New Advances in Physics, Vol. 12, No. 2, December 2018, pp. 89-97

On the Mössbauer Rotor Experiment as New Proof of Einstein's general theory of Relativity

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Abstract: *We review a correct interpretation of a historical experiment by Kündig on the transverse Doppler shift in a rotating system (Mössbauer rotor experiment). In fact, an experimental research group recently first reanalysed, and then replied such an experiment. The results of their reanalysing have shown that an interesting deviation of a relative redshift between emission and absorption resonant lines from the standard prediction, based on the relativistic dilatation of time, arises from a correct re-processing of Kündig's experimental data. Subsequent new experimental results by the reply of Kündig experiment, which have been realized by the same experimental research group, have remarkably shown a deviation from the standard prediction even higher. Here we use the power the Equivalence Principle (EP). The EP indeed states the equivalence between the gravitational "force" and the pseudo-force experienced by an observer in a non-inertial frame of reference. This is also the case of a rotating frame of reference. Hence, the EP permits to reanalyse the theoretical framework of the Mössbauer rotor experiment directly in the rotating frame of reference. This is a full a general relativistic treatment which permits to show that previous analyses missed an important effect of clock synchronization. If one adds this new effect, the correct general relativistic prevision agrees in a perfect way with the new experimental results. It is important to stress that this additional effect of clock synchronization has been missed in various previous papers in the literature. This generated some subsequent claim of invalidity of the relativity theory and/or some attempts to explain the experimental results through exotic effects. Our general relativistic interpretation shows, instead, that the new experimental results of the Mössbauer rotor experiment represents a new, strong and independent, proof of Einstein's general theory relativity (GTR). Remarkably, our results on the Mössbauer rotor experiment received an Honorable Mention at the Gravity Research Foundation 2018 Awards for Essays on Gravitation.*

We review a correct interpretation of a historical experiment by Kündig on the transverse Doppler shift in a rotating system, measured with the Mössbauer

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effect (Mössbauer rotor experiment) [3]. The Mössbauer effect was discovered by R. Mössbauer in 1958 [14]. For this effect, this German physicist was awarded the 1961 Nobel Prize in Physics. The effect consists in resonant and recoilfree emission and absorption of gamma rays, without loss of energy, by atomic nuclei bound in a solid. It is of fundamental importance for various researches in physics and chemistry. In this review paper, we will consider the so called Mössbauer rotor experiment. In this case, the Mössbauer effect acts by using an absorber orbited around a source of resonant radiation (or vice versa). The experiment's goal is to verify the relativistic time dilation for a moving resonant absorber (the source), inducing a relative energy shift between emission and absorption lines.

In [1, 2], the authors “rst reanalysed in [1] the data of a famous, historical experiment of Kündig on the transverse Doppler shift in a rotating system, measured with the Mössbauer effect [3]. In a latter time, they realized their own experiment on the time dilation effect in a rotating system [2]. In [1], the authors found that in the original experiment by Kündig [3] errors were present in the data processing. After the correction of the errors of Kündig, the experimental data gave the value [1]

$$\frac{\nabla E}{E} \simeq -k\frac{v^2}{c^2},$$

where $k = 0.596 \pm 0.006$, instead of the standard relativistic prediction $k = 0.5$ due to time dilatation. This was a puzzling issue. In [1] it is emphasized that, on one hand, the deviation of the coecient k in equation (1) from 0.5 exceeds by almost 20 times the measuring error. On the other hand, the revealed deviation cannot be attributed to the in”uence of rotor vibrations and/or other kinds of disturbing factors. The potential disturbing factors was indeed excluded by a very good methodology of Kündig [3]. Such a methodology is given by a first-order Doppler modulation of the energy of γ–quanta on a rotor at each fixed rotation frequency. Therefore, Kündig's experiment is today considered as being the most precise among other similar experiments [4–8]. In fact, the experimenters [4–8] measured only the count rate of detected γ–quanta as a function of rotation frequency. In [1] it has also been shown that the experiment in [8] confirms the supposition $k > 0.5$. Remarkably, the experiment in [8] contains much more data than the ones in [4–7]. In order to better investigating the results in [1], the authors realized their own experiment [2]. In [2], neither the scheme of the Kündig experiment [3], nor the schemes of other known experiments on the subject previously mentioned above [4–8] have been repeated. This permitted to obtain a completely independent information on

