IJCTA, 9(2-A), 2016, pp. 479-488

Mathematical Model of an Azimuthal-Elevation Tracking System of Small Scale Heliostat

M. Debbache*, A. Takilalte*, O. Mahfoud*, H. Karoua*, S. Bouaichaoui*, M. Laissaoui* and A. Hamidat*

ABSTRACT

The heliostat is an assembly of mirrors or a single mirror above a pedestal. It is oriented mechanically toward the displacement of the sun. It have a relative position depends to the sun and the tower position. For the high ratio of reflection solar rays to the top of a high tower, where the incident solar energy is converted to thermal energy, which is used to drive steam turbines and produce electricity. Heliostat has two motions, an azimuthal and an elevation motion. In this paper we have established a mathematical model and method to define the important control parameters which is the rotational speed and torque engine must be provided by an azimuthal-elevation tracking system to guide a small scale Heliostat.

Keywords: Heliostat, tracking system, mathematical model, control, etc.

1. INTRODUCTION

The huge world demand of the electricity, and the catastrophic effect of pollution on environment due by the uses of the fossil energy sources and the limitation of this sources, push us to look for a friendly and renewable energy sources to product the electricity. The sun is a kind of this sources of energy, many technologies used to exploit this source of energy such us the central solar tower- [1].

The central solar tower is an infrastructure used to produce the electricity by the concentration the solar reflected rays at the top of the tower, where an absorption system used to convert this rays to the heat energy to turn drive stream turbine. The reflection of sun rays have been done by the heliostats. The heliostat is a machine has a reflection area, it has a studied position in the field of the central solar, used to follow the sun motion by a tracking system [2-3]. Some recent control methods are discussed in [11-16].

Many types of solar tracking system with different accuracies exist , the tracking system can be implemented by using one-axis with higher accuracy, two-axis sun-tracking systems, in this work a mathematical model of a heliostat with biaxial tracking system has been presented, reserved to guide a heliostat by two independent stepper engines. An applied example has been presented in this work to define the torque provided by the stepper engines an theirs rotation velocity of a prototype of a heliostat designed by SolidWorks at Renewable Energy Development Center (CDER -Algiers).

2. MODEL DESIGN

A proposed design showed in Figure 1, is a prototype of a heliostat have been designed by SolidWorks at Renewable Energy Development Center (CDER-Algiers). It has 1m² reflection area,1m of height, 7.5m of tower's distance and 0deg of facing angle, used in a central solar of tower, have 10m of tower's height

* Centre de Développement des Energies Renouvelables, CDER, BP 62 Route de l'observatoire Bouzaréah, 16340, Algiers, Algeria

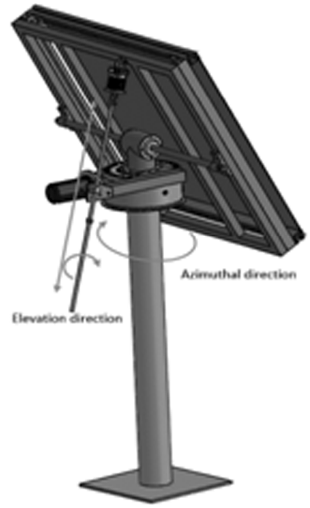

Figure 1: Heliostat design model

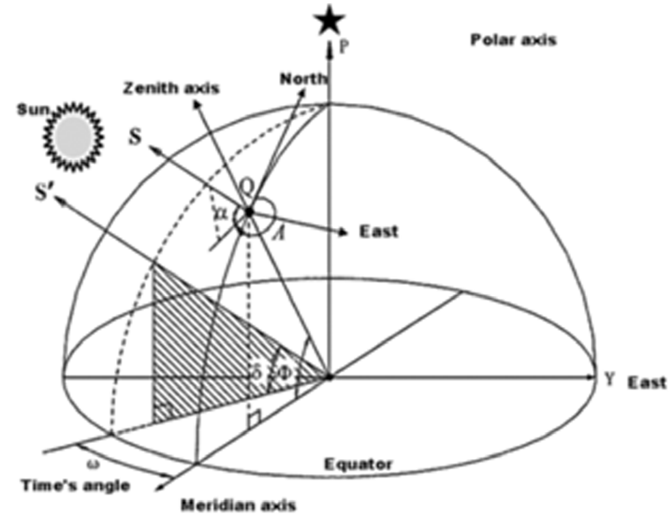

Figure 2: Locating the sun-related coordinate

guided by an azimuthal-elevation tracking system. The elevation mechanism is a screw-nut system controlled by a stepper engine where the screw is 0.025m in diameter,0.005m in thread step and 0.7m in length. The azimuthal mechanism is a screw- gear system controlled by a second stepper engine. The gear is 0.15m and screw is 0.03m in diameter and 0.008m in thread step.

