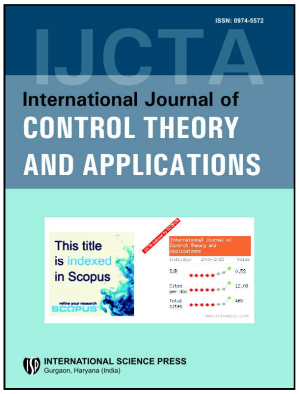

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A Minimal Cut-Set based Enumerative Approach for Two-Terminal Reliability Estimation

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Abstract: This paper presents an efficient minimal cut-set enumeration based algorithm to evaluate the two-terminal reliability of interconnection networks. The proposed algorithm not only effectively generates all the minimal cut-sets but also disjoint them to formulate the reliability expression. The minimal cut-sets so generated using the proposed approach are non-redundant in nature. The inclusion-exclusion principle is applied on these enumerated minimal cut-sets to form the reliability expression. The algorithm is well illustrated through an example. To show the efficiency and correctness of the proposed algorithm it is implemented on some sample benchmark interconnection networks.

Keywords: Interconnection Networks, Terminal Reliability, Minimal Cut-Sets, Probabilistic Graph

1. INTRODUCTION

Interconnection network plays an important role in many of the interconnected systems like traffic control, power control, clustered systems and parallel systems in particular. In parallel systems, it facilitates the efficient data transmission between the processing elements and memory modules. There are many factors that affect the design of interconnection of network viz. cost, reliability, flow, power etc [1]. Among which, reliability is usually selected as one of the most important indices of real world systems. Reliability engineering is found to be a key research area since the early development of engineering. An interconnection network is considered to be reliable for each couple of nodes in the network structure connected by multiple paths.

There are basically three different reliability measures are documented in the literature namely network reliability, two-terminal reliability and broadcast reliability. Network reliability (All-to-All) is defined as the probability that every node in the network is able to communicate with each other [15]. Two-terminal reliability (One-to- One) is the probability that at least one fault free path must exist between source node *s* to target node *t* [14]. Broadcast reliability (One-to-All) is the probability that all the nodes of the network are able to communicate from a given source node *s* [13]. Algorithms for computing these metrics belong to the NP-complete class [14], a family of NP-hard problems.

A network is represented by means of a graph whose nodes represent processing elements and edges represent the connecting links between processing elements. Both nodes and the edges have binary mutually exclusive states, working or failed. Many proposed algorithms for reliability estimation can be roughly classified as exact or approximation algorithms. Exact algorithms may be classified as pivotal decomposition using keystone components and enumeration of all possible cut-sets/path-sets and then disjoint them to formulate reliability expression [3-4].

To compute the connectivity and reliability of interconnection networks, various approaches are used viz. state enumeration, factoring theorem[4], binary decision diagram[2], sum of disjoint product[11], inclusion-exclusion principle[1], monte-carlo etc. Mostly the minimal path-set/cut-set algorithms are popular in the recent past due to its deterministic and exact estimation properties. The two-terminal reliability measure is one of the important, extensively researched measure of reliability for interconnection networks. Three types of algorithms are basically listed in the literature for searching of minimal paths are: symbolic expression based [5], direct search based [7], and augmentation based [6] algorithms. The symbolic expression based algorithms define the symbolic terms and operators, and develop their algebraic manipulations to produce the minimal path. However, it often generates redundant-paths which needs to be discarded from the final reliability expression. Augmentation based methods are not suitable when the size of the target network becomes large. Colbourn[7] proposed a good efficient algorithm of time complexity of $O(n\pi)$, where π is the number of minimal paths. Chen[8] has proposed a backtracking algorithm in which the overhead of eliminating minimal paths are avoided. Jasmon & Foong[9] proposed an approach in which minimal cut-sets can be directly generated without using minimal path-sets or inversion techniques. But they are not able to eliminate the generation of redundant terms. Therefore it require additional computations to generate a minimal solution. In [10], Goyal et al. has proposed a new method called SNEM for terminal reliability of communication networks. However, it does not produce a reliability expression. The improvement on MVI-SDP method and simplification of this approach was presented by Soh & Rai [11]. In [12], the authors have used to develop multistate minimal path vectors, which are the equivalent of minimal path-sets for multi-state networks.