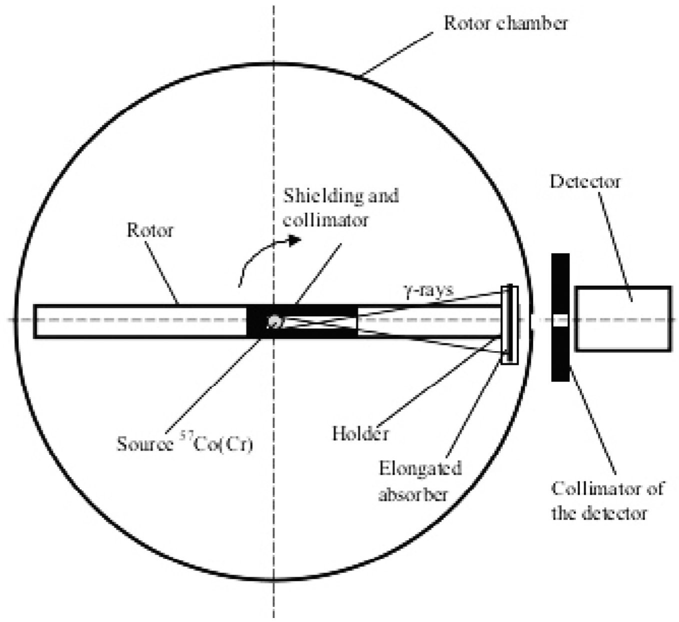

Figure 1: Scheme of the new Mössbauer rotor experiment, adapted from ref. [2]

the value of k in Eq. (1). The authors refrained indeed from the "rst-order Doppler modulation of the energy of γ–quanta [2]. This permitted to exclude the uncertainties in the realization of this method [2]. The standard scheme in [4–8] has been followed also in [2]. It means that the count rate of detected γ–quanta N as a function of the rotation frequency v has been measured. But, dierently from the experiments [4–8], in [2] the influence of chaotic vibrations on the measured value of k [2] has been evaluated. A method involving a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra has been developed [2]. The final result was a value is $k = 0.68 \pm 0.03$ [2]. This confirms that the coefficient k in Eq. (1) substantially exceeds 0.5. The reader can see the scheme of the new Mössbauer rotor experiment in Figure 1. Further technical details on this scheme can be found in [2].

In this review paper, we will use the power of the EP. It indeed states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference. The case of rotating frame of reference of the Mössbauer rotor experiment is a particular case. This will permit us to reanalyse the Mössbauer rotor experiment 's theoretical framework directly in the rotating frame of reference through a full general relativistic treatment [16, 17]. We will show that, in previous analyses in the literature, an important effect of clock synchronization has been missed. Thus, the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ [16, 17], which is in perfect agreement with the new experimental results in [2]. In other words, the general relativistic interpretation that we review in this paper shows that the new experimental results of the Mössbauer rotor experiment in [2] are a new, strong and independent, proof of the GTR. We must also stress emphasize that various papers in the literature (included ref. [4] published in Phys. Rev. Lett.) missed the additional effect of clock synchronization [1–8], [11–13]. This generated some claim of invalidity of relativity theory and/or some attempts to explain the experimental results through non-conventional or exotic effects [1, 2, 11, 12, 13].