3. MATHEMATICAL MODEL

3.1. Sun angle

All configuration of heliostat tracking system based on the sun position, where defined by two principals angles, altitude angle (α) and an azimuthal angle (A), Figure. 2 presents the coordinate system attached to the center of earth and her surface.

(S'): The sun vector is defined by the time angle (ω) and decline angle (δ) presented in Figure. 3. Where the decline angle equal[4]:

$$\delta = 23.45 \sin\left(\frac{360\left(284+N\right)}{365}\right) \tag{1}$$

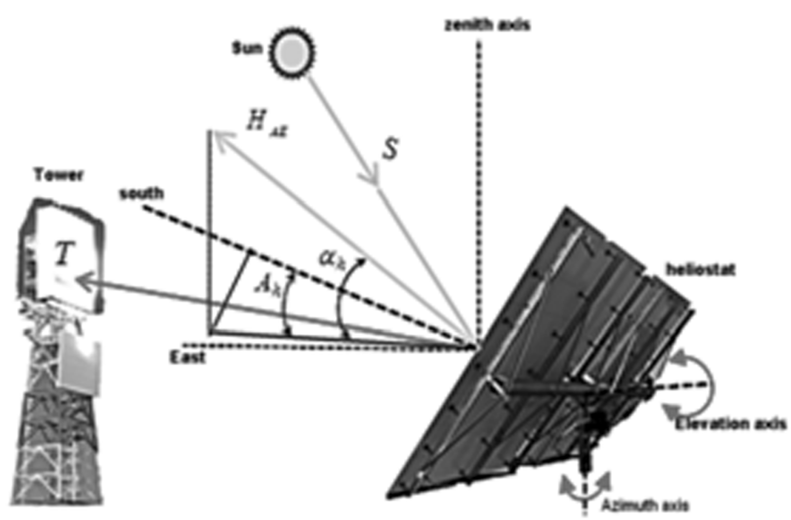

Figure 3: Angles of AE tracking system

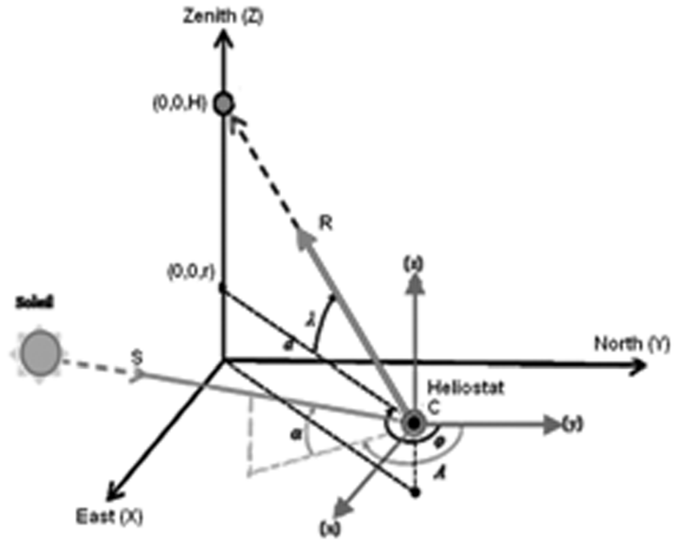

Figure 4: Incident and reflected rays

N: the rank number of days in the year. For example: N = 1 correspond the first January and N = 42 correspond the eleventh of February.

The time angle equal:

$$\omega = 15(TSV - 12) \tag{2}$$

TSV: The solar time in 24 hours, it can written by [5] [6]:

$$TSV = TL - DE + \left(\frac{E_t + 4L}{60} \right) \tag{3}$$

Where *TL* is the time given by the clock, *DE* is the deference in time to the Greenwich line, *L* is the site's longitude and *Et* is the correction of time equation.

$$TSV = TL - DE + \left(\frac{E_t + 4L}{60} \right) \tag{4}$$

Where

$$JD = (360/365)\,(N - 81) \tag{5}$$

In the position (*Q*) situated in the earth surface at the altitude angle (Φ), the position of the vector (*S*) on the coordinate system attached to surface of the earth defined by the altitude angle (α) and the azimuthal angle (*A*) of the sun it have been presented in Figure. 3[3].