The methods so far discussed in the literature has many limitations like some of the approaches are only suitable for a limited size networks, many of the approaches suffer from binary tree explosion or intractable. Although many approaches are found in the literature for reliability evaluation of interconnection networks however, only few works are reported for directed networks. The two-terminal reliability is a highly requiring factor for directed interconnection networks because of the reliable packet delivery from source to destination. The dynamic interconnection networks viz. MIN, OMEGA, Shuffle Exchange are directed in nature in which two-terminal reliability is a important factor for consideration.

The rest of the paper is organized as follows. The Section 2 of this paper proposes a new algorithm for minimal cut-set enumeration, Section 3 elaborately explains the proposed algorithm through a suitable example, Section 4 presents the implementation of the proposed algorithm with the discussion on the result obtained and finally the Section 5 concludes the paper with a further scope for intend researchers.

2. PROPOSED ALGORITHM

A new algorithm is proposed in this section for enumerating all minimal cut-sets of the given interconnection network.

2.1. Objective

The objective of this algorithm is to enumerate all the non-redundant potential minimal cut-sets of an interconnection network.

2.2. Notations

G (N, L)	:	A graph *G* having *N* number of nodes and *L* number of links.
Ψ	:	Former matrix of *G*
p	:	Reliable edges for successful transmission from '*s*' to '*t*'.
q	:	Unreliable edges whose removal makes the network disconnected.
þ	:	Inherent ancestor.
R	:	Two-Terminal Reliability of the network
x, y	:	Nodes having ancestor relationship.
Remove (Ψ, k)	:	Remove *k* number of links from set Ψ.
Former (G, q)	:	Find the ancestor of *q* from graph *G* .
Add (C, temp)	:	Add the cut-set *temp* to the cut-set set *C*.

2.3. Definitions

The following definitions help in increasing the readability of the proposed algorithm.

Cut-set is the minimal number of edges that can be removed to make the network disconnected.

Minimal cut-set is a cut 'C', if there is no subset of 'C' which is also a cut.

Two-terminal reliability is the probability of successful data transmission from an input node *s* to an output node *t*.

Former matrix is a matrix having information about the ancestor of an edge in graph G.

Origin is the starting point from which all the minimal cut sets are generated i.e. all zeros in their corresponding columns of Former matrix.

2.4. Algorithm For Finding Minimal Cut-sets

This section presents the algorithm for finding all minimal cuts.

Input: (G)

Processes:

1. *Initialize* $\mathcal{C} \leftarrow \emptyset$*, k* $\leftarrow$ *1, temp* $\leftarrow \emptyset$
2. *Construct Former matrix of G.*
3. *Select Origin* (Ψ) $\leftarrow$ *Columns having only 0 entries in G.*
4. $\mathcal{C} \leftarrow \Psi$
5. *Repeat for each element in* $\mathcal{C}$ *till no change occurs.*

    temp $\leftarrow \emptyset$*, k* $\leftarrow$ *1*

    q $\leftarrow$ *Remove* (Ψ*, k)*

    p $\leftarrow$ *Former (G, q)*

    temp $\leftarrow$ *temp U {p} U {*Ψ *~q}*

    if temp $\subset \Psi$ *then*

        discard temp   *//Removes duplicate entry* *else*

$\forall (x, y) \in temp$

if $x \text{þ} y \wedge y \text{þ} x$ *then*

$\Psi \leftarrow \Psi \cup temp$

6. $k = k+1$
7. $\mathcal{C} \leftarrow Add(C, temp)$

Output: Minimal cut-sets in $\mathcal{C}$.

2.5. Illustration

Consider a directed network with 6-nodes and 10-links (Fig. 1). Here '*s*' is the source node and '*t*' is the terminal node. The Former matrix of Fig. 1 is constructed and presented in step 1 of illustration.

