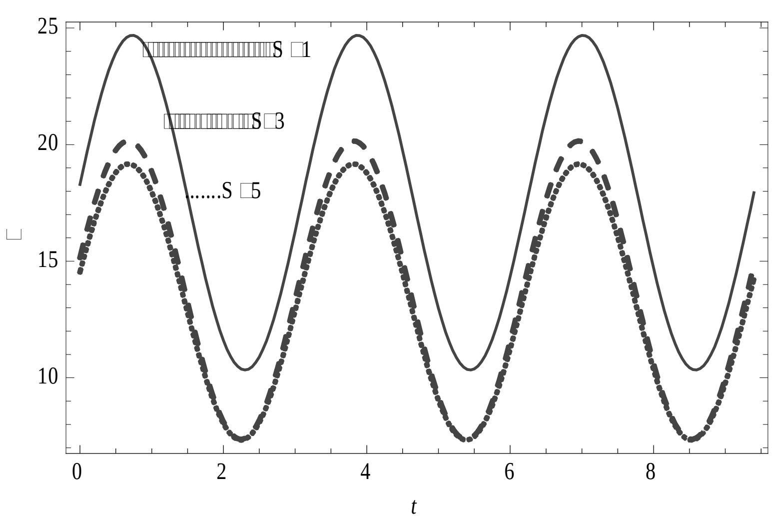

Figure 37: The Skin-Friction Distribution is Plotted Against *t* for Several Values of *S*, with $P_r = 0.7, t = \frac{\pi}{4}, L = 3, V = 0.2, W = 2., G_c = 5, K = 0.01, G_r = 10, S_c = 2$

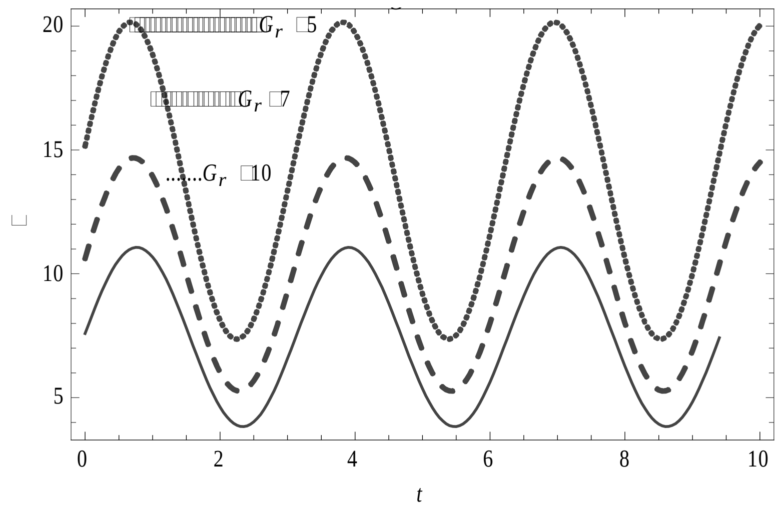

Figure 38: The Skin-Friction Distribution is Plotted Against *t* for Several Values of G_r, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $V = 0.2$, $W = 2.$, $G_c = 5$, $K = 0.01$, $S = 3$, $S_c = 2$

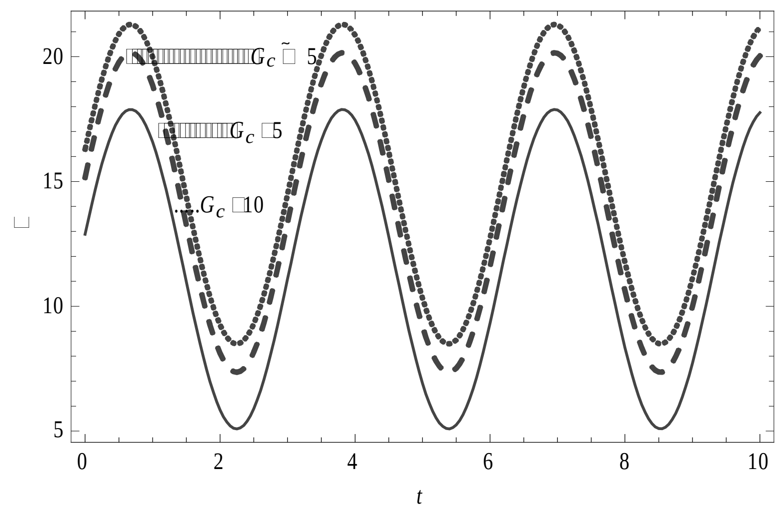

Figure 39: The Skin-Friction Distribution is Plotted Against *t* for Several Values of G_c, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $L = 3$, $V = 0.2$, $W = 2.$, $G_r = 10$, $K = 0.01$, $S = 3$, $S_c = 2$