Where

$$\alpha = \sin^{-1}\left(\sin\delta \sin\Phi + \cos\delta\, \cos\omega \cos\Phi\right) \tag{6}$$

$$A = \cos^{-1}\left(\frac{\sin\delta \cos\Phi - \cos\delta \cos\omega \sin\Phi}{\cos\alpha} \right) \tag{7}$$

If: $\quad sin(\omega) = 0 \Rightarrow A + 2\pi - A$

3.2. Heliostats angle

The heliostat position to solar tower is defined by the front angle (φ) and focal angle (λ) which have a relationship with the distance between a heliostat and tower, see Figure. 4[3,7].

$$\lambda = arctg\left(\frac{H_t - r}{d}\right) \tag{8}$$

Where H_t is the tower's high and R is the heliostat's high. To define the configuration of the azimuthal-elevation tracking system in any position of a heliostat in solar central field. The coordinate system attached to the underground and heliostat surface plans. Which the normal vector of the heliostat (H_{AE}) is defined by the angle azimuthal angle (A_h) and the elevation angle (α_h), this angles can be derived exclusively by a reflection law, concerning the position of the sun vector, the vector of target position at the tower and the normal vector of the heliostat[8].

$$A_h = \sin^{-1}\left(\frac{\cos\alpha \sin A + \cos\lambda \sin\varphi}{\cos^2\alpha \cos^2\lambda - 2\cos\alpha\cos\lambda\cos(A+\varphi)}\right) \tag{9}$$

$$\alpha_h = \sin^{-1}\left(\frac{\sin\alpha + \sin\lambda}{\sqrt{2\left(1+\sin\alpha\sin\lambda - \cos\alpha\cos\lambda\cos(A+\varphi)\right)}}\right) \tag{10}$$

3.3. The rotation velocities

The heliostat states have been determined by the azimuthally and the elevation angle, where the variation of the two angles have been discussed in the term of the time necessary to refreshing the value of this parameters for deferent position in solar central field and deferent annual states. Figure. 12, 13 present an example of the azimuthally and the elevation rotation velocity distribution of the heliostat have 7.5 m of distance from the solar tower and 0deg of facing angle. In Figure. 5, TRCPE is the time necessary of the variation of the elevation angle and TRCPA for the azimuthal angle, we consider this two factors as a step time of position variation to determine the elevation an azimuthal velocity. Where maximum of TRCPA is almost 90 seconds at two time instances between 8H00 to 9H00, and 16H00 to 18H00, and his minimum is at between 12H00 to 13H00 with the value of 50 seconds. For TRCPE increases from 25 seconds in the morning and get the maximum between 12H00 to 13H00, and it starts decrease in afternoon to return to same value of the starting at the morning, so the heliostat rotation velocities equal:

$$\Omega_{EL} = \frac{\Delta\alpha_h}{TRCPE} \tag{11}$$

$$\Omega_{AZ} = \frac{\Delta A_h}{TRCPA} \tag{12}$$

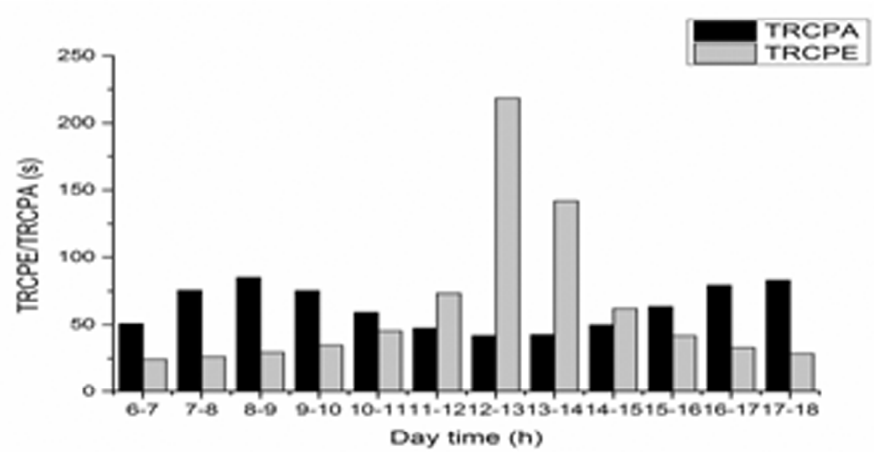

Figure 5: The variation of TRCPE/TRCPA.

Then the rotational velocities provided by the elevation and azimuthal stepper engine are defined by:

$$\Omega_{EL,engine} = \frac{\Omega_{EL} l p_{ELscrew}}{2\pi} \tag{13}$$

$$\Omega_{AZ,engine} = \frac{\Omega_{EL} l p_{AZscrew}}{2\pi} \tag{14}$$

Where l is the length of heliostat reflection area and p is a screw step (Figure. 11).

