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# A MULTI-CRITERIA DECISION METHOD USING FUZZY TOPSIS TECHNIQUE FOR RANKING COUNTRIES BASED ON ICT DEVELOPMENT INDEX (IDI)

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## 1. INTRODUCTION

In 2008, United Nations International Telecommunication Union (ITU) published a series of annual ICT Development reports by the name of "Measuring the Information Society" in which the ICT Development Index(IDI) is computed for each country as an index to measure and compare development in Information and Communication Technology (ICT) performance within and across countries and it was first presented in 2009. The IDI was formerly known as the Digital Opportunity Index (DOI).

IDI is computed from the average of the three access, use and skills dimension indices with different allocated weight- which are calculated by the method of Fuzzy entropy Shannon- and this article uses one method of Multi Criterion Decision Making (MCDM) method, Fuzzy TOPSIS, as an alternative of average method for ranking countries in terms of ICT development.

In connection with the point previously mentioned, section 2 describes the three dimensions and the eleven indicators forming the IDI, then the methodology used to calculate the IDI in 2010 and 2011 international IDI reports is introduced. Then we mention some of important criticisms the IDI has been encountered during the past years. In this paper, the assessments of alternatives are described as Fuzzy numbers, therefore we use related numbers for each criterion and each country for three years of 2007, 2008 and 2010. In section 3 we give an introduction to Fuzzy numbers. In section 4, first the Fuzzy entropy Shannon approach for determining the weight of each criterion, then the Fuzzy TOPSIS method for ranking the alternatives are described in detail. A numerical example is illustrated in section 5 with the conclusion in section 6.

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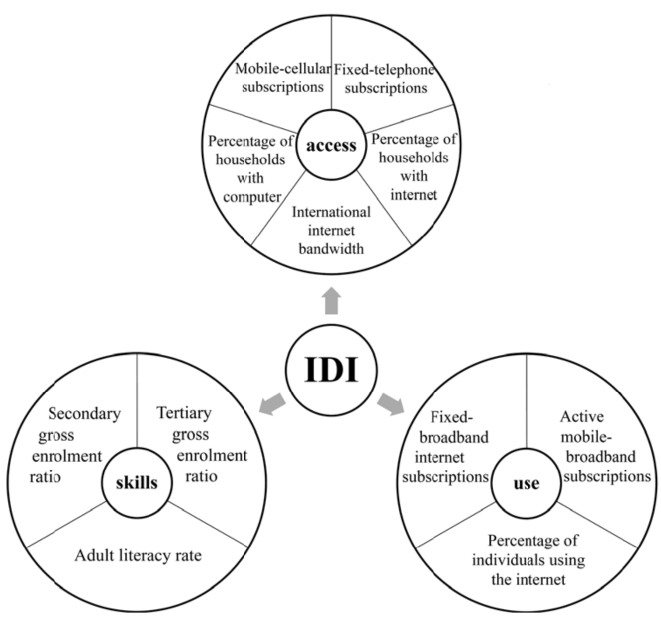
## 2. THE ICT DEVELOPMENT INDEX

The ICT Development Index is based on 11 ICT indicators, classified into 3 sub-indices: Access, Use and Skills.

- The access sub-index describes ICT readiness for each country of the world and includes five underpinning and access indicators as the following: Fixed-telephone subscriptions per 100 inhabitants, Mobile-cellular subscriptions per 100 inhabitants, International internet bandwidth Bit/S per internet user, Percentage of households with computer and Percentage of households with the internet.
- The use sub-index explains ICT intensity of each country and consists of 3 intensity and usage indicators as the following: Percentage of individuals using the internet, Fixed (wired)-broadband internet subscriptions per 100 inhabitants and Active mobile-broadband subscriptions per 100 inhabitants.
- The skills sub-index shows ICT skills or ability of each country and has 3 proxy indicators as the following: Secondary gross enrolment ratio, Tertiary gross enrolment ratio and Adult literacy rate[1].

Figure 1 shows sub-indices and indicators of the IDI:

**Figure 1: Sub-indices and 11 indicators of IDI**



### 2.1. The IDI Methodology

This section outlines the methodology in the "measuring the information society" reports to compute the IDI, and provide some details on different steps involved. The IDI is a combined index of three different dimensions measured by the ITU to rank countries in terms of ICT development:

First of all it is important to understand on what basis the indicators are selected for this index. The indicators are selected based on some specific criteria, including pertinence for the IDI objectives, accessibility of data and the results of diverse statistical analyses such as the Principal Component Analysis (PCA).[1] According to the reports, Every 11 indicators of the IDI are first normalized and put on an interval between o and 1 in order to ensure that the data set uses the same unit of measurement, because some of the indicators are represented as a percentage of the household or population, with a maximum number of 100, while others may have values more than 100, such as mobile-cellular subscriptions or international internet bandwidth. The selected normalization method for the IDI was "distance to a reference measure". The reference method is the ideal value that could be reached for each variable[2]. The rescaled indicator is:

$$Standardised\ value = \frac{actual\ value}{ideal\ value} \tag{1}$$

For all indicators the ideal value is 100 except for four indicators including International internet bandwidth Bit/S per internet user, Mobile-cellular subscriptions per 100 inhabitants, Fixed-telephone subscriptions per 100 inhabitants and Fixed (wired)-broadband internet subscriptions per 100 inhabitants which was calculated by adding 2 standard deviations to the mean value of the indicator. The ideal values for these 4 indicators are 280’377, 170, 60 and 60 respectively. For the indicator "International Internet Bandwidth bit/s per Internet User" the data were first converted to a logarithmic (log) scale to decrease the effect of the large number of outliers at the high end of the value scale, therefore the ideal value of 280’377 bit/s per internet user is equivalent to 5.45 when transformed to a log scale[2].

