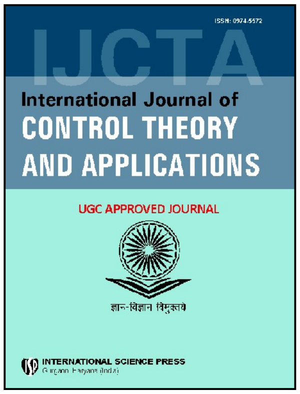

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Discretization of Uncertain Linear Systems with Time Delay

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Abstract: This paper is concerned with the problem of discretization of linear uncertain systems with external delay. The parameter uncertainties are assumed to be norm bounded and appear in the state matrices and the time delay is assumed to be constant. For obtaining discrete-time approximations of multivariable continuous uncertain system with time delay, three different methods for is discussed based on the Euler's and Tustin's approximations. The proposed methods are dependent on the size of the delay and the selected sampling period also the uncertainties values. An illustrative example is simulated to demonstrate the efficiency of the developed methods.

Keywords: discrete-time approximation, uncertain systems with time delay, Tustin's approximation, explicit Euler method, implicit Euler method.

1. INTRODUCTION

Delays phenomena appear naturally in modelling many processes encountered in physics, mechanics, biology etc. Moreover, even if the process itself contains no delay, the closed loop system can cause significant delays from the sensors, actuators, computation time, etc. However the presence of the delay in a dynamic system is a cause of instability and performance degradation. So in the last years, the study of systems with delays has received special attention and many basic researches depending on the type of considered systems and application domain were conducted in the literature [1, 2, 3].

Therefore, the analysis and control of either continuous or discrete time-delay systems have long been a focused topic of theoretical as well as practical importance.However, the discretization of continuous systems with time delay has not been extensively studied [4].

With the rapid progress of the large scale integration of semiconductor devices and the resulting availability of cheap computers, there is a renewed interest in the approximation of continuous time multivariable systems in discrete time [5]. Such of models have applications in computer simulation, as well as the identification of the system through the data samples the input-output. Other applications of these methods are possible in the digital adaptive control and computer control of complex processes.Consequently the problem of discrete time approximation of multivariable continuous-time systems has been considered in [5] by using five different methods. In [11] the problem of discretization for uncertain linear systems with time-delay, which contains a norm bounded uncertainty parameter, was studied, by using two different methods: the state transition method and the method based on the trapezoidal rule for integration. In [9] an approximation of linear system with time-delay using two different methods: the Euler's and Tustin's method of discretization has been proposed.

Regarding this concept, however, the exciting papers were restricted to discretize the linear regular system with delay using the Euler and Tustin methods, or they have been restricted to discretizing uncertain systems with constant delay by using other approximation techniques.

All of these approaches require a very small sampling period to be considered accurate. But, in control applications where large sampling periods are inevitably introduced,this may not be the case due to physical and technical limitations [6, 7]. Given that the uncertain system have attracted a lot of researches from mathematics and control communities due to the fact that uncertain systems can better describe the behaviour of some physical systems than other systems [8].

Motivated by the satisfying aforementioned results, we propose in this paper to extend those results obtained in [6, 10, 12] to discretize an uncertain linear continuous system with constant delay in the state by using the Euler and Tustin methods of approximation [9,10,11,12]. But, it is interesting to remember that the problems of discretization of this class of systems are more complicated than those for regular systems.Therefore, in this paper we consider three different methods for obtaining the discrete-time approximation. These are:

i) The Forward Euler method

ii) The Backward Euler method

iii) Tustin's method

Our goal is to propose a new idea of approximation of uncertain linear systems with external delay, using three different methods of discretization in order to conclude the superiority or the inferiority of each in terms of stability. On the one hand, our aim is to conclude on the effect of choice of the sampling period on the stability of the system and on the other hand to conclude on the effect of the uncertainty parameter about the digitization of the system with delay. Some recent control methods are described in [20-25].

The rest of this paper is organized as follows. In section2, the three proposed methods of discretization are represented for discrete-approximation of uncertain continuous system with external delay.

In section 3, numerical examples are given to illustrate the effectiveness and merits of our methods and, furthermore, the effects of the variation of the uncertainty parameter on the efficiency of those.Finally, the last section includes a comparison between the three proposed techniques of discretization as well as the study of the effect of the variation of uncertainty parameters on the discretization of the continuous uncertain system with delay, and at the end of the paper a conclusion is derived.

2. DISCRETIZATION OF UNCERTAIN CONTINUOUS SYSTEM WITH EXTERNAL DELAY

In modern control systems, information is digitally processed which requires a sampling of the signals. One speaks in this case of sampled or discrete systems [5, 10]. In practice it is difficult to characterize a physical system, somodelling is a tool increasingly used to configure or analyze systems. The purpose of a model is the quantitative and qualitative evaluation of the description of the behaviour and the proper functioning of a system. In reality, the systems are often a coupling between the discrete and continuous. It is therefore necessary to switch between continuous to discrete and vice versa [10, 12]. Therefore, we focus on the discretization class continuous systems uncertain with constant delay on the state [13, 14].Thus, the state representation of an uncertain linear system with external delay is given, in the continuous case, by:

$$\begin{cases} \dot{x}(t) = \left[A_0 + \Delta A_0(t)\right]x(t) + \left[A_1 + \Delta A_1(t)\right]x(t-\tau) + Bu(t) \\ y(t) = Cx(t) \\ x(t) = \phi(t), t \in \left[-\tau, 0\right] \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input and the scalar $\tau > 0$ is the delay of the system. $\phi(t)$ is the initial condition of the system. $A_{0,}A_1$ and B are known real constant matrices of suitable dimensions. $\Delta A_0(t)$ and $\Delta A_1(t)$ are the time-varying parameter uncertainties, and are assumed to be of the form:

$$\Delta A_i(t) = D_i F(t) E_i \quad (2)$$

where D_0, D_1, E_0 and E_1 are constant matrices, and $F(t) \in R^{ixj}$ is the time-varying system, which is assumed to be of the form:

$$F(t) = diag\left\{F_1(t), ..., F_r(t)\right\} \quad (3)$$

where

$$F_i(t)^T x F_i(t) \leq I, i = 1, ..., r \quad (4)$$

$\Delta A_0(t)$ and $\Delta A_1(t)$ are said to be admissible if the both (2) and (4) are hold.