3.4. Torque engine

The start moving of the heliostat requires a value of a torque engine greater than the loads value (P) applied by the heliostat structure in all instance of changing the mirror position obtained by the synchronizing of the elevation and azimuthal mechanism in motion. The elevation mechanism moves the reflection area by screw- nut system controlled by a stepper engine where the torque in up and down moving is defined by the equations[9,10]:

Down:

$$C_{EL,engine} = \frac{F_{El} D}{2}\left(\frac{f\pi D - p}{\pi D + fp}\right).S \tag{15}$$

Up:

$$C_{EL,engine} = \frac{F_{El} D}{2}\left(\frac{p + f\pi D}{\pi D - fp}\right)S \tag{16}$$

In Figure 7, F_{El} is the elevation force, p is the screw step of the elevation mechanism, D is the diameter of screw, f is the friction coefficient and s is the security factor mostly choose it equal (1.2~1.3).

For the azimuthal mechanism presented in Figure 8 and 9, the torque engine is defined by the same equation of the moving up in elevation mechanism but here the active force is the azimuthal force acting in screw-gear system used (F_{AZ}), the torque applied in the opposite way to return to the start point is the maximum torque can be provided by the azimuthal engine with maximum velocity[10].

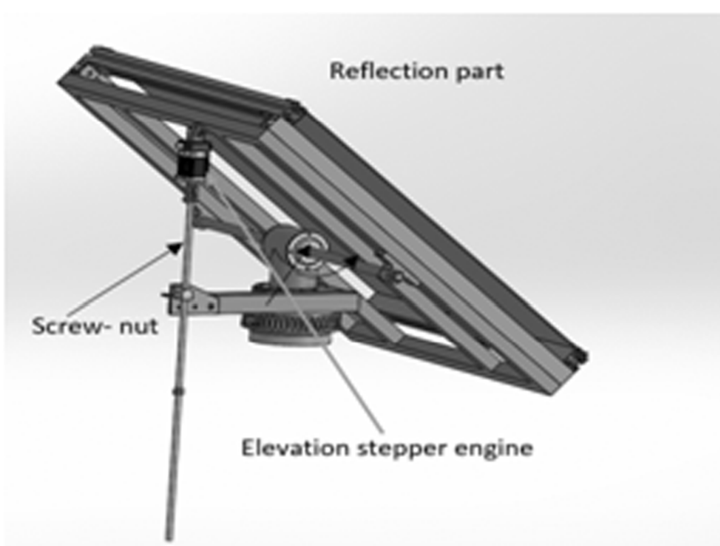

Figure 6: The elevation mechanism

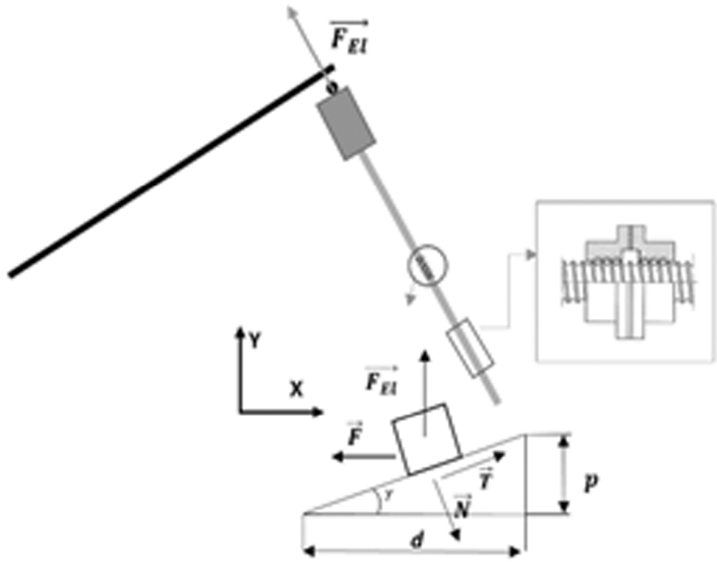

Figure 7: The model of elevation mechanism

Figure 8: The azimuthal mechanism

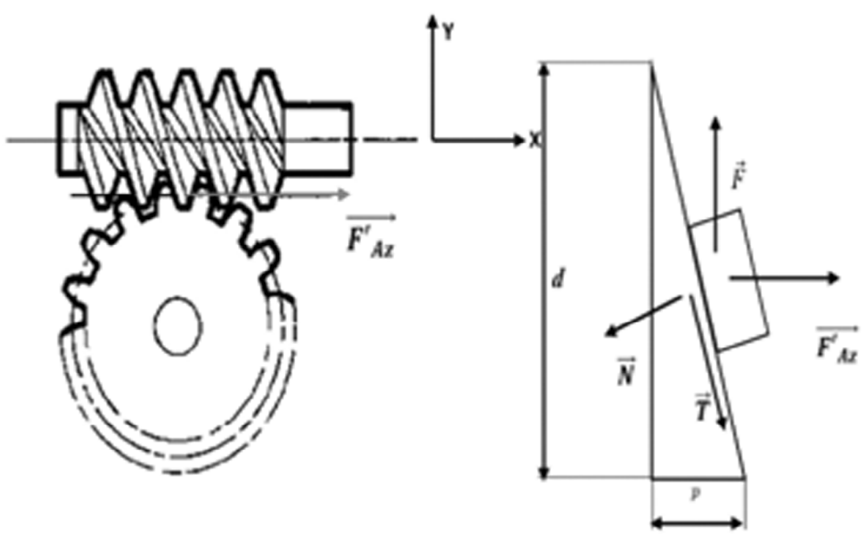

Figure 9: The model of azimuthal mechanism

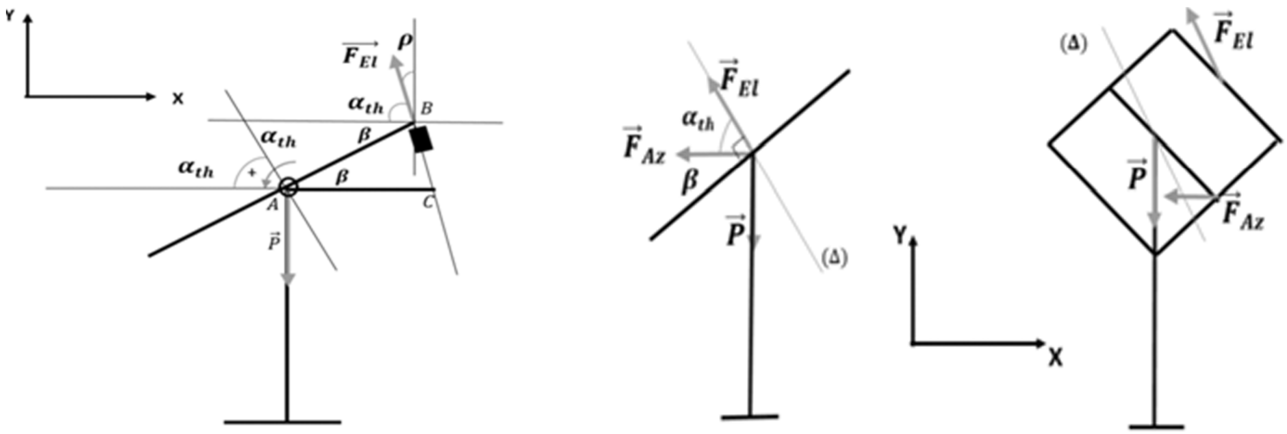

Figure 10: Kinematic model of the elevation and Azimuthal mechanism

$$C_{AZ,engine} = \frac{F_{AZ'} D}{2}\left(\frac{p + f\pi D}{\pi D - fp}\right).S \tag{17}$$