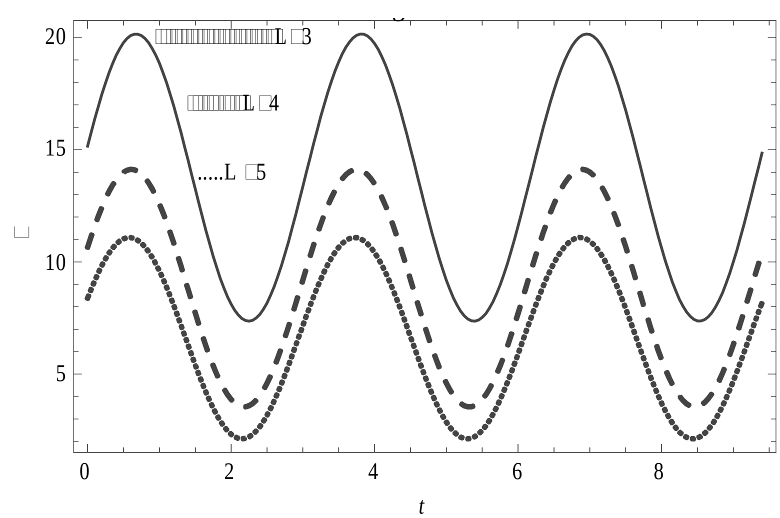

Figure 40: The Skin-Friction Distribution is Plotted Against *t* for Several Values of *L*, with $P_r = 0.7$, $t = \frac{\pi}{4}$, $G_c = 5$, $V = 0.2$, $W = 2.$, $G_r = 10$, $K = 0.01$, $S = 3$, $S_c = 2$

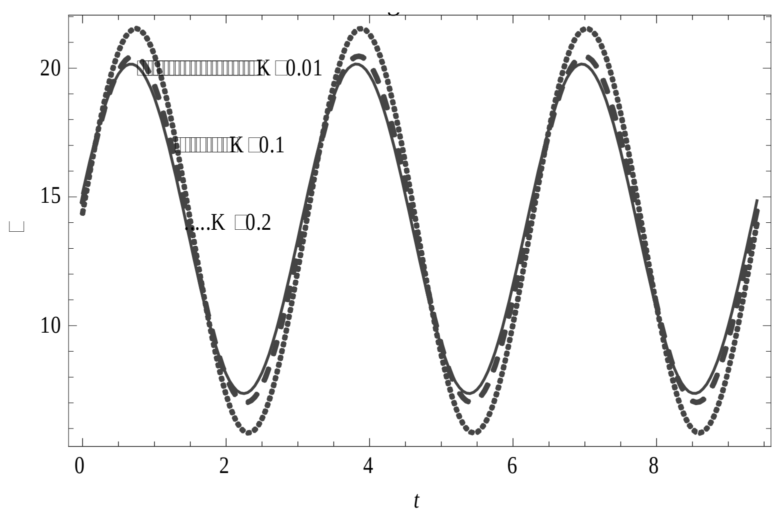

Figure 41: The Skin-Friction Distribution is Plotted Against *t* for Several Values of *K*, with $P_r = 0.7, t = \frac{\pi}{4}, G_c = 5, V = 0.2, W = 2., G_r = 10, L = 3, S = 3, S_c = 2$