$A_0 x(t)$: Original Term state, $A_1 x(t-\tau)$: Delayed Term state; with $\tau = hT$: a multiple delay the sampling period is an integer *h*. *C* represents the system's output matrix, *T* is the sampling period chosen suitably. *x*, *u*, and y respectively are the state vector, the vector and the control vector output.

For system (1) we define the transfer matrix of the system in the Laplace domain by this report:

$$H(p) = \frac{Y(p)}{X(p)} \quad (5)$$

The size of this transfer matrix *H(p)* is linked to the size of outputs vectors *y* and control vectors *u*.

If *y* has a size *l* and *u* has a size *m*, *H(p)*will be sized *l x m*.

So the problem that may occur is the transition from an internal representation given by (1) to the external representation given by (5) in the case where the system contains a time delay.

This passage will be done, in the case of the uncertain continuous system with time delay on the state we are dealing with, by using the Laplace transform on the equation (1).

So we obtain this following equation:

$$\begin{cases} pX(p) = \left[A_0 + \Delta A_0\right]X(p) + \left[A_1 + \Delta A_1\right]X(p)e^{-\tau p} + BU(p) \\ Y(p) = CX(p), \\ X(p) = \phi(p) \end{cases} \quad (6)$$

This equation can also be written as follows:

$$X(p) = \left[pI - \{(A_0 + \Delta A_0(p)) + (A_1 + \Delta A_1(p))e^{-\tau p}\}\right]^{-1} BU(p) \quad (7)$$

then:

$$Y(p) = C\left[pI - \{(A_0 + \Delta A_0(p)) + (A_1 + \Delta A_1(p))e^{-\tau p}\}\right]^{-1} BU(p) \tag{8}$$

Hence we obtain the following transfer matrix:

$$H(p) = \frac{Y(p)}{U(p)} = C\left[pI - \left\{(A_0 + \Delta A_0(p)) + (A_1 + \Delta A_1(p))e^{-\tau p}\right\}\right]^{-1} B \tag{9}$$

The matrix $C\left[pI - \left\{(A_0 + \Delta A_0(p)) + (A_1 + \Delta A_1(p))e^{-\tau p}\right\}\right]^{-1} B$ called transfer matrix, this is a transfer function matrix, that is to say a rational function in *p*.

In the progress of the discrete-time models, we have toassume a suitable sampling interval, denoted by *T*. A suitablecriterion for the choice of *T* isthat *wT* be less than 0.5 [5], where *w* is the magnitude of the eigen value of *A* farthest from the origin of the s-plane [10].In order to switch between continuous to discrete domain the first consisting of the discretization is not to lose or at least lose a minimum of information by sampling [5, 11]. For this to be possible we have to verify the validity of Shanon theorem since in the case of non compliance with this theorem the sampling process loses information, and frequencies above half the sampling frequency of the signal are removed. The methods of discretization we treated can be classified as follows, and they are represented in the following section.

2.1. The Forward Euler Method (Or Explicit Euler Method)

The forward Euler's method is one such numerical method and is explicit [9]. Explicit methods calculate the state of the system at a later time from the state of the system at the current time without the need to solve algebraic equations.These approximations on the z-transform function matrix exploit this relationship: $z = e^{pT}$

The idea is to approximate this relationship by a linear relationship between *z* and *p*. So the linear approximation of the first order function exponential gives:

$$e^{pT} \approx 1 + pT \tag{10}$$

that can also written as follow:

$$p = \frac{z-1}{T} = \frac{1-z^{-1}}{Tz^{-1}} \tag{11}$$

this technique of discretization is the result of a derivative between two sampling period [7]:

$$L^{-1}(pX(p)) = \frac{dx(t)}{dt} \approx \frac{x(t+T) - x(t)}{T} = z^{-1}\left(\frac{z-1}{T}X(z)\right) \tag{12}$$

To obtain the transfer function matrix we have replaced p given in (10) by: $\frac{1-z^{-1}}{Tz^{-1}}$

So we obtain the following discrete matrix:

$$H(z) = C\left[\left(\frac{1-z^{-1}}{Tz^{-1}}\right)I - \left\{(A_0 + \Delta A_0) + (A_1 + \Delta A_1)e^{-\tau p}\right\}\right]^{-1} B \tag{13}$$

The Forward Euler method is based on a truncated Taylor series expansion. So by applying an approximation Taylor expression near *0* of the term $e^{-\tau(\frac{1-z^{-1}}{Tz^{-1}})}$ to the first order approximation we can obtain:

$$e^{-\tau(\frac{1-z^{-1}}{Tz^{-1}})} = 1-\tau\frac{z-1}{T} \tag{14}$$

Hence the equation (13) is as follows:

$$H(z)=C\left[(\frac{1-z^{-1}}{Tz^{-1}})I-\{(A_0+\Delta A_0)+(A_1+\Delta A_1)(1-\tau\frac{z-1}{T})\}\right]^{-1}B \tag{15}$$

and also can be described as follows:

$$H(z)=C\left[\frac{I}{T}z-\frac{I}{T}-\{(A_0+\Delta A_0)+(A_1+\Delta A_1)(1-hz+h)\}\right]^{-1}B \tag{16}$$

2.2. The Backward Euler Method (Implicit Euler Method)

This method is also called Backward rectangular rule, This method is an implicit one which contrary to explicit methods finds the solution by solving an equation involving the current state of the system and the later one [12, 13, 18].