In the next step, every standard value for each indicator is multiplied by its weight. The weights are based on the PCA results obtained when the index was first computed.[2] Weights used for indicators and sub-indices included in the IDI are as the following:

1. Principal Component Analysis was used to examine the fundamental nature of the data. A moredetailed description of the analysis is available in annex 1 to ITU (2009).
2. For more details, see annex 1 to ITU (2009)

**Table 1**
**Weights used for indicators and sub-indices included in the IDI according to IDI reports**

| | *Weights (indicators)* | *Weights (sub-index)* |
|---|---|---|
| **ICT access** | | |
| Fixed-telephone subscriptions per 100 inhabitants | 0.2 | 0.4 |
| Mobile-cellular subscriptions per 100 inhabitants | 0.2 | |
| International internet bandwidth Bit/S per internet user | 0.2 | |
| Percentage of households with computer | 0.2 | |
| Percentage of households with internet | 0.2 | |
| **ICT use** | | |
| Percentage of individuals using the internet | 0.33 | 0.4 |
| Fixed (wired)-broadband internet subscriptions per 100 | 0.33 | |
| inhabitants | 0.33 | |
| Active mobile-broadband subscriptions per 100 inhabitants | | |
| **ICT skills** | | |
| Secondary gross enrolment ratio | 0.33 | 0.2 |
| Tertiary gross enrolment ratio | 0.33 | |
| Adult literacy rate | 0.33 | |

*Source:* ITU Report (2011)

In the next step, the numbers calculated for each indicator in the previous step are added together separately for every sub-index and then each 3 computed sub-index numbers are multiplied by their weight.

Finally the calculated numbers in the former step are added together and the result is multiplied by 10 to rescale on a scale from 1-10 in order to compare the values of the indicators and sub-indices between all countries.

## 2.2. Criticisms of the IDI

The IDI has become one of the most extensively used indicators for comparison of ICT development between countries because of its multidimensional calculation. However the IDI has received some criticisms over the past few years.

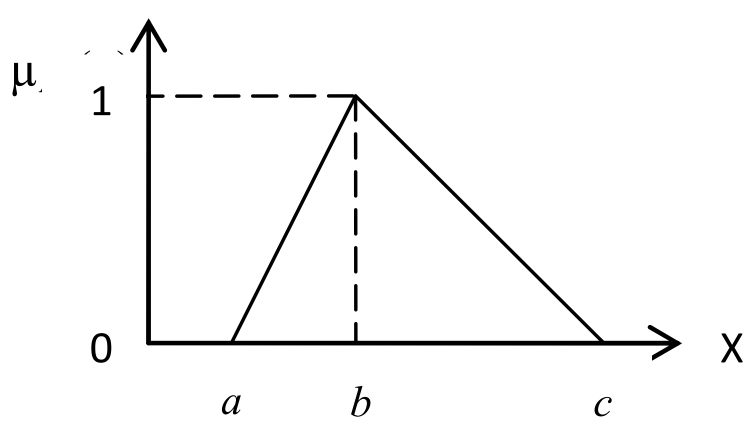
Jeffrey James (2010) criticized the IDI methodology in two ways. In the first place, he argued that the ICT Development Index is not suitable for the purpose of comparing countries, because it concentrates on double-counting of input and output, confuses means and ends, adds dependent and independent variables together and adds its component parts rather than multiply them. For instance, consider the entry ‘percentage of households with a computer’,it is really an input to the number of 'Internet users per 100 inhabitants'. The confusion of means and ends has some highly undesirable properties that are likely to cause policy mistakes of various kinds. The clearest undesirable property is that countries with the same usage of information technology will be ranked according to the inputs of the process (such as computers or Internet connections). In particular, countries with a relatively high ratio of input to outputs will be ranked above countries that make more effective use of their inputs. The second weakness of IDI methodology is that the entries in the table 2 are largely obvious. What is not clear however is why the first two entries, Fixed-telephone subscriptions per 100 inhabitants and Mobile-cellular subscriptions per 100 inhabitants,appear under 'access' while they fall clearly in the use category[3].

Another group of criticisms is addressed in this study. In IDI methodology obtained from IDI reports, weighted average is applied for calculating the IDI number for each country. In our opinion, weighted average is not necessarily the best way to measure the final state because it is not an accurate method. Because of this, a new technique is necessary to be introduced. In this paper we rank the selected countries using Fuzzy TOPSIS technique.

## 3. PRELIMINARIES IN FUZZY

The Fuzzy sets theory, which was introduced by Zadeh (1965), is related to such problems with uncertain and imprecise data[4]. In this paper we use triangle fuzzy numbers in our calculations. Triangle fuzzy number is a certain type of fuzzy set. A triangle fuzzy number can be denoted by a triplet as $\tilde{A}=(a,b,c)$. Fuzzy number, $\tilde{A}$, is defined by a membership function $\mu\tilde{A}(x)$. Figure 1 is a diagram of membership function of a triangle fuzzy number.