If one follows [9, 16, 17], a transformation from an inertial frame, in which the space-time is Minkowskian, to a rotating frame of reference in cylindrical coordinates, can be used. In the starting inertial frame, we have the line-element [9, 16, 17]

$$ds^2 = c^2 dt^2 - dr^2 - r^2\, d\phi^2 - dz^2. \tag{2}$$

Let us consider the following transformation to a frame of reference $\{t', r', \phi' z0\}$ which rotates with an uniform angular rate ω with respect to the starting inertial frame [9, 16, 17]

$$t = t' \quad r = r' \qquad \phi = \phi' + \omega t' \quad z = z'. \tag{3}$$

Eq. (2) becomes the famous Langevin metric in the rotating frame [9, 16, 17]

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2\, dt'^2 - 2\omega r'^2 d\phi' dt' - dr'^2 - r'^2 d\phi'^2 - dz'^2. \tag{4}$$

Despite it is simple to grasp, the transformation (3) is highly illustrative of the GTR general covariance. It indeed shows that one can start to work in a "simpler" frame and then transforming to a more "complex" one [16, 17]. We consider light propagating in the radial direction ($d\phi' = dz' = 0$) [16, 17]. Hence, the line element (4) becomes [16]

$$ds^2 = \left(1 - \frac{r'^2\omega^2}{c^2}\right) c^2 dt'^2 - dr'^2. \tag{5}$$

Through the EP, one interprets the line element (5) in terms of a curved spacetime in presence of a static gravitational field [10, 15, 16, 17]. This gives a pure general relativistic interpretation of the pseudo-force that an observer in a rotating, non-inertial frame of reference experiences [16, 17]. We set the origin of the rotating frame in the source of the emitting radiation [16, 17]. Thus, we obtain a first contribution arising from the gravitational redshift [16, 17]. This is easily computed using Eq. (25.26) in [10], which, in the twentieth printing 1997 of [10], reads

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}} = | g_{00}(r_1') |^{-\frac{1}{2}} - 1. \tag{6}$$

This equation gives the redshift of a photon emitted by an atom at rest in a gravitational field and received by an observer at rest at infinity. Here, we will use a slightly different equation with respect to Eq. (25.26) in [10]. In fact, here we consider a gravitational field which increases with increasing radial coordinate r'. Instead, Eq. (25.26) in [10] concerns a gravitational field which decreases with increasing radial coordinate [16, 17]. In addition, we set the zero potential in $r' = 0$ rather than at infinity, and we use the proper time τ rather than the wavelength λ [16, 17]. Therefore, from Eq. (5), we have [16, 17]

$$z_1 \equiv \frac{\nabla\tau_{10} - \nabla\tau_{11}}{\tau} = 1 - | g_{00}(r_1') |^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{1 - \frac{(r_1')^2\omega^2}{c^2}}}$$

$$= 1 - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \simeq -\frac{1}{2}\frac{v^2}{c^2}. \tag{7}$$

Here, $\nabla\tau_{10}$ is the delay of the emitted radiation, $\nabla\tau_{11}$ is the delay of the received radiation, $r'_1 \simeq c\tau$ is the radial distance between the source and the detector and $v = r'_1\omega$ is the tangential velocity of the detector [16, 17]. Then we have the first contribution, say $k_1 = \frac{1}{2}$, to k [16, 17]. Let us again emphasize that it is the power of the EP which enabled us to use a pure general relativistic treatment in previous analysis [16, 17].

Now, let us consider the following key point [16, 17]. We computed the variations of proper time $\nabla\tau_{10}$ and $\nabla\tau_{11}$ in the origin of the rotating frame which is, in turn, located in the source of the radiation [16, 17]. On the other hand, the detector moves with respect to the origin in the rotating frame [16, 17]. Hence, the clock in the detector needs to be synchronized with the clock in the origin. We will see that this gives a second, additional, contribution to the redshift [16, 17]. This new contribution has been missed in previous analyses [1–8], [11–13]. In order to calculate this additional contribution, we use Eq. (10) of [9]. Such an equation gives the proper time increment $d\tau$ on the moving clock having radial coordinate r' for values $v \ll c$