Figure 10 presents a simple model was estimated to define the forces, which are the elevation (F_{El}) force and the azimuthal force (F_{AZ}), can be applied to move a heliostat. The application of the first Newton low gives:

$$\sum F_y = 0 \quad F_{El} = P / \cos(\rho) \tag{18}$$

Where:

$$\rho = \frac{\pi}{2} - \alpha_{th} \tag{19}$$

With the same method to define the maximum value of the azimuthal force, the results were:

$$\sum F_x = 0 \Rightarrow F_{AZ} = F_{El}.\cos(\alpha_{th}) \tag{20}$$

That gives:

$$F_{AZ} = -P.\cos(\alpha_{th}) / \cos(\rho) \tag{21}$$

$$F_{AZ} = -P.\cos(\alpha_{th}) / \cos\left(\frac{\pi}{2} - \alpha_{th}\right) \tag{22}$$

The maximum elevation force equal the potential load value where the heliostat reflation area take the horizontal position which called the security position where the elevation angle equal $\pi/2$, and for azimuthal force the maximum being at an elevation angle equal $\pi/4$ .we define the effective azimuthal force (F'_{AZ}) acting in screw-gear gives from the equivalent torque applied in screw-gear system where:

$$C_{gear} = F'_{AZ} d_{gear} = F_{AZ} l \tag{23}$$

$$F'_{AZ} = \frac{F_{AZ} l}{2 d_{gear}} \tag{24}$$

4. RESULTS AND ANALYSIS

In this paper,we have established the Matlab programming codes to determine the variation of the azimuthal and elevation angles, velocities and torques for the proposed heliostat design in the solstice summer day in

Ghardaïa where the latitude angle is 32.4deg and the longitudinal angle is 3.8deg [10] [11] , the results showed in Figure. 11 and 12 give the range of the variation of the both angles, [-55.78: 41.53deg] for the azimuthal angle, [29.3:55.36deg] for the elevation angle in down moving and [55.36:39.27deg] in up moving.