**Figure 2: Triangle fuzzy number, Ã**



The membership function of fuzzy number $\mu_{\tilde{A}}$ (x) is defined as:

$$U(x)= \begin{cases} \frac{x-a}{b-a}, & if \quad a \le x \le b; \\ \frac{c-x}{c-b}, & if \quad b \le x \le c; \\ 0, & if \quad else \end{cases} \tag{2}$$

For this Fuzzy number, $(x-a)/(b-a)$ is defined as left membership function ($f_A^L$) and $(x-c)/(b-c)$ is right membership function ($f_A^R$).

If we consider $\tilde{A}=(a_1,b_1,c_1)$, where $a_{1<}b_{1<}c_{1,}$ and $\tilde{I}=(a_2,b_2,c_2)$, where $a_{2<}b_{2<}c_{2,}$ as two fuzzy numbers and $\lambda$ as a real number, some main algebraic operations of fuzzy numbers can be shown as follows:

$$\tilde{A}\oplus\tilde{I} = (a_1+a_2,b_1+b_2,c_1+c_2) \tag{3}$$

$$\tilde{A}-\tilde{I} = (a_1-a_2,b_1-b_2,c_1-c_2) \tag{4}$$

$$\tilde{A}\oplus\tilde{I} = (a_1\times a_2,b_1\times b_2,c_1\times c_2), \text{ if } a_{1\geq 0,}\ a_{2\geq 0} \tag{5}$$

$$\lambda\otimes\tilde{I}= \begin{cases} (\lambda\times a_2,\lambda\times b_2,\lambda\times c_2) & ,\lambda \geq 0 \\ (\lambda\times c_2,\lambda\times b_2,\lambda\times a_2) & ,\lambda<0 \end{cases} \tag{6}$$

$$\tilde{A}\varnothing\tilde{I} = \left(\frac{a_1}{c_2},\frac{b_1}{b_2},\frac{c_1}{a_2}\right) \text{ ,if } a_{1\geq 0,}\ a_{2\geq 0} \tag{7}$$

Diamond in 1988 [5] developed a method in order to define the distance between fuzzy numbers. Let A = (a,b,c) and I = (d,e,f) be fuzzy numbers. Then the distance square between A and I is:

$$d^2(A,I) = (Center\ Difference)^2 + (Left-Side\ Difference)^2 + (Right-Side\ Difference)^2 = (b-e)^2 + [(b-e)-(a-d)]^2 + [(b-e)-(c-f)]^2 \tag{8}$$

Chen in 2000 [6], defined the distance between A and I as:

$$d(A,I) = \left\{\frac{1}{3}\left[(a-d)^2+(b-e)^2+(c-f)^2\right]\right\}^{1/2} \tag{9}$$

According to the Lee and Li in 1988 [7] mean and standard deviation of the triangular fuzzy number A = (a,b,c) are defined as:

$$\mu_A = \frac{1}{4}(a + 2b + c) \tag{10}$$

$$\sigma_A = \frac{1}{80}\left(3a^2 + 4b^2 + 3c^2 - 4ab - 4bc - 2ac\right) \tag{11}$$

## 4. RESEARCH METHODOLOGY

In this paper the weights of each 11 criteria of IDI are calculated by the use of Fuzzy Shannon's Entropy. After that, in order to rank the alternatives, which are some selected countries, Fuzzy TOPSIS is utilized. Finally, we compare the ranks of countries using mentioned method with the published reports of ITU by the name of "measuring the information society" for the years 2007, 2008 and 2010.

### 4.1. The Fuzzy Shannon's Entropy

In this paper we prioritize the indicators constructing the IDI by using Fuzzy Shannon's Entropy. The steps for Fuzzy Shannon's Entropy are explained as follows:

**Step 1.** Construct the fuzzy decision matrix

$$FDM = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ . & . & \dots & . \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix}; \ \tilde{W} = [w_1, w_2, \dots, w_n], X_{ij} = [a_{ij}, b_{ij}, c_{ij}], \quad i = 1,2,\dots,m, \quad j = 1,2,\dots,n \tag{12}$$

Where $W_i$ is the weight for each criterion and $X_{ij}$ are all fuzzy numbers.

**Step 2.** Construct the normalized fuzzy decision matrix (NFDM).

In this paper the raw data are normalized using the following method in order to bring the different criteria scales into a comparable scale between 0 and 1. The normalized fuzzy decision matrix is given by:

$$\tilde{u}_{ij} = \left( \frac{a_j}{\sum_{j=1}^{n} c_j}, \frac{b_j}{\sum_{j=1}^{n} c_j}, \frac{c_j}{\sum_{j=1}^{n} c_j} \right) \tag{13}$$

**Step 3.** Compute entropy $E_i$ as the following:

$$E_i = -k \sum_{j=1}^{m} u_{ij} \ln u_{ij}, \qquad \mathrm{i} = 1,2,\dots,\mathrm{n} \tag{14}$$

Where k is the entropy constant which is equal to $(\ln m)^{-1}$ and $u_{ij} \ln u_{ij}$ is defined as 0 if $u_{ij} = 0$.