$$d\tau = dt'\left(1 - \frac{r'^2\omega^2}{c^2}\right) \tag{8}$$

Now, let us insert the condition of null geodesics $ds = 0$ in Eq. (5). The result is [16, 17]

$$cdt' = \frac{dr'}{\sqrt{1 - \frac{r'^2\omega^2}{c^2}}}. \tag{9}$$

We chose the positive sign in the square root because the radiation is propagating in the positive r direction [16, 17]. Combining eqs. (8) and (9), we get [16, 17]

$$cd\tau = \sqrt{1 - \frac{r'^2\omega^2}{c^2}}dr'. \tag{10}$$

One well approximates Eq. (10) as [16, 17]

$$cd\tau \simeq \left(1 - \frac{1}{2}\frac{r'^2\omega^2}{c^2} + ...\right)dr'. \tag{11}$$

This last equation enables to find the second contribution of order $\frac{v^2}{c^2}$ to the variation of proper time as [16, 17]

$$c\nabla\tau_2 = \int_0^{r_1'}\left(1 - \frac{1}{2}\frac{(r_1')^2\omega^2}{c^2}\right)dr' - r_1' = -\frac{1}{6}\frac{(r_1')^3\omega^2}{c^2} = -\frac{1}{6}r_1'\frac{v^2}{c^2}. \tag{12}$$

Recalling that the radial distance between the source and the detector is given by $r_1' \simeq c\tau$, the additional contribution of order $\frac{v^2}{c^2}$ to the redshift is [16, 17]

$$z_2 \equiv \frac{\nabla \tau_2}{\tau} = -k_2 \frac{v^2}{c^2} = \frac{1}{6} \frac{v^2}{c^2}. \tag{13}$$

Then, one obtains $k_2 = \frac{1}{6}$. Thus, Eqs. (7) and (13) permit to obtain the total redshift as [16, 17]

$$z \equiv z_1 + z_2 = \frac{\nabla \tau_{10} - \nabla \tau_{11} + \nabla \tau_2}{\tau} = -(k_1 + k_2) \frac{v^2}{c^2} \tag{14}$$

$$= -\left(\frac{1}{2} + \frac{1}{6}\right) \frac{v^2}{c^2} = -k \frac{v^2}{c^2} = -\frac{2}{3} \frac{v^2}{c^2} = 0.\overline{6} \frac{v^2}{c^2}.$$

This last result is completely consistent with the result $k = 0.68 \pm 0.03$ in [2].

It is important to notice that the additional factor $-\frac{1}{6}$ in Eq. (13) arises from clock synchronization [16, 17]. Therefore, its theoretical absence in [1–8], [11–13] implies the incorrect comparison of clock rates between a clock at the origin and one at the detector [16, 17]. Some strange consequences have been wrong claims of invalidity of relativity theory and/or some attempts to explain the experimental results in [2] through non-standard and/or "exotic" effects [1, 2, 11, 12, 13]. Instead, such strange claims and attempts must be rejected. We stress that, also in the discussion of the effect of clock synchronization, a pure general relativistic treatment has been used.

Ref. [9] has been evoked for a discussion of the Langevin metric. This line-element is often used to take into account the GTR effects in Global Positioning Systems (GPS). Thus, one arrives to the following interesting realization [16, 17]. The additional component of $-\frac{1}{6}$ in Eq. (13) to the total value of k is analogous to the correction that one considers in GPS when one needs to account for the difference between the time measured in a frame co-rotating with the Earth geoid and the time measured in a non-rotating (locally inertial) Earth centred frame [16, 17]. The same works for the difference between the proper time of an observer at the surface of the Earth and at in"nity [16, 17]. In fact, if we simply consider the gravitational redshift due to the Earth's gravitational field, without considering the effect of the Earth's rotation, GPS cannot work

[16, 17]! The lesson here is that the proper time elapsing on the orbiting GPS clocks cannot be simply used to transfer time from one transmission event to another. Path-dependent effects must be indeed taken into due account. The same happened in the above discussion of clock synchronization [16, 17]. This means that the additional value $-\frac{1}{6}$ in Eq. (13) must not be considered as being an obscure mathematical or physical detail. It is, instead, a fundamental ingredient that one must take into due account [16, 17]. If the reader needs further details on the analogy between the results presented in this review paper and the use of the GTR in GPS, we suggest ref. [16].