Figure 13, 14 show the variation of the engines velocity in the range of [0.021:0.032rd/s] for the azimuthal motion with a mean value 0.026rd/s, and [0.005:0.027rd/s] for the elevation motion with a mean value 0.0188rd/s, the mean value equivalent in degrees to 1.5deg/s for azimuthal velocity and 1.08deg/s for the elevation velocity, this results proves with the commercial engines properties that the value 0.9deg/s is the best velocity choice for the high accuracy.

Figure. 14, 15 present the variation of the azimuthal and elevation torque engine, where the azimuthal engine starts moving the heliostat reflection area with maximum value (7N.m) for reason of the passing by the critical position at elevation angle value 45deg in moving up motion where the potential effect is the maximum. A low value at the instances between [12H00/13H00] where the reflection area position far to the critical position and after starts up to the final value (4N.m) at the instance 18H00 passing by the critical position in moving down motion.

For the elevation torque, the engine works by two phases, the first phase when the engine moves up the reflection area with a start torque value (3N.m) to the low value (1.75N.m) at the instance 13H00 where the

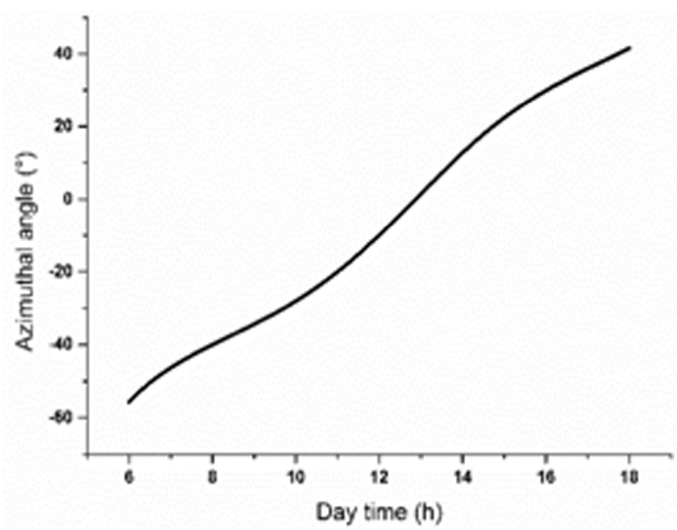

Figure 11: The heliostat azimuthal angle

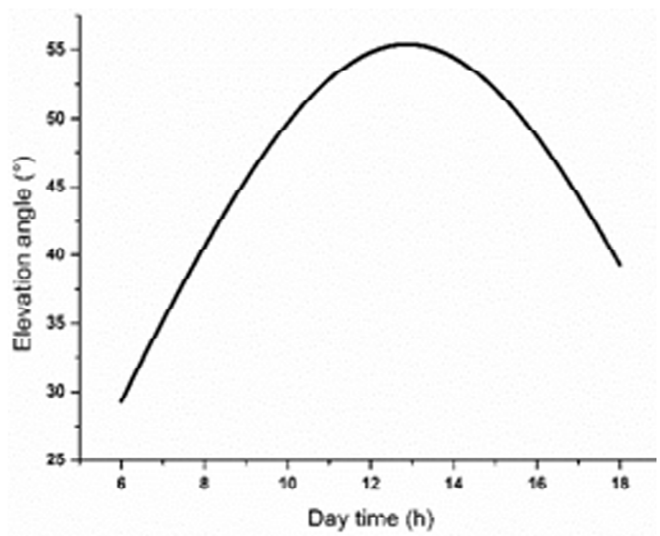

Figure 12: The heliostat elevation angle

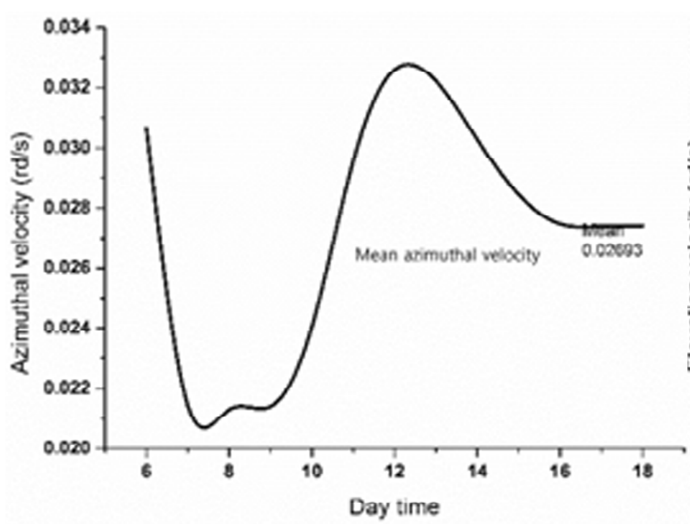

Figure 13: The azimuthal velocity

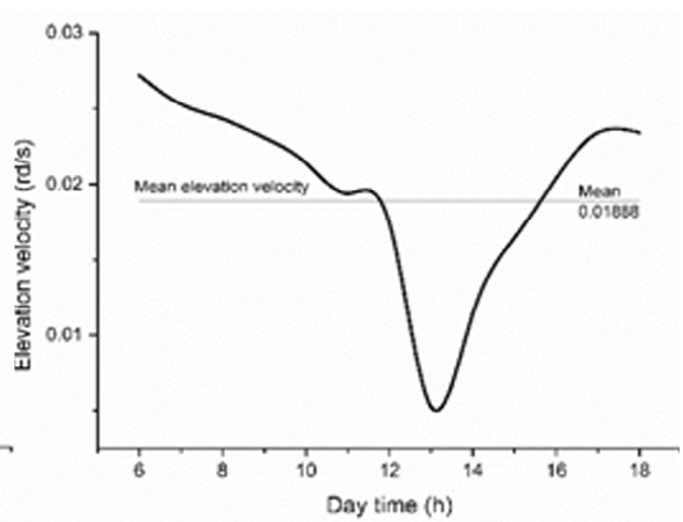

Figure 14: The elevation velocity

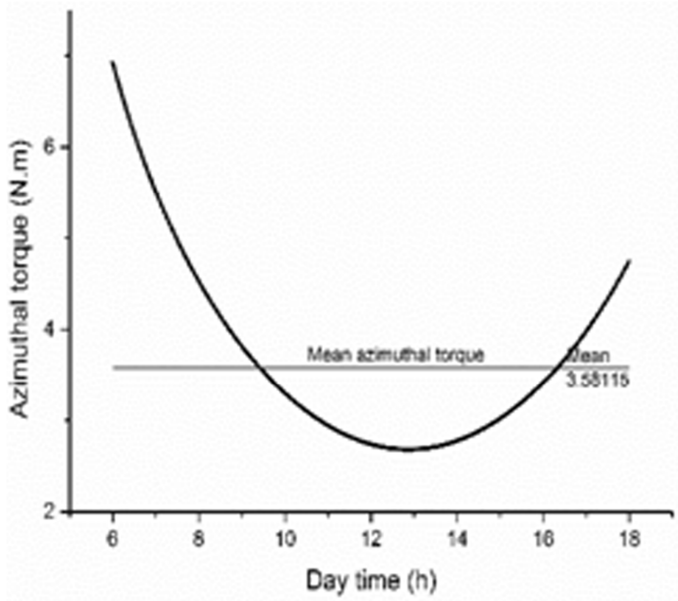

Figure 15: The azimuthal torque engine

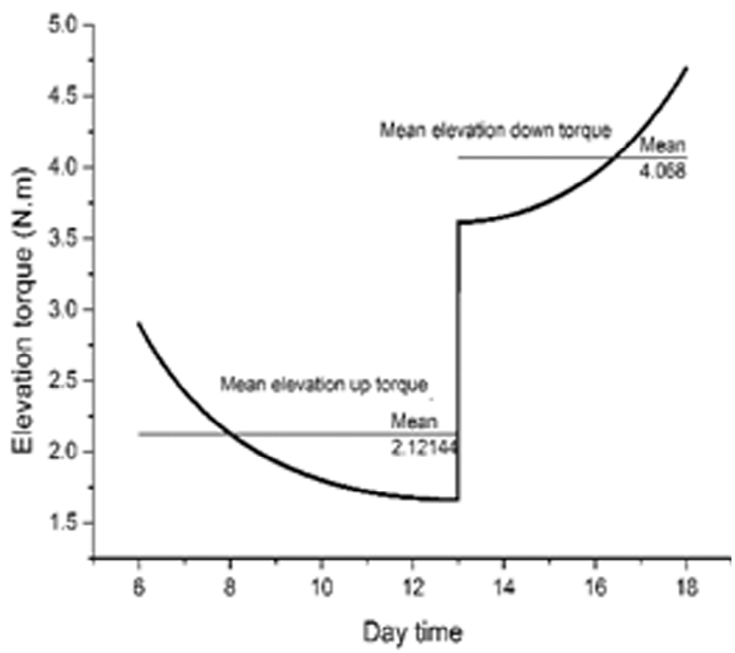

Figure 16: The elevation torque engine

elevation angle get the maximum (55.36deg) the engine starts enter in the second phase of moving down of the reflection area with a value of torque greater than the values of the first phases (3.5N.m) and continues increase to the value (4.5N.m) at the final instance 18H00.in this phase the engine resists the maximum loads applied by the heliostat reflection area.