This method of discretization is the result from the approximation (13) of the derivative that can be calculated between two sampling periods [9]:

$$L^{-1}(pX(p))=\frac{dx(t)}{dt}\approx\frac{x(t)-x(t-T)}{T}=z^{-1}(\frac{z-1}{zT}X(z)) \tag{17}$$

and also can be written as:

$$p=\frac{z-1}{zT}=\frac{1-z^{-1}}{T} \tag{18}$$

Therefore using this method of discretization and also using the last equality given in (17) to the transfer function matrix given in (10) we obtain the discrete transfer function as follows:

$$\mathrm{H(z)=C}\left[(\frac{1-z^{-1}}{T})I-\{(A_0+\Delta A_0)+(A_1+\Delta A_1)e^{-\tau(\frac{1-z^{-1}}{T})}\}\right]^{-1}B \tag{19}$$

and by using the Taylor expansion near zero term $e^{-\tau(\frac{z-1}{Tz})}$ to a first order approximation we obtain the following inequality:

$$e^{-\tau(\frac{z-1}{Tz})}=1-\tau\frac{z-1}{zT} \tag{20}$$

the expression H(z) becomes:

$$H(z)=C\left[\frac{I}{T}-\frac{I}{Tz}-\{(A_0+\Delta A_0(t))+(A_1+\Delta A_1(t))(1-h+\frac{h}{z})\}\right]^{-1}B \tag{21}$$

2.3. Bilinear Transformatin Method

This method is also called Tustin's method or the trapezoidal rule in digital control community [5, 13]. So as enounced in the introduction approximation a good discrete-time approximation for a continuous-time linear system is obtained through the bilinear z-transformation if the sampling interval T is selected suitably so that $wT \leq 0.5s$, where w is the magnitude of the pole of the transfer function of the continuous-time system farthest from the origin of the s-plane [5, 10]. This transformation is given by:

$$z = \frac{1 + p\frac{T}{2}}{1 - p\frac{T}{2}} \tag{22}$$

or

$$z^{-1} = \frac{2 - pT}{2 + pT} \tag{23}$$

where $z = e^{pT}$ in ordinary transformation in z.

Also the inequality (23) can be written as follows:

$$p = \frac{2}{T}\frac{z-1}{z+1} = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{24}$$

So after replacing p by its equivalent given in (24) in the transfer matrix we obtain:

$$H(z) = C\left[\frac{2\ I}{T(1+z^{-1})} - \frac{2\ I}{T(z+1)} - \{(A_0 + \Delta A_0) + (A_1 + \Delta A_1)e^{-2\frac{\tau}{T}(\frac{1-z^{-1}}{1+z^{-1}})}\}\right]^{-1} B \tag{25}$$

by applying an approximation Taylor expansion near0of the term $e^{-2\frac{\tau}{T}(\frac{1-z^{-1}}{1+z^{-1}})}$ to the first order approximation, we obtain this equality:

$$e^{-2\frac{\tau}{T}(\frac{1-z^{-1}}{1+z^{-1}})} = 1 - \tau\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{26}$$

Hence the transfer function matrix obtained by using the Tustin method can be represented in the discrete case as follows:

$$H(z) = C\left[\frac{2\ I}{T\ (1+z^{-1})} - \frac{2\ I}{T\ (z+1)} - \{(A_0 + \Delta A_0) + (A_1 + \Delta A_1)\ (1 - \frac{2\ h}{(1+z^{-1})} + 2\frac{h\ z^{-1}}{1+z^{-1}})\}\right]^{-1} B \tag{27}$$

3. NUMERICAL EXAMPLES

In this section, three different examples of application of the proposed approach are presented. These examples show the validity of the discretizationmethod applied to the classes of systems with uncertainties, and T represents the sampling period chosen equal to $0.1s$. In a first example, we consider the system with uncertainty parameter and constant delay on the state (1) described by:

$$\begin{cases} \dot{x}(t) = [A_0 + \Delta A_0]x(t) + [A_1 + \Delta A_1]x(t-0.5) + Bu(t) \\ y(t) = C\,x(t) \end{cases} \tag{28}$$

with:

$$A_0 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}; A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}; D = 0$$

$$\Delta A_0(t) = \Delta A_1(t) = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix};$$

where

$$D_0 = D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}; \quad E_0 = E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \tau = 0.5s$$

Knowing that $F(t) \le 1;$ for all of t

so: $\Delta A_0(t) = D_0\,F(t)\,E_0 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}$ and $\Delta A_1(t) = D_1\,F(t)\,E_1 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}$

In this case we dealt an example of linear uncertain continuous system with constant delay τ , which appears in the state and equal to *0.5s*

First with applying the first discretization method previously: the explicit Euler method proposed for this class of system we got the transfer function matrix enunciated in (13):

The simulation results of the step response of the system in the continuous case and in the discrete case are shown in Figure 1, knowing that the sampling period chosen is equal to *0.1 s*.

