DESINGNING GROUP ACCEPTANCE SAMPLING PLANS FOR THE GENERALISED EXPONENTIAL DISTRIBUTION USING MINIMUM ANGLE METHOD

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ABSTRACT: In this paper, minimum angle method is introduced to find the parameters of a group acceptance sampling plan in which the truncated lifetimes follows a generalized exponential distribution. The values of operating ratio corresponding to the producer's risk and consumer's risk are calculated and using minimum angle method and the minimum angle θ is found. Tables are constructed and examples are provided.

Keywords: Generalized exponential distribution, Group acceptance sampling: producer's risk, Operating characteristics, Producer's risk, Minimum angle method.

1. INTRODUCTION

The sampling procedure is turned out to be a life testing, when the quality characteristics are related to product lifetime. It is very often not to observe a failure for a highly reliable product within available experimental time duration.

The ordinary acceptance sampling plan for different distributions have been developed by many researchers including, Kantam *et al*., [1]. Baklizi [2], Balakrishnan *et al*., [3] and Lio *et al*., [4] and [5]. However, it requires more cost time and observation to collect the sample items for making a decision of either accepting or rejecting the lot of products. An acceptance sampling plan involves quality contracting on product orders between the producers and consumers. In order to fix the acceptance number in a sampling plan is very difficult. By the minimum angle criteria the optimum value of the acceptance number was designed. In this paper designing group acceptance sampling plan under generalized exponential distribution using minimum angle method is presented. GASP plans having minimum angle by keeping the producer's risk below 5% and consumer's risk below 10% for specified AQL and LQL were presented.

2. GENERALIZED EXPONENETIAL DISTRIBUTION

The cumulative distribution function (cdf) of the generalized exponential distribution is given by

$$G_{T\{(\delta,\lambda)\}}(t) = \left(1 - e^{-\frac{t}{\lambda}}\right)^{\delta} \quad (1)$$

Where λ and θ are scale and shape parameter. If some other parameters are involved then they are assumed to be known, the shape parameter is very important factor for designing the minimum angle group acceptance sampling plan. We assume that the distribution function depends on time only through the ratio t/λ. The median of this distribution for $\delta = 2$ is given by $\mu = 1.2279\,\lambda$.

It is further observed that the generalized exponential distribution can be used quite effectively in many circumstances, in place of lognormal or generalized Rayleigh distribution also. The closeness properties with other distributions, Statistical inferences, order statistics, have been discussed by several authors. The readers are referred to the recent review article by Gupta and Kundu [6] for a current account on the generalized exponential distribution. It is also observed in different studies that generalized exponential distribution might fit better than Weibull or gamma distribution in some cases.

3. OPERATING CHARACTERISTICS FUNCTION

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the mean from its true value μ. This function is called Operating Characteristic (OC) function of the sampling plan. When the sample size $n = rg$ is known, we can able to find the probability of acceptance of a lot when the quality of the product is sufficiently good. Using the probability of acceptance the corresponding to producer's risk and consumer's risk the values are tabulated to calculate the minimum angle method.

Notation:

g – Number of groups
r – Number of items in a group
n – Sample size
c – Acceptance number
t_0 – Termination time
a – Test termination time multiplier
m – Shape parameters
β – Consumer's risk
P – Failure probability
$L(p)$ – Probability of acceptance
μ – Mean life
μ_0 – Specified life
θ – Minimum angle
δ – Shape parameter

4. GROUP ACCEPTANCE SAMPLING (GASP) UNDER EXPONENTIAL DISTRIBUTION

A product is considered as good and acceptable for consumer's use, if the true value μ which is the median of the lifetime distribution of a product is not smaller than the specified median value μ_0. If the actual value μ is smaller than value μ_0. The following GASP is proposed based on the truncated life test:

1. Select the number of groups *g* and allocate predefined r items to each group so that the sample size for a lot will be $n = gr$.
2. Select the acceptance number *c* for a group *and* the experiment time t_0.
3. Perform the experiment for the *g* groups simultaneously and record the number of failures for each group.
4. Accept the lot if atmost *c* failures occur in each of all groups.
5. Terminate the experiment if more than *c* failures occur in any group and reject the lot.

The probability of rejecting a good lot is called the producer's risk, whereas it is represented as α. The probability of accepting a bad lot is known as the consumer's risk, which is represented as β. We will determine the number of groups *g* in the proposed sampling plan so that the consumer's risk does not exceed the value β = .01. Since the lot size is large enough, we can use the binomial distribution to develop the GASP. According to the GASP the lot of products is accepted only if there are atmost c failures observed in each of the g groups. The

$$L(p) = \left(\sum_{i=0}^{r} \binom{r}{i} p^i (1-p)^{r-i} \right)^g \tag{2}$$

Where *p* is the probability that an item in a group fails before the termination time $t_o = a\mu_0$.