**Step 4.** Calculate the degree of diversification $D_i$ as bellow:

$$D_i = 1 - E_i \tag{15}$$

**Step 5.** Set $W_t$ as the degree of attribute i according to following formula:

$$W_i = \frac{D_i}{\sum_{s=1}^{n} D_s}, \qquad i = 1,2,\dots,n \tag{16}$$

**Step 6.** Set $W_t'$ where λ for each criterion is shown in table (2) which are extracted from IDI annual reports.

$$W_i' = \lambda \otimes W_i, \qquad i = 1,2,\dots,n \tag{17}$$

**Step 7.** Calculate the final weight of each criterion based on the following formula:

$$W_i'' = \frac{W_i'}{\sum_{s=1}^{n} W_s'}, \qquad i = 1,2,\dots,n \tag{18}$$

### 4.2. The Fuzzy TOPSIS Method

TOPSIS is one of the most classical methods for solving MCDM problems and was first developed by Hwang and Yoon and was used in determining the ranking of the countries based on different indices [8]. It is based on the principles that the chosen alternative should have the longest distance from the negative-ideal solution, and the shortest distance from the positive-ideal solution. In classical TOPSIS the rating and weight of the criteria are known precisely[9]. However, under many real situations crisp data are inadequate to model real life situation since human judgments are vague and cannot be estimated with exact numeric values[10]. To solve the problem of vagueness and uncertainty existing in information from human judgments, fuzzy set theory has been used in many MCDM methods including TOPSIS.

Chen and Hwang in 1992 [11] first used fuzzy numbers to introduce fuzzy TOPSIS for the first time. Triantaphyllou and Lin (1996)[12] developed a specific kind of fuzzy TOPSIS method in which pertinent closeness for each alternative is calculated using fuzzy arithmetic operations.

The various steps of fuzzy TOPSIS are presented as follows:

**Step 1:** Determine the weighting of evaluation criteria

Assume that a committee of $k$ decision makers ($D^1, D^2, \ldots, D^k$) is responsible for evaluating $m$ alternatives ($A_1, A_2, \ldots, A_m$) under $n$ criteria ($C_1, C_2, \ldots, C_n$). Criteria are classified into benefit (B) and cost (C). Suppose $x_{ij}^t = (a_{ij}^t, b_{ij}^t, C_{ij}^t)$; $x_{ij}^t \in R^+$; $i = 1,2,\ldots,m;$ $j = 1, 2,\ldots,n;$ $t = 1,2,\ldots k,$ is a triangular fuzzy number and the score assigned to alternative $A_i$ by decision maker $D^t$ for criterion $C_j$. In addition, let $w_j^t = (e_j^t, f_j^t, g_j^t)$; $w_j^t \in R^+$; $j = 1,2,\ldots,n;$ $t = 1,2,\ldots,k$ is a triangular fuzzy number and the weight assigned to criterion $C_j$ by decision maker $D^t$ (Dr. Safari)

**Step 2:** Construct the fuzzy decision matrix

$$FDM = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ . & . & \ldots & . \\ x_{m1} & x_{m2} & \ldots & x_{mn} \end{bmatrix} ; \ \widetilde{W} = [w_1, w_2, \ldots, w_n] \tag{19}$$

Where $W_i$ is the weight for each criterion.

**Step 3:** Normalize the fuzzy decision matrix

The normalized fuzzy decision matrix denoted by $\tilde{U}$ is shown as the following[13]:

Before any calculation we normalize all data in order to ensure that the data set uses the same unit of measurement.

$$\begin{aligned} c_j^+ &= \underset{i}{Max}\, c_{ij} \quad , \quad j \in B; \\ a_j^- &= \underset{i}{Min}\, a_{ij} \quad , \quad j \in C; \\ \widetilde{u}_{ij} &= \left( \frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right) \quad , \quad j \in B; \\ \widetilde{u}_{ij} &= \left( \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right) \quad , \quad j \in C \end{aligned} \qquad (20$$

Using the normalization method mentioned above, all the triangle fuzzy numbers are rescaled to similar ranges, from 0 to 1.

**Step 4:** Construct the Weighted Normalized Fuzzy Decision Matrix

The Weighted Normalized Fuzzy Decision Matrix can be shown as the following equation via matrix $\tilde{V}$:

$\tilde{V} = [\tilde{V}_{ij}]_{m\times n}$, i = 1,2,…,n; j=1,2,…,m

$$\tilde{V} = \tilde{U}\ (\times)\tilde{W} \tag{21}$$

**Step 5:** Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS)

As the members of $\tilde{V}$ are normalized positive triangular fuzzy numbers and belong to the range of interval [0, 1], the fuzzy positive-ideal solution ($S^+$) and the fuzzy negative-ideal solution ($S^-$) can be determined as:

$$\begin{aligned} S^+ &= \left(\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+\right), \tilde{v}_j^+ = \left(\underset{i}{Max}\,\tilde{v}_j^a\ ,\ \underset{i}{Max}\,\tilde{v}_j^b\ ,\ \underset{i}{Max}\,\tilde{v}_j^c\right) \\ S^- &= \left(\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\right), \tilde{v}_j^- = \left(\underset{i}{Min}\,\tilde{v}_j^a\ ,\ \underset{i}{Min}\,\tilde{v}_j^b\ ,\ \underset{i}{Min}\,\tilde{v}_j^c\right) \end{aligned} \tag{22}$$