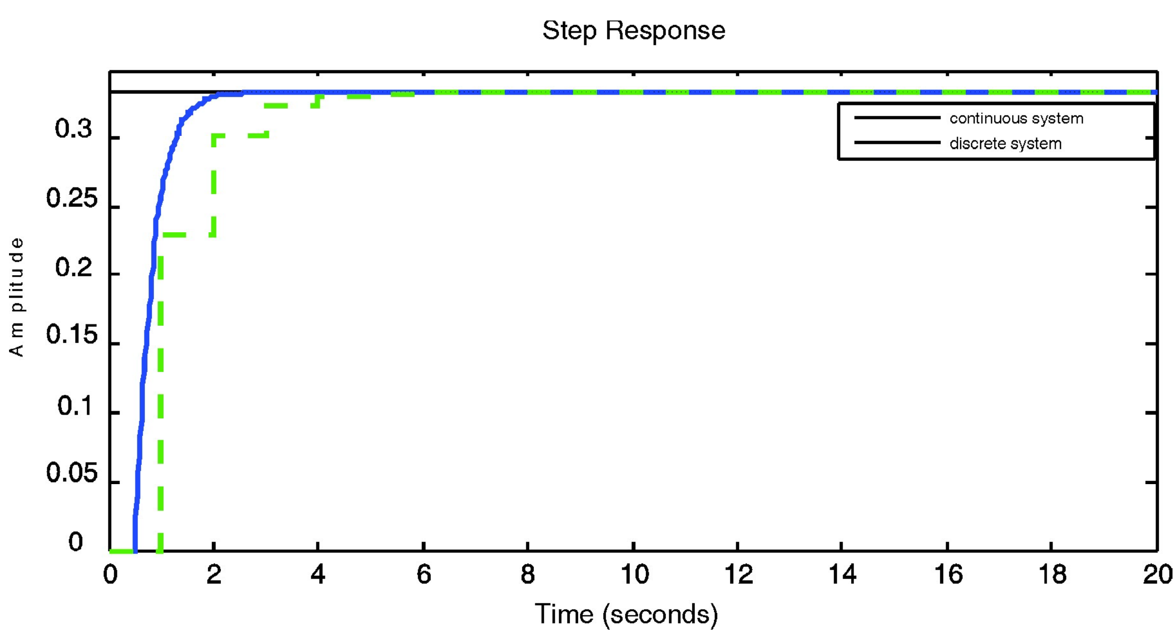

Figure 1: Discretization of our system using the explicit Euler method

Then,with applying the implicit Euler implicit method of discretization to the considered system in the state representation (28), and keeping the same sampling period $T = 0.1s$, we obtain the transfer function matrix in the discrete case defined in (21) and knowing that $F(t) \leq 1;$ for all of t.

Simulation result of the response of our system to a step in the continuous case and in the discrete case using the second method of Euler is shown in the Figure bellow.

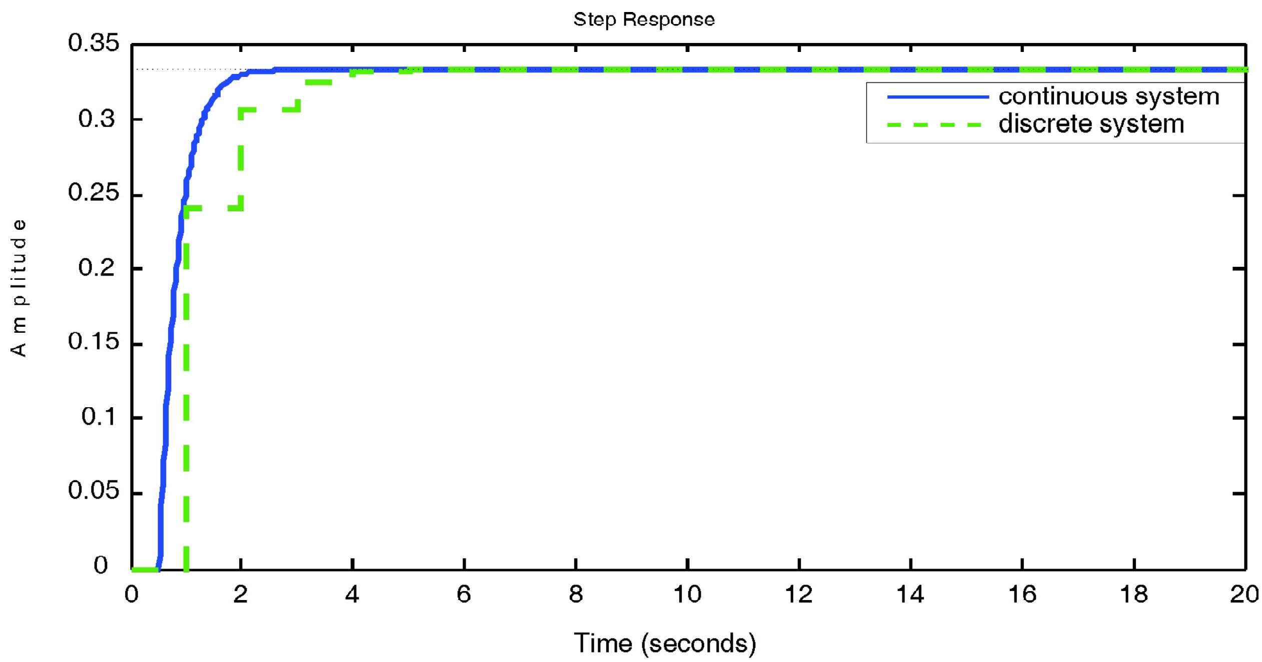

Figure 2: Discretization of the system by the implicit Euler method

Finally applying the bilinear transformation method to the considered system in equation (28), and keeping the same sampling period, we have obtained the transfer function matrix in the discrete obtained by using theTustin method obtained in (27).The simulation of the step response of our system in the continuous case and in the discrete case gave us the following results which are represented in the Figure 3.