CONCLUSION REMARKS

In this review paper, it has been shown that the power of the EP, stating the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference), can be used to reanalyse, from a pure general relativistic point of view, the theoretical basis of the new Mössbauer rotor experiment in [2], directly in the rotating frame of reference. Our results show that previous analyses missed an additional effect of clock synchronization. Thus, by adding this effect, the correct general relativistic prevision gives $k \simeq \frac{2}{3}$, in perfect agreement with the new experimental results in [2]. Therefore, in this review paper we have shown that the general relativistic interpretation of the new experimental results of the Mössbauer rotor experiment must be considered a new, strong and independent, proof of Einstein's GTR. The importance of the results presented in this review paper is emphasized by the issue that various papers in the literature (included ref. [4] published in Phys. Rev. Lett.) missed the effect of clock synchronization [18], [1113]. This generated strange claims of invalidity of relativity theory and/or some attempts to explain the experimental results in [2] through non-standard and/or exotic effects [1, 2, 11, 12, 13]. Instead, such strange claims and attempts must be ultimately rejected. Remarkably, our results on the Mössbauer rotor experiment received an Honorable Mention at the Gravity Research Foundation 2018 Awards for Essays on Gravitation [17].

Acknowledgements

It is a plesure thanking the EIC of NAP, Prof. B. G. Sidharth, for inviting us to write this review paper.

References

[1] A. L. Kholmetskii, T. Yarman and O. V. Missevitch, Phys. Scr. 77, 035302 (2008).

[2] A. L. Kholmetskii, T. Yarman, O.V. Missevitch and B. I. Rogozev, Phys. Scr. 79, 065007 (2009).

[3] W. Kündig, Phys. Rev. 129, 2371 (1963).

[4] H. J. Hay *et al,* Phys. Rev. Lett. 4, 165 (1960).

[5] H. J. Hay, in Proc. 2nd Conf. Mössbauer Effect, ed A Schoen and D M T Compton (New York: Wiley) p 225 (1962).

[6] T. E. Granshaw and H. J. Hay, in Proc. Int. School of Physics, 'Enrico Fermi' (New York: Academic) p 220 (1963).

[7] D. C. Champeney and P. B. Moon, Proc. Phys. Soc. 77, 350 (1961).

[8] D. C. Champeney, G. R. Isaak and A. M. Khan, Proc. Phys. Soc. 85, 583 (1965).

[9] N. Ashby, Liv. Rev. Rel. 6, 1 (2003).

[10] C. W. Misner , K. S. Thorne, J. A. Wheeler, Gravitation, Feeman and Company (1973).

[11] A. Kholmetskii, T. Yarman, O. Missevitch, Eur. Phys. Jour. Plus, 128, 42 (2013).

[12] V. O. de Haan, Jour. Comp. Meth. Scien. Eng., 13, 51 (2013).

[13] A. L. Kholmetskii, T. Yarman, M. Arik and O.V. Missevitch, AIP Conf. Proc. 1648, 510011 (2015).

[14] R. L. Mössbauer, Zeitschrift für Physik A (in German) 151, 124 (1958).

[15] L. Landau and E. Lifsits, Classical Theory of Fields (3rd ed.), London: Pergamon (1971).

[16] C. Corda, Ann. Phys. 355, 360 (2015).

[17] C. Corda, accepted for publication in Int. Journ. Mod. Phys. D, pre-print in arXiv:1805.06228.