**Step 6:** Calculate the distance of each alternative from FPIS and FNIS

In this step the distance of each alternative from $S^+$ and $S^-$ is determined using the following formula:

$$\begin{aligned} d_i^+ &= \sum_{j=1}^{n} d\left(\tilde{v}_{ij}\ ,\tilde{v}_j^+\right),\ \text{i=1,2,…,m; j=1,2,…,n} \\ d_i^- &= \sum_{j=1}^{n} d\left(\tilde{v}_{ij}\ ,\tilde{v}_j^-\right),\ \text{i=1,2,…,m; j=1,2,…,n} \end{aligned} \tag{23}$$

Where $d_i^+$ implies the distance between each alternative and ideal positive solution and $d_i^-$ implies the distance between each alternative and ideal negative solution. In this paper, the Diamond method (Equation 8) is used to calculate the distance of each alternative from fuzzy positive and negative ideal solutions. The Diamond method is as the following:

$$\begin{aligned} d(\tilde{v}_{ij}, \tilde{v}_j^+) = \Big\{ &\Big[\tilde{v}_j^b - \underset{i}{Max}\ \tilde{v}_j^b\Big]^2 + \\ &\Big[\tilde{v}_j^b - \underset{i}{Max}\ \tilde{v}_j^b\Big]^2 + \Big[(\tilde{v}_j^b - \underset{i}{Max}\ \tilde{v}_j^b) - (\tilde{v}_j^a - \underset{i}{Max}\ \tilde{v}_j^a)\Big]^2 + \\ &\Big[(\tilde{v}_j^b - \underset{i}{Max}\ \tilde{v}_j^b) - (\tilde{v}_j^c - \underset{i}{Max}\ \tilde{v}_j^c)\Big]^2 \Big\}^{1/2} \end{aligned} \tag{24}$$

$$d(\tilde{v}_{ij}, \tilde{v}_j^-) = \Big\{ \big[ \tilde{v}_j^b - \underset{i}{Min}\ \tilde{v}_j^b \big]^2 + \big[ (\tilde{v}_j^b - \underset{i}{Min}\ \tilde{v}_j^b) - (\tilde{v}_j^a - \underset{i}{Min}\ \tilde{v}_j^a) \big]^2 + \big[ (\tilde{v}_j^b - \underset{i}{Min}\ \tilde{v}_j^b) - (\tilde{v}_j^c - \underset{i}{Min}\ \tilde{v}_j^c) \big]^2 \Big\}^{1/2}$$

**Step 7:** Compute the closeness coefficient ($CC_i$) of each alternative:

The closeness coefficient is defined to determine the ranking order of all alternatives once the $d_i^+$ and $d_i^-$ of each alternative $A_i$ have been calculated. The closeness coefficient shows the distances between the fuzzy positive ideal solution and the fuzzy negative ideal solution concurrently. The closeness coefficient of each alternative is calculated as [14]:

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, \; i = 1, 2, ..., m \tag{25}$$

As it is obvious from the above equation, the closer an alternative $A_i$ is to the $S^+$ and the farther it is from $S^-$, the more $S_i$ approaches to 1.

**Step 8:** Ranking the alternatives to select the best one

According to the closeness coefficient, which was computed in the previous step, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives[14,15].Then the optimal alternative can be ranked by a decision maker.

## 5. DATA ANALYSIS

As mentioned before, in the first place, we determine the weight of each criterion using Fuzzy Shannon's Entropy, then we apply the Fuzzy TOPSIS technique for ranking countries in terms of IDI mentioned in the 2009, 2010 and 2011 reports of IDI. In this part, we demonstrate the application of these methods by numerical examples. In our calculations there are 69 alternatives (countries) and 11 criteria. Data are taken from IDI reports of 3 years of 2007, 2008 and 2010. The criteria include Fixed-telephone subscriptions per 100 inhabitants ($C_1$), Mobile-cellular subscriptions per 100 inhabitants ($C_2$), International internet bandwidth Bit/S per internet user ($C_3$), Percentage of households with computer ($C_4$), Percentage of households with the internet ($C_5$), Percentage of individuals using the internet ($C_6$), Fixed (wired)-broadband internet subscriptions per 100 inhabitants ($C_7$), Active mobile-broadband subscriptions per 100 inhabitants ($C_8$), Secondary gross enrolment ratio ($C_9$), Tertiary gross enrolment ratio ($C_{10}$), and Adult literacy rate

($C_{11}$). In addition, alternatives include 69 countries which were investigated in the 2009, 2010 and 2011 IDI reports.

### 5.1. Fuzzy Shannon's Entropy

**Step 1.** First we determine the fuzzy decision matrix. Data are taken from the 2007, 2008 and 2010 reports of IDI. Data are fuzzy triangle numbers in the form of (a,b,c), where the first, second and third components display the 2009, 2010 and 2011 related numbers in IDI reports, respectively. The aggregate fuzzy decision matrix (FDM) for Shannon's Entropy is as below:

**Table 2**
**Fuzzy Decision Matrix**

| *FDM* | $C_1$ | $C_2$ | $C_3$ | … | $C_{11}$ |
|---|---|---|---|---|---|
| Australia | (38.9, 43.6, 47.1) | (101, 102.5, 102.8) | (8035, 9728, 41361) | … | (99, 99, 99) |
| Austria | (38.7, 39.4, 40.8) | (18.6, 129.7, 145.8) | (30116, 41127, 73744) | … | (99, 99, 99) |
| ⋮ | ⋮ | ⋮ | ⋮ | … | ⋮ |
| Venezuela | (18.4, 22.9, 24.4) | (86.1, 96.2, 97.7) | (3016, 5509, 6779) | … | (94.5, 95.2, 95.2) |