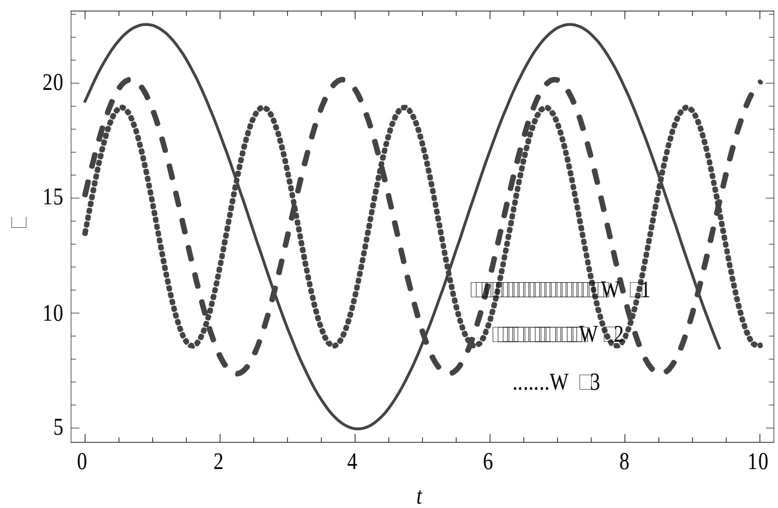

Figure 42: The Skin-Friction Distribution is Plotted Against *t* for Several Values of *W*, with $P_r = 0.7, t = \frac{\pi}{4}, G_c = 5, V = 0.2, K = 0.01., G_r = 10, L = 3, S = 3, S_c = 2$

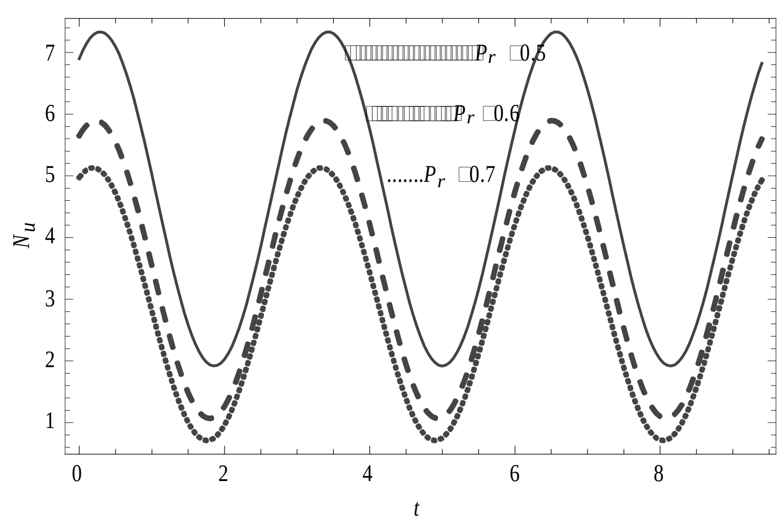

Figure 43: The Nusselt Number Distribution is Plotted Against *t* for Several Values of P_r, with $K = 0.01, t = \frac{\pi}{4}, G_c = 5, V = 0.2, W = 2., G_r = 10, L = 3, S = 3, S_c = 2$

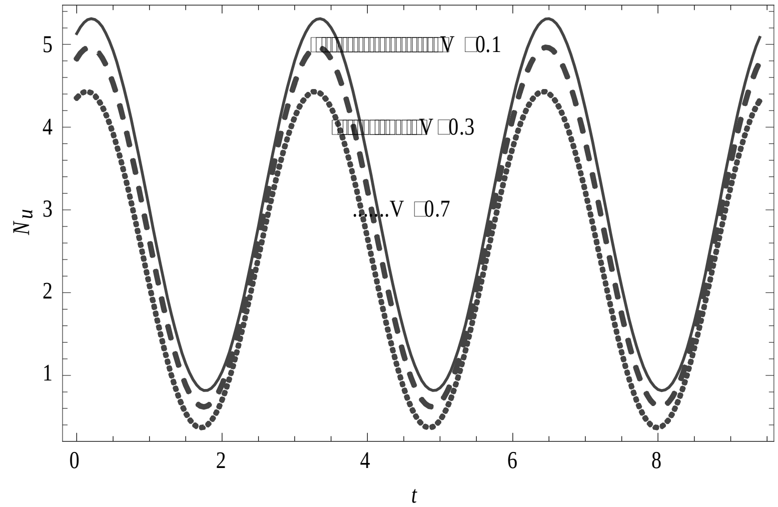

Figure 44: Nusselt Number Distribution is Plotted Against *t* for Several Values of *V*, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $W = 2.$, $G_r = 10$, $L = 3$, $S = 3$, $S_c = 2$