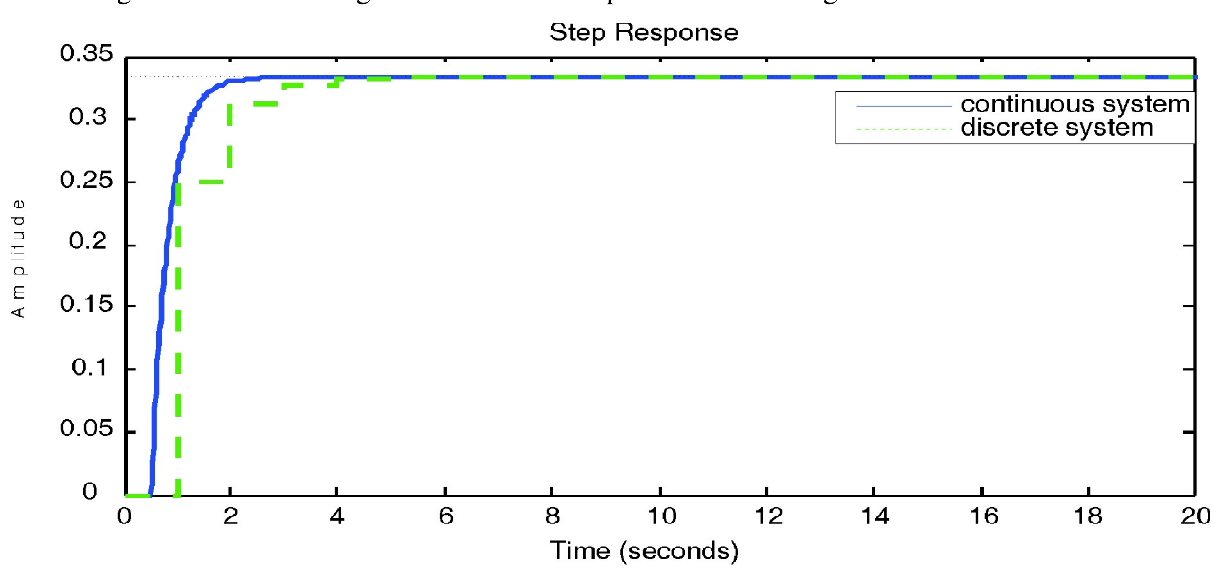

Figure 3: Discretization of our system by the Tustin method

In the simulation, during the discretization of the uncertain system with delay, using the three different methods it was found that a small variation of the parameter uncertainty leads to loss of the discretization.

Indeed, with varying the parameter uncertainty, the discrete system does not suitably follow the evolution of the system in the continuous case.

Therefore, using those methods of approximation, it was found that a variation of 2.5% of the uncertainty parameter leads to the obtaining of a discrete system with different evolution than that of the system in the continuous case. With keeping the same sample period chosen previously *(T=0.1s)*, the simulation results in the step responses of the continuous system and approximate discrete system in the case of varying the parameter uncertainty with 2,5% and using the proposed methods of discretization are depicted in the figure 4.

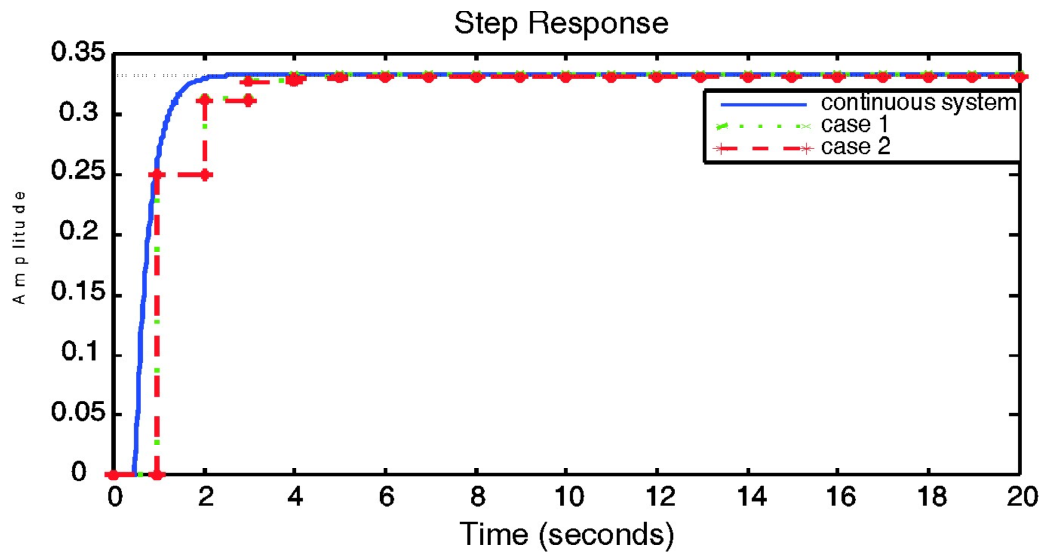

Figure 4: (a) Discretization of our system using the explicit Euler method with case 1 represents the discrete system without varying the uncertainty parameter and case 2 represents the discrete system in the case of varying the uncertainty parameter