5. CONCLUSIONS

To determine the control parameters of the heliostat tracking system should be know the technical parameters, design solution implanted in this system and the mathematical model. This work is a particular example to define the control parameters, which are the rotation velocity and the torque engine. Where we established a mathematical modeland method have been established for a small scale heliostat controlled by an azimuthal-elevation tracking system.

REFERENCES

[1] Y. T. Chen, A. Kribus, B. H. Lim, C. S. Lim, K. K. Chong, J. Karni, R. Buck, A. Pfahl, and T. P. Bligh, "Comparison of two sun tracking methods in the application of a heliostat field," *Journal of Solar Energy Engineering*, **126**, 638-644, 2004.

[2] M. Guo, F. Sun, Z. Wang, and J. Zhang, "Properties of a general azimuth–elevation tracking angle formula for a heliostat with a mirror-pivot offset and other angular errors," *Solar Energy*, **96**, 159-167, 2013.

[3] A. Gama, M. Haddadi and A. Malek, "Etude et réalisation d'un concentrateur cylindro-parabolique avec poursuite solaire aveugle," *Review of Renewable Energy*, **11**, 437-451, 2008.

[4] K. K. Chong and M. H. Tan, "Range of motion study for two diûerent sun-tracking methods in the application of heliostat ûeld," *Solar Energy*, **85**,1837-1850, 2011.

[5] M. Iqbal, *An Introduction to Solar Radiation*, Academic Press, Toronto, Canada, 1983.

[6] R. Merat and A. Moreau, *Physique Appliquée*, Nathan Technique, Paris, France, 1994.

[7] O. Aliman, I. Daut, M. Isa, and M. R. Adzman, "Simplification of sun tracking mode to gain high concentration solar energy,"*American Journal of Applied Sciences*, **4**(3), 171-175, 2007.

[8] A. Gamil, S. U.-H. Gilani, and H. H. Al-kayiem, "Simulation of incident solar power input to fixed target of central receiver system in Malaysia," *Proc. 2013 IEEE Conference on Sustainable Utilization and Development in Engineering and Technology*, **CSUDET-2013**, 92-97, 2013.

[9] A. Montenon, *Analyse, Mutualisation et Optimisation par la Commande de la Consommation Energétique des Héliostats Autonomes des Centrales à Concentration Solaire*, Doctorate Thesis, Université de Toulouse, France, 2013.

[10] A. V. Bardakas, V. K. Chasiotis, D. A. Tzempelikos, and A. E. Filios, "Design and implementation of a three axis digitally controlled traverse system for flow surveys in a drying chamber," *International Journal of Engineering & Technology,***3**(3),350-356, 2014.

[11] S. Vaidyanathan, "A novel 3-D conservative chaotic system with sinusoidal nonlinearity and its adaptive control", *International Journal of Control Theory and Applications*, **9** (1), 115-132, 2016.

[12] S. Vaidyanathan and S. Pakiriswamy, "A five-term 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control method", *International Journal of Control Theory and Applications*, **9** (1), 61-78, 2016.

[13] S. Vaidyanathan, K. Madhavan and B.A. Idowu, "Backstepping control design for the adaptive stabilization and synchronization of the Pandey jerk chaotic system with unknown parameters", *International Journal of Control Theory and Applications*, **9** (1), 299-319, 2016.

[14] A. Sambas, S. Vaidyanathan, M. Mamat, W.S.M. Sanjaya and R.P. Prastio, "Design, analysis of the Genesio-Tesi chaotic system and its electronic experimental implementation", *International Journal of Control Theory and Applications*, **9** (1), 141-149, 2016.

[15] S. Vaidyanathan, S. Sampath and A.T. Azar, "Global chaos synchronisation of identical chaotic systems via novel sliding mode control method and its application to Zhu system," *International Journal of Modelling, Identification and Control*, **23** (1), 92-100, 2015.

[16] I. Pehlivan, I.M. Moroz and S. Vaidyanathan, "Analysis, synchronization and circuit design of a novel butterfly attractor", *Journal of Sound and Vibration*, **333** (20), 5077-5096, 2014.