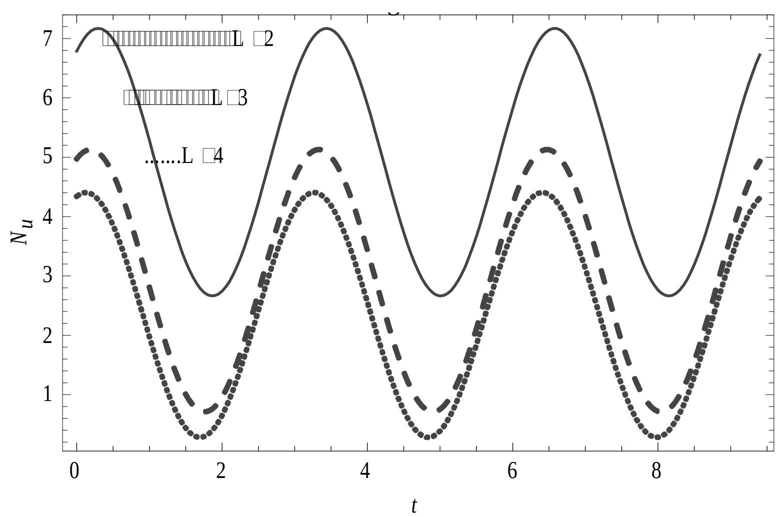

Figure 45: Nusselt Number Distribution is Plotted Against *t* for Several Values of *L*, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $W = 2$, $G_r = 10$, $V = 0.2$, $S = 3$, $S_c = 2$

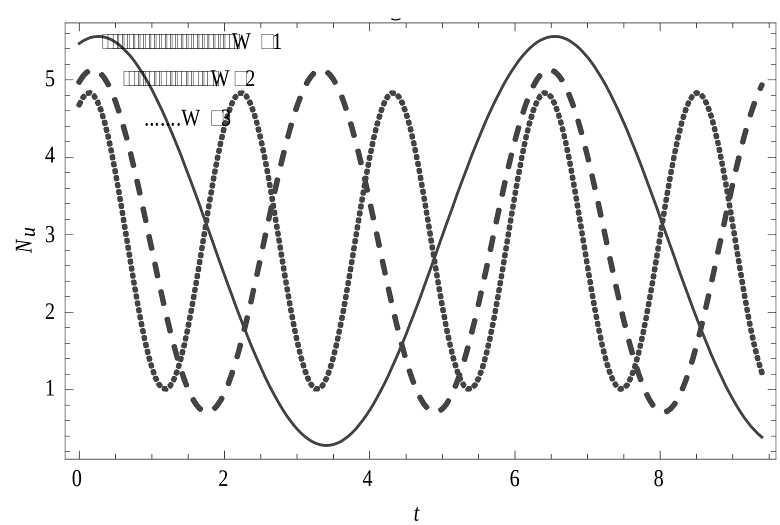

Figure 46: Nusselt Number Distribution is Plotted Against *t* for Several Values of *W*, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $L = 3$, $G_r = 10$, $V = 0.2$, $S = 3$, $S_c = 2$

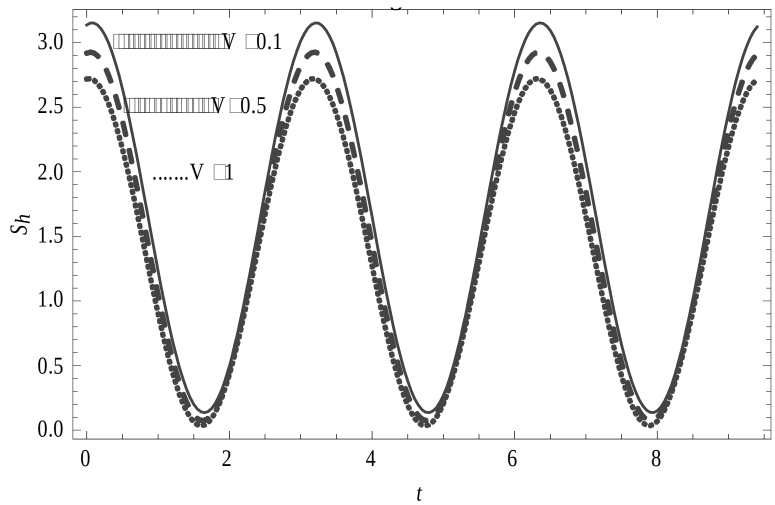

Figure 47: Sherwood Number Distribution is Plotted Against *t* for Several Values of *V*, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $W = 2.$, $G_r = 10$, $L = 3$, $S = 3$, $S_c = 2$