**Step 2.** In this step we normalize the fuzzy decision matrix based on equation (13). The normalized fuzzy decision matrix (NFDM) is as below:

**Table 3**
**Normalized Fuzzy Decision Matrix**

| *NFDM* | $C_1$ | $C_2$ | $C_3$ | … | $C_{11}$ |
|---|---|---|---|---|---|
| Australia | (0.0167, 0.0187, 0.0202) | (0.0122, 0.0124, 0.0124) | (0.0006, 0.0007, 0.0031) | … | (0.015, 0.015, 0.015) |
| Austria | (0.0166, 0.0169, 0.0175) | (0.0022, 0.0157, 0.0176) | (0.0022, 0.0031, 0.0055) | … | (0.015, 0.015, 0.015) |
| ⋮ | ⋮ | ⋮ | ⋮ | … | ⋮ |
| Venezuela | (0.0079, 0.0098, 0.0105) | (0.0104, 0.0116, 0.0118) | (0.0002, 0.0004, 0.0005) | … | (0.0143, 0.0144, 0.0144) |

**Step 3.** In this step we calculate $E_i$ for each criterion according to equation (14) as follows:

**Table 4**
**Entropy $E_i$**

| | $C_1$ | $C_2$ | $C_3$ | … | $C_{11}$ |
|---|---|---|---|---|---|
| $E_i$ | (0.8765, 0.9254, 0.9691) | (0.8340, 0.9114, 0.9938) | (0.1121, 0.2058, 0.3783) | … | (0.9887, 0.9953, 0.9992) |

**Step 4.** In this step we calculate $D_i$ for each criterion according to equation (15) as follows:

**Table 5**
**Degree of diversification $D_i$**

| | $C_1$ | $C_2$ | $C_3$ | … | $C_{11}$ |
|---|---|---|---|---|---|
| $D_i$ | (0.1235, 0.0746, 0.0309) | (0.1660, 0.0886, 0.0062) | (0.8879, 0.7942, 0.6217) | … | (0.0113, 0.0047, 0.0008) |

**Step 5.** In this step we calculate $W_i$ for each criterion according to equation (16) as follows:

**Table 6**
**Degree of attribute Wi**

| | $C_1$ | $C_2$ | $C_3$ | … | $C_{11}$ |
|---|---|---|---|---|---|
| $W_i$ | (0.0420, 0.036, 0.0342) | (0.0565, 0.0427, 0.0069) | (0.3021, 0.3832, 0.6890) | … | (0.0039, 0.0023, 0.0009) |

**Step 6.** In this step we calculate $W'_t$ for each criterion according to equation (17) as follows:

**Table 7**
**Initial weight of each criterion**

| | $C_1$ | $C_2$ | $C_3$ | … | $C_{11}$ |
|---|---|---|---|---|---|
| $W_i$ | (0.0420, 0.0360, 0.0342) | (0.0565, 0.0427, 0.0069) | (0.3021, 0.3832, 0.6890) | … | (0.0039, 0.0023, 0.0009) |
| $\lambda$ | 0.08 | 0.08 | 0.08 | … | 0.0667 |
| | (0.0034, 0.0029, 0.0027) | (0.0045, 0.0034, 0.0005) | (0.0242, 0.0307, 0.0551) | … | (0.0003, 0.0002, 0.00006) |

**Step 7.** In this step we calculate $W''_t$ the final weight for each criterion according to equation (18) as follows:

**Table 8**
**Final weight of each criterion**

| *Criterion* | *Weight* | *Criterion* | *Weight* |
|---|---|---|---|
| $C_1$ | (0.033803864, 0.029290399, 0.031121125) | $C_7$ | (0.120227833, 0.115844427, 0.100862724) |
| $C_2$ | (0.045450534, 0.034786325, 0.006239492) | $C_8$ | (0.29111016, 0.283863175, 0.09951661) |
| $C_3$ | (0.24311186, 0.311983378, 0.62663816) | $C_9$ | (0.011201385, 0.007483128, 0.002297261) |
| $C_4$ | (0.50793895, 0.042364621, 0.025430191) | $C_{10}$ | (0.031844209, 0.02742989, 0.026521429) |
| $C_5$ | (0.069949186, 0.062407876, 0.040061234) | $C_{11}$ | (0.002585718, 0.001532089, 0.00066591) |
| $C_6$ | (0.099921352, 0.083014692, 0.040645857) | | |

## 5.2 Fuzzy TOPSIS

In this part we apply the Fuzzy TOPSIS technique for ranking countries in terms of IDI mentioned in the 2009, 2010 and 2011 IDI reports. We follow the steps as follows:

**Step 1:** The weights of the criteria are calculated by Fuzzy Shannon's Entropy up to now, and these values can be used in Fuzzy TOPSIS in order to rank the alternatives which are 69 countries in this paper. The calculated weights of the criteria can be seen in table 2.