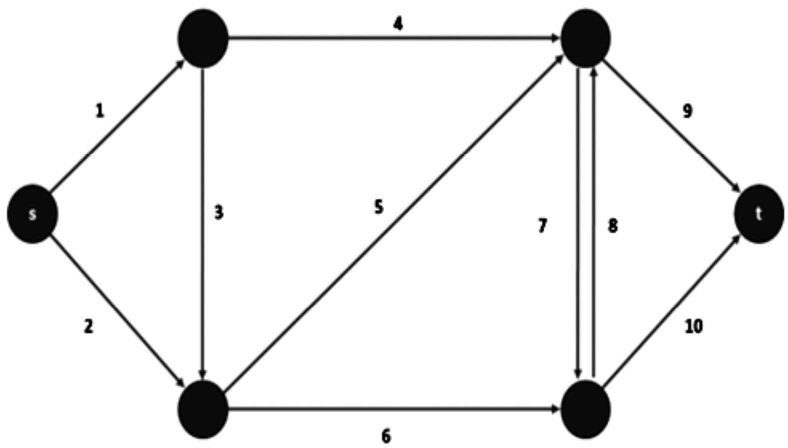


Figure 1: A network with 6 nodes and 10 links

Table 1
Former matrix construction

	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0	0	0
5	0	1	1	0	0	0	0	0	0	0
6	0	1	1	0	0	0	0	0	0	0
7	0	0	0	1	1	0	0	1	0	0
8	0	0	0	0	0	1	1	0	0	0
9	0	0	0	1	1	0	0	1	0	0
10	0	0	0	0	0	1	1	0	0	0

Step 1:

Step 2: Initially, Origin set Ψ = {9, 10} guarantees network failure.

So, {9, 10} may be considered as one of the potential cut-set. For arc 9, the potential cut-set Ψ 1={4, 5, 8, 10} by discarding {9}. Similarly, for arc 10, the potential cut-set Ψ 2 = {6, 7, 9} by discarding {10}.

Step 3: Considering Ψ 1= {4, 5, 8, 10}, the potential cut-set for arc 10 is Ψ 3= {4, 5, 8, 6, 7}. Arc 7 and 8 are having inherent ancestor relationship (þ), so they are eliminated.

Therefore Ψ 3 = {4, 5, 6}.

Similarly, considering Ψ 2 = {6, 7, 9}, the potential cut-set for arc 9 is Ψ 4 = {6, 7, 4, 5, 8} and after eliminating arc7 and arc 8, Ψ 4 = {4, 5, 6} which is duplicate cut-set in

Step 4: The set containing all the minimal cut-sets of the network are: $\mathcal{C}$ = {(9, 10), (4, 5, 8, 10), (6, 7, 9), (4, 5, 6)}.

By applying the inclusion-exclusion principle on the generated cut-sets of set $\mathcal{C}$, we obtain the unreliability expression as follows:

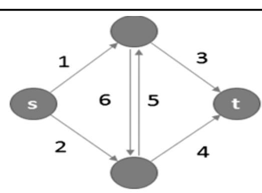
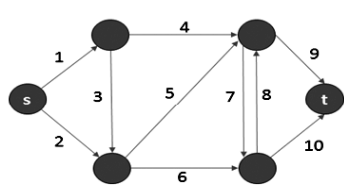
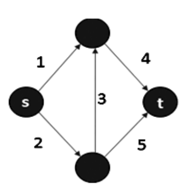
$$UR = q^2 + q^4 + 2\,q^3 - q^6 - q^5 - q^5 - q^7 - q^7 - q^6 + q^{12}$$

For simplicity, considering the value of q to be 0.1 and $p = 1\text{-}q$, the two-terminal reliability of the example network is computed to be 0.9558.