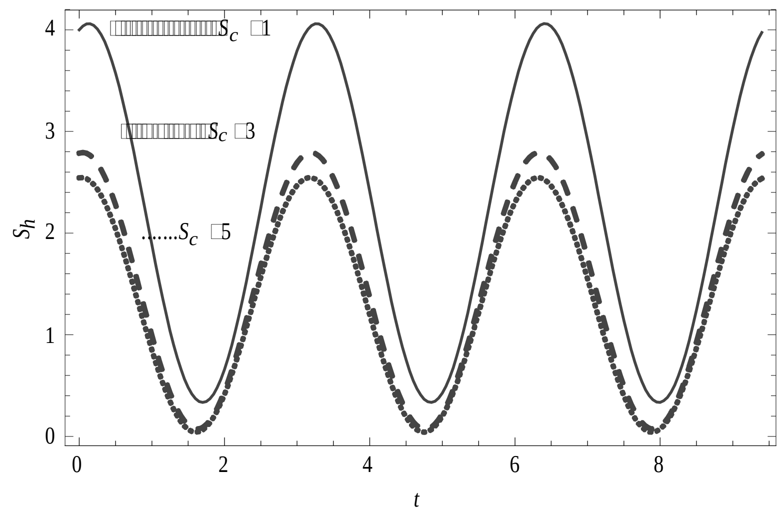

Figure 48: Sherwood Number Distribution is Plotted Against *t* for Several Values of S_c, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $W = 2.$, $G_r = 10$, $L = 0.5$, $S = 3$, $V = 0.2$

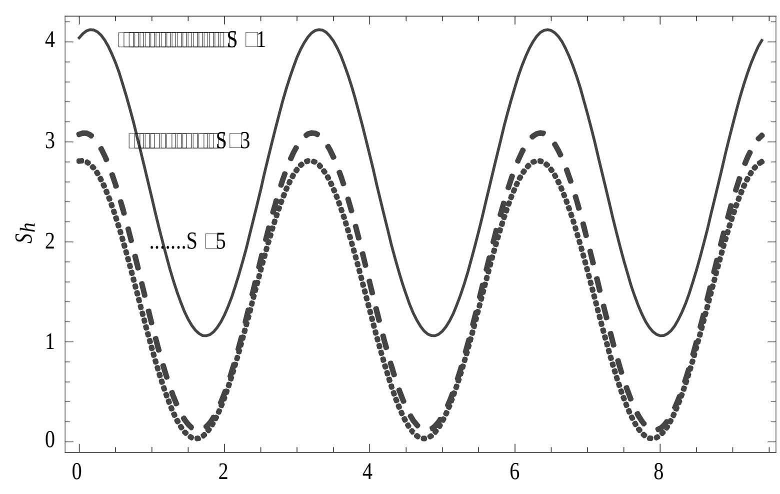

Figure 49: Sherwood Number Distribution is Plotted Against *t* for Several Values of *S*, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $W = 2.$, $G_r = 10$, $L = 3$, $S_c = 2$, $V = 0.2$

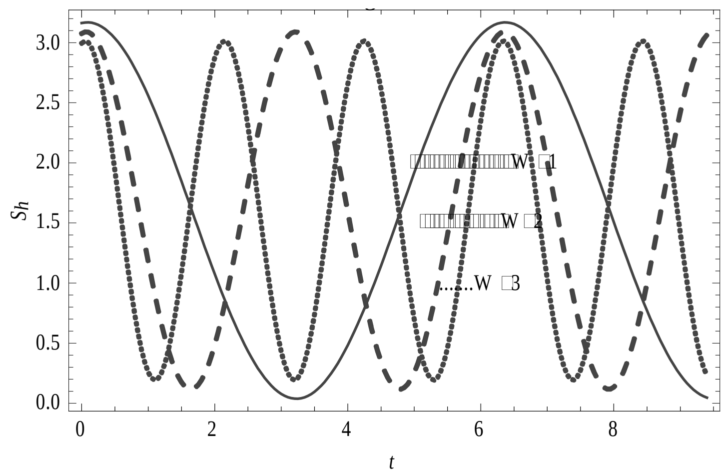

Figure 50: Sherwood Number Distribution is Plotted Against *t* for Several Values of *W*, with $K = 0.01$, $t = \frac{\pi}{4}$, $G_c = 5$, $P_r = 0.7$, $S = 3.$, $G_r = 10$, $L = 3$, $S_c = 2$, $V = 0.2$

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