**Step 2**. In this step we construct the Fuzzy decision matrix (FDM).

**Table 9**
**Sample Fuzzy Decision Matrix**

| *Fuzzy Decision Matrix* | | | | | |
|---|---|---|---|---|---|
| *Country* | *Fixed-telephone subscriptions* | *Mobile-cellular subscriptions* | … | *Tertiary gross enrolment ratio* | *Adult literacy rate* |
| Australia | (38.9,43.6,47.1) | (101,102.5,102.8) | … | (72.8,77,77.5) | (99,99,99) |
| Austria | (38.7,39.4,40.8) | (18.6,129.7,145.8) | … | (51,54.7,56.4) | (99,99,99) |
| ⋮ | ⋮ | ⋮ | … | ⋮ | … |
| Italy | (35.7,36.8,49) | (135.4,150.8,153.1) | … | (67.2,68.8,72) | (98.9,98.9,99) |
| ⋮ | ⋮ | ⋮ | … | ⋮ | ⋮ |
| Venezuela | (18.4,22.9,24.4) | (86.1,96.2,97.7) | … | (58,78.6,78.6) | (94.5,95.2,95.2) |

**Step 3.** In this step we normalize the fuzzy decision matrix based on equation (20). The sample normalized fuzzy decision matrix for Italy is as below:

**Table 10**
**Sample Normalized Fuzzy Decision Matrix**

| *Normalized Fuzzy Decision Matrix* | | | | | |
|---|---|---|---|---|---|
| *Country* | *Fixed-telephone subscriptions* | *Mobile-cellular subscriptions* | … | *Tertiary gross enrolment ratio* | *Adult literacy rate* |
| Australia | (0.5903,0.6616,0.7147) | (0.4893,0.4966,0.498) | … | (0.7137,0.7549,0.7598) | (0.992,0.992,0.992) |
| Austria | (0.5873,0.5979,0.6191) | (0.0901,0.6283,0.7063) | … | (0.5,0.5363,0.5529) | (0992,0.992,0.992) |
| ⋮ | ⋮ | ⋮ | … | ⋮ | ⋮ |
| Italy | (0.5417,0.5584,0.7436) | (0.656,0.7306,0.7418) | … | (0.6588,0.6745,0.7059) | (0.991,0.991,0.992) |
| ⋮ | ⋮ | ⋮ | .. | ⋮ | ⋮ |
| Venezuela | (0.2792,0.3475,0.3703) | (0.4172,0.4661,0.4734) | … | (0.5686,0.7706,0.7706) | (0.9469,0.9539,0.9539) |

**Step 4.** In this step we construct the weighted normalized fuzzy decision matrix according to equation (21).

**Table 11**
**Sample Weighted Normalized Fuzzy Decision Matrix**

| *Weighted Normalized Fuzzy Decision Matrix* | | | | | |
|---|---|---|---|---|---|
| *Country* | *Fixed-telephone subscriptions* | *Mobile-cellular subscriptions* | … | *Tertiary gross enrolment ratio* | *Adult literacy rate* |
| Australia | (0.0199,0.0194,0.0222) | (0.0222,0.0173,0.0031) | … | (0.2273,0.0207,0.0201) | (0.0026,0.0015,0.0007) |
| Austria | (0.0198,0.0175,0.0192) | (0.041,0.0218,0.0044) | … | (0.2273,0.0207,0.0201) | (0.0026,0.0015,0.0007) |
| ⋮ | ⋮ | ⋮ | … | ⋮ | ⋮ |
| Italy | (0.0183,0.0164,0.0231) | (0.0298,0.0254,0.0046) | … | (0.021,0.0185,0.0187) | (0.0026,0.0015,0.0007) |
| ⋮ | ⋮ | ⋮ | … | ⋮ | ⋮ |
| Venezuela | (0.0094,0.0102,0.0115) | (0.0189,0.0162,0.0029) | … | (0.0181,0.0211,0.0204) | (0.0024,0.0015,0.0006) |

**Step 5.** In this step we determine the fuzzy positive-ideal solution (FPIS) and the fuzzy negative-ideal solution (FNIS) which is calculated from all alternatives (69 countries) across all 11 criteria according to equation (22).

**Table 12**
**FPIS & FNIS**

| *Ideal solution* | *Fixed-telephone subscriptions* | *Mobile-cellular subscriptions* | … | *Tertiary gross enrolment ratio* | *Adult literacy rate* |
|---|---|---|---|---|---|
| $S^+$ | (0.0301,0.0283,0.0311) | (0.0364,0.0303,0.0062) | … | (0.0295,0.0264,0.0265) | (0.0026,0.0015,0.0007) |
| $S^-$ | (0.0004,0.0005,0.0008) | (0.0041,0.0071,0.0019) | … | (3E-19,2.7E-19,5.2E-05) | (0.0014,0.0008,0.0004) |

**Step 6.** In this step we calculate the distance of each alternative from FPIS ($d_i^+$) and FNIS ($d_i^-$) based on equation (23). The distances for Italy as a sample are as following:

**Table 13**
**Distance of alternatives from FPIS & FNIS**

| *Distances from FPIS & FNIS* | | |
|---|---|---|
| *Country* | *$di^+$* | *$di^-$* |
| Australia | 0.812427 | 0.532933 |
| Austria | 0.963732 | 0.400799 |
| ⋮ | ⋮ | ⋮ |
| Italy | 0.905992 | 0.433728 |
| ⋮ | ⋮ | ⋮ |
| Venezuela | 1.206752 | 0.136481 |

**Step 7.** In this step we compute the closeness coefficient ($CC_i$) of each alternative according to equation (25). The closeness coefficient for Italy as a sample is as following:

**Table 14**
**Closeness Coefficient of each alternative**

| *Closeness Coefficient* | |
|---|---|
| *Country* | *$CC_i$* |
| Australia | 0.396127 |
| Austria | 0.293727 |
| ⋮ | ⋮ |
| Italy | 0.323746 |
| ⋮ | ⋮ |
| Venezuela | 0.101607 |

**Step 8.** Finally the alternatives can be ranked in the ascending order of the $CC_{i.}$ The top ten countries ranked through fuzzy TOPSIS technique and the IDI ranking of three years of 2007, 2008 and 2010 are listed as bellow:

**Table 15**
**Top 10 counties ranked through Fuzzy TOPSIS technique**

| *Countries* | *Ranking through Fuzzy TOPSIS* |
|---|---|
| Luxembourg | 1 |
| Korea (Rep.) | 2 |
| Japan | 3 |
| Singapore | 4 |
| Australia | 5 |
| Sweden | 6 |
| Hong Kong, China | 7 |
| Denmark | 8 |
| New Zealand | 9 |
| Netherlands | 10 |

## CONCLUSION

As it was mentioned in section 2.2 of this paper, one of the main criticisms of the IDI is concerned with the method of IDI methodology. In the 2007, 2008 and 2010 IDI reports,the ICT union used the weighted average method to calculate the IDI for each country. This method is not a perfect way to calculate the IDI and contains some drawbacks such as double-counting of input and output or the misuse of Fixed-telephone subscriptions per 100 inhabitants and Mobile-cellular subscriptions per 100 inhabitants in access category while they are better compatible with the use category. On the other hand there are lots of efficient multiple criterion decision making methods for ranking alternatives across a set of criteria. One of the best MCDM approaches is Fuzzy TOPSIS technique which is applied in this paper as a method to compute the IDI. TOPSIS technique compares the alternatives with the positive and the negative ideal solutions. By proposing this technique the error of using weighted average to calculate IDI is eliminated to some degree and the result is more accurate. The results of ranking countries by the Fuzzy TIPSIS technique shown in table 15 are somewhat different from the 2007, 2008 and 2010 ranking. The reason of this difference is that the weights of each criterion which are calculated by the Fuzzy Shannon method in this paper are somewhat different from the weights used in ICT reports of IDI which are shown in table 1. Since MCDM methods are known as formal approaches applied in a wide area for selecting or ranking alternatives in terms of various criteria, they can be useful for the objective of proper ranking the countries in annual ICT development reports. As a recommendation, comprising the results of other MCDM techniques with Fuzzy TOPSIS technique and the IDI ranking results could be useful for the purpose of selecting the best technique for ranking countries in terms of IDI.

## *References*

1. ITU, Measuring the Information Society, Geneva, 2009.
2. ITU, Measuring the Information Society, Geneva, 2011.
3. James J (2012) The ICT Development Index and the digital divide: How are they related?. *Technol. Forecast. Soc. Change* 79:587-594. doi: 10.1016/j.techfore. 2011.08.010
4. Zadeh La (1965) Fuzzy Sets. *Information and Control* 8:338-353.
5. Diamond P (1988) *Fuzzy Least Squares. Information Sciences* 46:141-157. doi: 10.1016/0020-0255(88)90047-3
6. Chen CT (2000) Extensions of the TOPSIS for group decision- making under fuzzy environment. *Fuzzy Sets and Systems* 114:1-9. doi: 10.1016/s0165-0114(97)00377-1
7. Lee ES, Li RJ (1988) Comparison of Fuzzy numbers based on the probability measure of Fuzzy events. *Comput. Math. Applic* 15:887-896. doi: 10.1016/0898-1221(88)90124-1
8. Moghimi R, Anvari A (2013) An integrated fuzzy MCDM approach and analysis to evaluate the financial performance of Iranian cement companies. *Int. J. Adv. Manuf. Technol* 71:685-698. doi: 10.1007/s00170-013-5370-6
9. Mahdavi I, Heidarzade A, Sadeghpour-Gildeh B, Mahdavi-Amiri N (2009) A general fuzzy TOPSIS model in multiple criteria decision making. *Int. J. Adv. Manuf. Technol* 45:406-420. doi: 10.1007/s00170-009-1971-5
10. Büyüközkan G, Feyzioğlu O, Nebol E (2008) Selection of the strategic alliance partner in logistics value chain. *Int. J. Production Economics*. 113:148-158. doi:10.1016/j. ijpe.2007.01.016
11. Chen SJ, Hwang CL (1992) *Fuzzy multi attribute decision making, lecture notes in economics and mathematical system series.* Springer-Verlag New York 375.
12. Triantaphyllou E, Lin CT (1996) Development and Evaluation of Five Fuzzy Multiattribute Decision-Making Methods. *Int. J. Approx. Reason* 14:281-310. doi:10.1016/0888-613x(95)00119-2

13. Chu TC (2002) Selecting Plant Location Via a Fuzzy TOPSIS Approach. *Int. J. Adv. Manuf. Technol* 20:859-864.doi: 10.1007/s001700200227
14. Jafarnejad Chaghooshi A, Fathi MR, kashef M (2012) Integration of Fuzzy Shannon's Entropy with Fuzzy TOPSIS for industrial robotic system selection. *JIEM* 5:102-114.
15. Chen CT (2000) Extension of the TOPSIS for group decisionmaking under fuzzy environment. *Fuzzy Sets Syst* 114:1–9.doi:10.1016/S0165-0114(97)00377-1