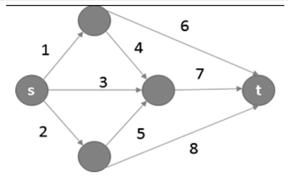
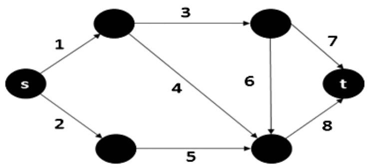
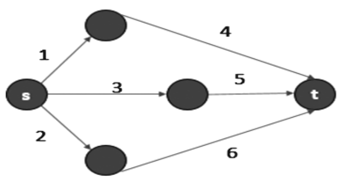
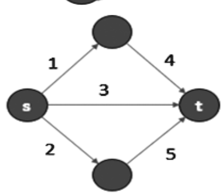
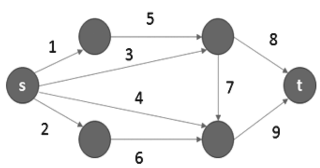
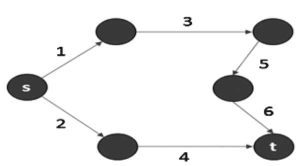
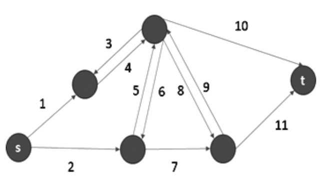
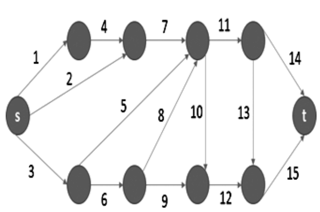
3. RESULT AND DISCUSSION

The proposed algorithm has been implemented on some sample benchmark networks and the results are recorded. The recorded results are presented in Table-II. In all the cases the reliability of the edges are assumed to be 0.9. However, the proposed approach can also be applicable for heterogeneous link reliability.

Table 2
Application of the proposed algorithm on some sample benchmark networks

Sl. No	*Interconnection Network*	*Minimal Cuts*	*Two-Terminal Reliability*
1		{(3, 4), (1, 5, 4), (3, 2, 6), (1, 2)}	0.978141
2		{(9, 10), (4, 5, 8, 10), (6, 7, 9), (4, 5, 6)}	0.987922
3		{(4, 5), (1, 3, 5), (4, 2), (1, 2, 3)}	0.978141

(contd...Table 2)

Sl. No	*Interconnection Network*	*Minimal Cuts*	*Two-Terminal Reliability*
4		{(6, 7, 8), (1, 7, 8), (3, 4, 5, 6, 8), (6, 7, 2), (1, 3, 4, 5, 8), (1, 7, 2), (3, 4, 5, 6, 2), (1, 2, 3, 4, 5)}	0.995966
5		{(7, 8), (3, 8), (4, 5, 6, 7), (3, 4, 5, 6)}	0.979904
6		{(4, 5, 6), (1, 5, 6), (4, 3, 6), (4, 5, 2), (1, 3, 6), (1, 2, 5), (2, 3, 4), (1, 2, 3)}	0.992028
7		{(3, 4, 5), (1, 3, 5), (2, 3, 4), (1, 2, 3)}	0.996006
8		{(8, 9), (4, 6, 7, 8), (3, 5, 9), (3, 4, 5, 6, 7)}	0.9888302
9		{(4, 6), (2, 6), (4, 5), (2, 5)}	0.960599
10		{(3, 10, 11), (4, 5, 10, 11), (5, 8, 11), (3, 7, 9, 10), (4, 5, 8, 11), (4, 5, 7, 10), (4, 5, 7), (4, 5, 7, 8), (5, 7)}	0.9869815
11		{(14, 15), (11, 15), (12, 13, 14), (11, 12, 13)}	0.988140

From Table-II it can be observed that, a network having 5 nodes and 6 links (Sl. No. 6 of Table-II) generates 8 numbers of minimal cut-sets and the computed reliability is 0.992028. By taking another network having 4 nodes, 5 links (Sl. No. 3 of Table-II) generates only 4 numbers of minimal cut-sets and the reliability evaluated is 0.978141. Similarly, the proposed algorithm is also implemented on relatively larger size networks of 10 nodes and 15 links (Sl. No. 11 of Table-II) to yield two-terminal reliability value of 0.988140. The above observations show that the proposed algorithm efficiently computes the two-terminal reliability of the inputted interconnection networks. It can also be observed that the proposed algorithm is also applicable to large size networks as well.

CONCLUSIONS

The presented work in this paper enumerates all the non-redundant minimal cut-sets for exact evaluation of two-terminal reliability. The proposed algorithm is also implemented on some sample benchmark networks to prove its correctness and efficiency. The proposed algorithm is applicable for all kinds of directed interconnection networks including homogenous and heterogeneous link capacity, regular and general interconnection networks. It leaves a scope for the intend researchers for further investigation to implement it on undirected networks as well.

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