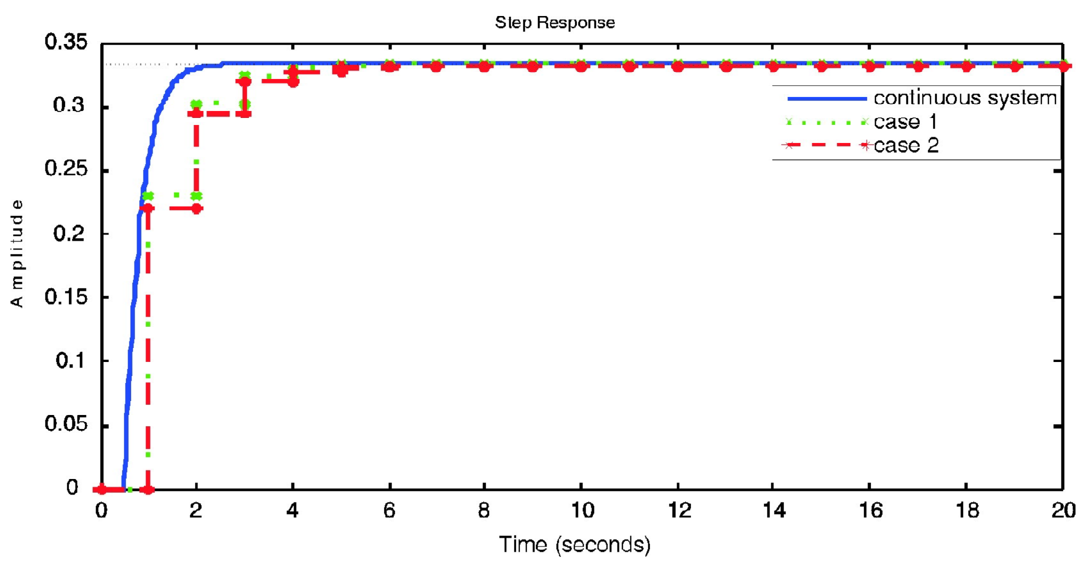

Figure 4: (b) Discretization of our system using the implicit Euler method with case 1 represents the discrete system without varying the uncertainty parameter and case 2 represents the discrete system in the case of varying the uncertainty parameter

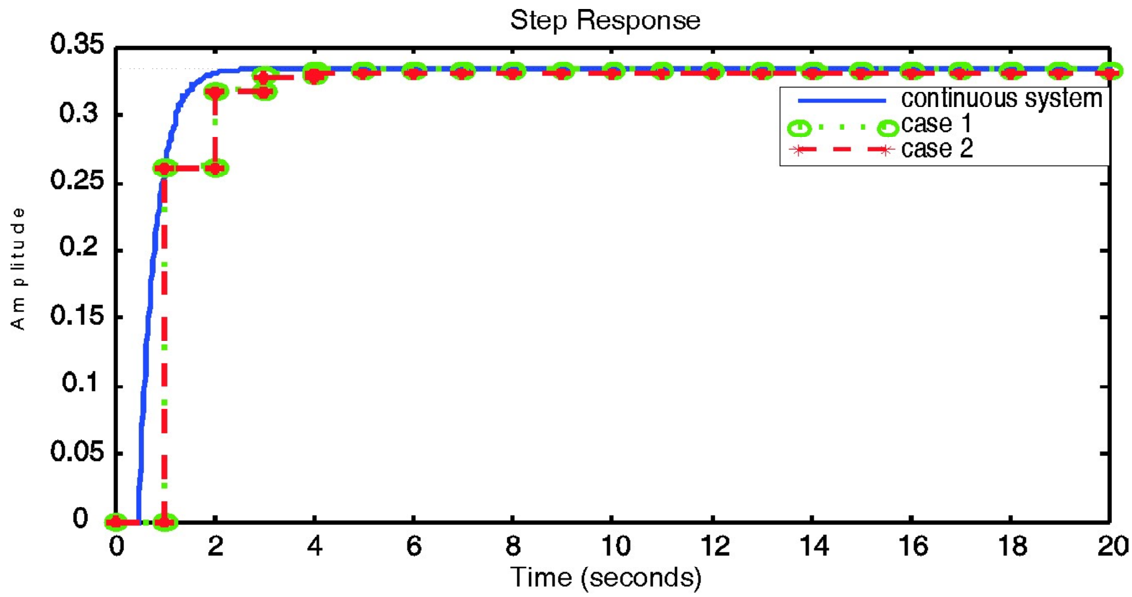

Figure 4: (c) Discretization of our system using the Tustin method with case 1 represents the discrete system without varying the uncertainty parameter and case 2 represents the discrete system in the case of varying the uncertainty parameter

Therefore it can be seen that by keeping the same sampling period, the variation of the uncertainty parameter of *2.5%* can affect the discretization of the system.

For the same methods of discretization, to followproperly the continuous system in the case of varying the parameter uncertainty with *2.5%* it is necessary to grow properly the sampling period for achieving the better discretization.So with taking a sample period *T* equal to *0.4s* we have obtained the following results that show the good evolution of the discrete system compared to continuous time in the case of varying the parameter uncertainty, by using the different methods as shown in the figures bellow.

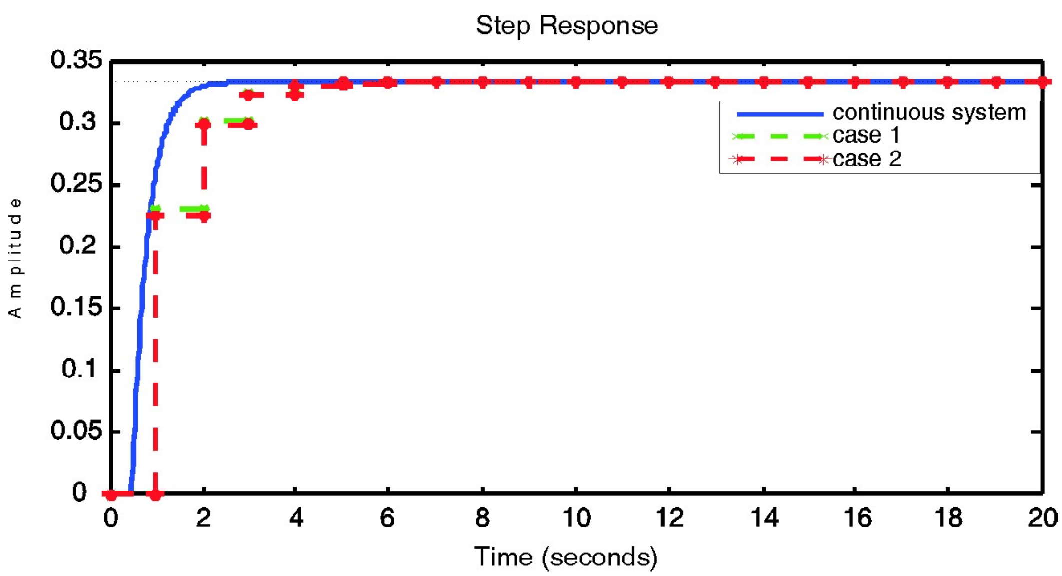

Figure 5: (a) Discretization of our system using the explicit Euler method with case 1 represents the discrete system without varying the uncertainty parameter and case 2 represents the discrete system in the case of varying the parameter uncertainty

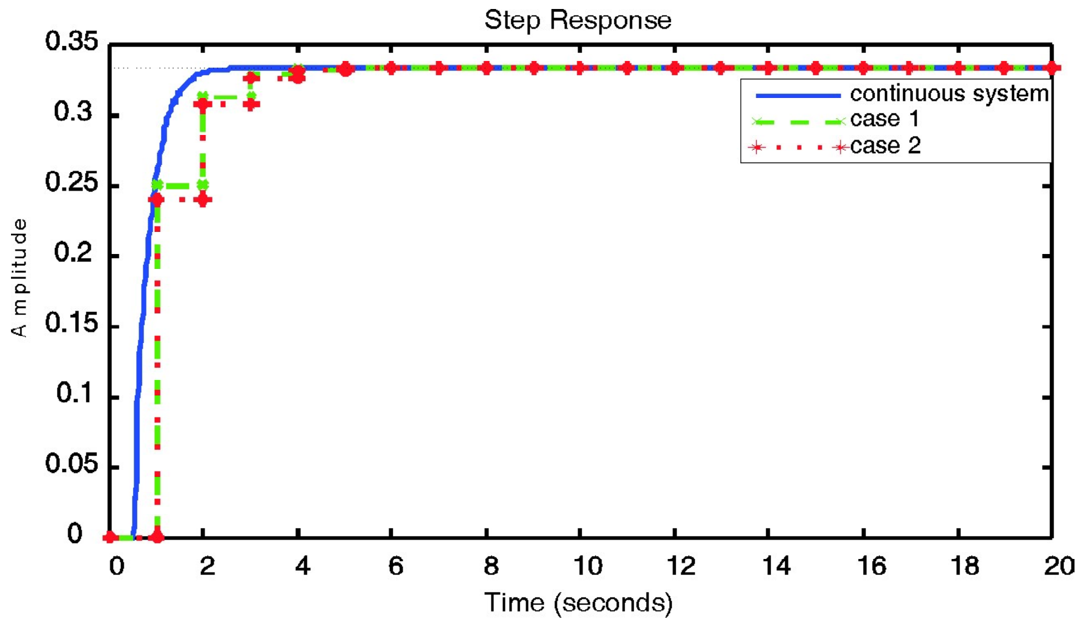

Figure 5: (b) Discretization of our system using the implicit Euler method with case 1 represents the discrete system without varying the uncertainty parameter and case 2 represents the discrete system in the case of varying the parameter uncertainty

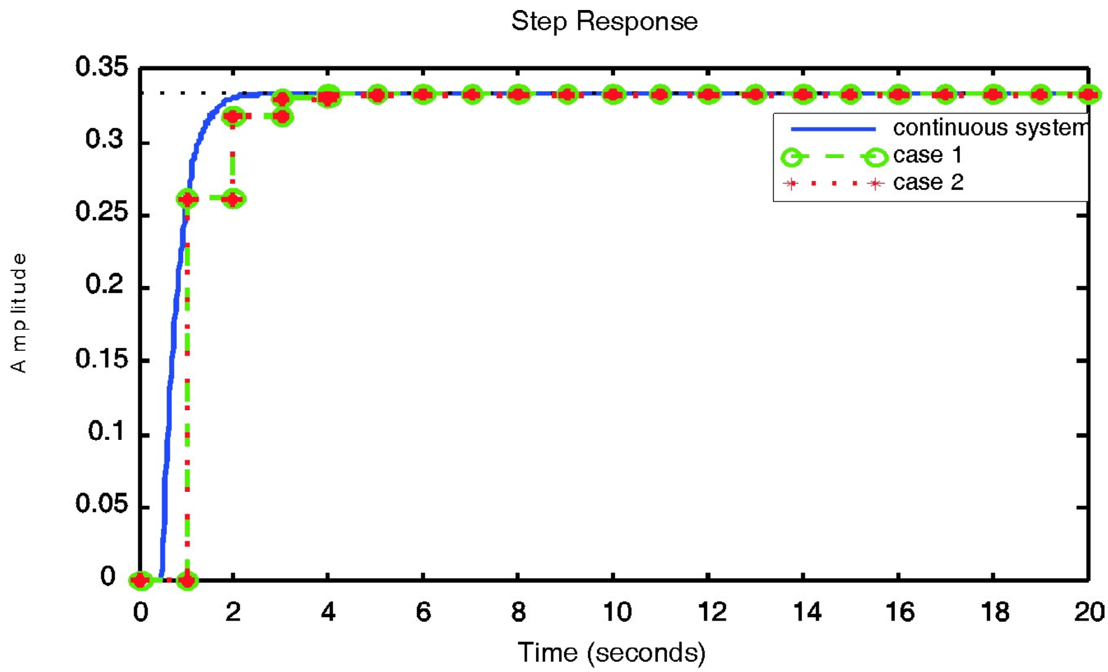

Figure 5: (c) Discretization of our system using the Tustin method with case 1 represents the discrete system without varying the uncertainty parameter and case 2 represents the discrete system in the case of varying the parameter uncertainty

So we can conclude the following three main issues

- In the case that we keep the same sampling period and for better discretization we have to not exceed a rate of change of the uncertainty parameter equal to 0.048 i.e. the matrix admissible used not to destroy the discretization of the continuous signal is as follows for different methods of discretization.

$$\Delta A_0(t) = \Delta A_1(t) = D_0\, F(t)\, E_0 = D_1\, F(t)\, E_1 = \begin{pmatrix} 0.1952 & 0 \\ 0 & 0.152 \end{pmatrix}$$

So if we vary the uncertainty parameter more than 2.5% we risk losing the stability of the discrete system for the different methods used

- With varying the parameter of uncertainty, it can be seen in those figures, that the representation of discrete time system follows the evolution of the continuous system after a delay which corresponds to the value of the state delay which is equal to 0.5 s
- In the case of varying the uncertainty parameter of rate of 2.5%,to achieve the same progress achieved in discrete time as achieved in continuous time, we have to grow properly the sampling time, otherwise we risk having a bad discretization and sometimes an unstable discrete system

4. COMPARISON BETWEEN THE USED METHODS OF APPROXIMATION

The simulation results of this class of systems has shown the validity of the three methods in terms of stability, in fact we see that the three methods of discretization did not destroy the stability of the original system, so the transition from the analogue domain to the digital domain using one of these methods has not led to the loss of the information, except that each of these methods has shown its superiority or inferiority to the other.The explicit Euler method (Forward difference method) is somewhat less accurate than the Euler's implicit method of discretization (Backward difference method) but it is easier to use, so, the drawback is due to limitations on the size of the sampling period to ensure numerical stability. Furthermore, it is well-known in numerical analysis that the forward Euler approximation is more sensitive for the choice of the sampling period in terms of numerical stability than the backward Euler approximation [10, 16, 17, 18]. Both methods are much easier at programming and in calculation comparing to the Tustin method. We also see that the bilinear approximation method is the most accurate of the three methods because it gave a better approximation.

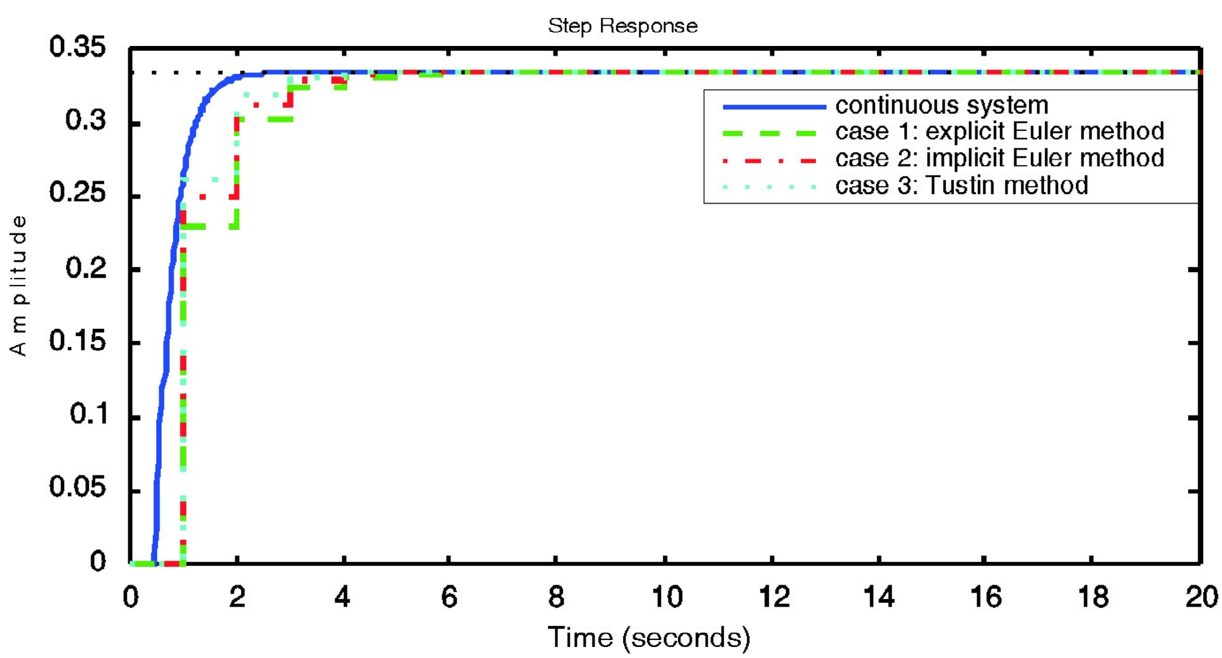

Figure 6: Comparison between the different methods of approximations

5. CONCLUSION

In this paper three methods of discretization based on Euler's and Tustin approximations are proposed for classes of continuous and uncertain linear systems with constant delay on the state

Simulation results showed that the bilinear approximation method is the better although it is difficult to calculate, the implicit discretization method's may be used to replace explicit discretization method's in the case where the stability of these requirements impose rigorous conditions to the size of the sampling step. It is also clear that the value of 0.5s delay appeared in the simulation using the three methods of discretization without damaging the stability of the initially stable system [19].

The effect of a delay and the parameter of uncertainty on the dynamics of a system depend not only on its value but also the characteristics of the system. Indeed, the presence of delay and a parameter of uncertainty can affect the stability of the system, since its presence can cause complex behaviours namely oscillations, instability and degradation in performance. For best approximation of a continuous uncertain system with delay, it is allowed to choice one of the proposed methods but it isn't allowed to vary the value of the parameter uncertainty arbitrarily because we risk losing the evolution of the continuous time system.

Therefore in this paper we have proposed a new method of discretization of uncertain systems with time-delay based on two techniques of approximation the Euler and Tustin methods, on the other hand in the paper [12] stated before the discretization of this class of system was carried out by other methods of discretization: State transition method and the method based on trapezoidal rule of integration.So with comparing our work to the work presented in paper [12] we conclude the superiority of our methods at the level of calcul and precision.

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