The probability *p* for the generalized exponential distributions with δ = 2 is given by

$$p = G_{T\{(2,\lambda)\}}(t_0) = \left(1 - e^{-1.2279\frac{\mu_0}{\mu}}\right)^{\delta}. \tag{3}$$

The minimum number of groups required can be determined by considering the consumer's risk when the true median life equals the specified median life when $\mu = \mu_0$. In the case of zero failure test that is for $c = 0$ the number of groups can be determined by the minimum integer satisfying the following inequality

$$g > \frac{In\beta}{r In(1 - p_0)}.$$

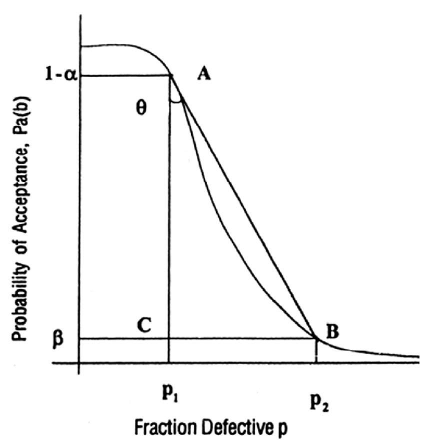
5. Minimum Angle Method

The practical performance of a sampling plan is revealed by it operating characteristic curve. Norman Bush *et al.*, [8] have used different techniques involving comparison of some portion of the OC curve to that of the ideal curve. The approach of minimum angle method by considering the tangent of the angle between the lines joining the points (AQL, $1 - \alpha$) (LQL β) is shown in Figure where p_1 = AQL, p_2 = LQL. By employing this method one can get a better discriminating plan with the minimum angle. Tangent of angle made by lines *AB* and *AC* is

$$\tan \theta = BC/AC$$

$$\tan \theta = (p_2 - p_1)/(Pa(p_1) - Pa(p_2)) \quad (4)$$

The smaller the value of this $\tan \theta$, closer is the angle θ approaching zero and the chord *AB* approaching *AC*, the ideal condition through (AQL, $1 - \alpha$). This criterion minimizes simultaneously the consumer's and producer's risks. Thus both the producer and consumer favour the plans evolved by the criterion.



Minimum angle for given p_1 and p_2

6. Designing GASP for the Generalized Exponential Distribution Using Minimum Angle Method

- First calculate the mean ratio μ/μ_0 corresponding to d_1 and d_2, Where the mean ratio, $d_1 = \mu_1/\mu_0$, be the acceptable reliability level (ARL) at the producer's risk and the mean ratio, $d_2 = \mu_2//\mu_0$ which is equal to 1, be the lot tolerance reliability level (LTRL) at the consumer's risk.
- Select the values for termination ratio a, *r* for given shape parameter $\delta = 2$.
- Locate the value of mean ratio corresponding to the probability of acceptance of GASP along with producer's and consumer's risk.

- Find tan θ from the table.
- Calculate the value $\theta = \tan^{-1}(\tan \theta)$
- Select the parameter of the sampling plan corresponding to the smallest value of θ.

7. CONSTRUCTION OF TABLES

The Tables are constructed using *OC* function for GASP plans *e* the probability of failure under Exponential distribution is given by the equation (3). Using the above values the minimum angle tan θ is calculated using the equation (4) Tables 1 and 2 give the proposed values of tan θ for various values *c* and *g* corresponding to the mean ratio and α below 5% and β below 10% for the given p_1 and p_2. Numerical value in these tables reveals the following facts.

The parameter $n = rg$ and θ can be obtained from the selected table corresponding to μ/μ_0 *a*, *r* and *g* along with producer's risk and consumer's risk.

Example 1: Suppose one want to design GASP under generalized Exponential distribution for given α = .05, β = .01, $\mu/\mu_0 = 4$, and *a* = .7 *r* = 9 among the various values of θ the Minimum angle corresponds to *c* = 2 and *g* = 4 the value $\theta = 17.011^0$ Thus, the desired sampling plan has parameters as (4, .7, 2, 4) as mean ratio, Number of items, Acceptance number, Number of groups respectively.

Example 2: For given $\mu/\mu_0 = 8$, *a* = .9 and *r* = 9 Minimum angle corresponds to *c* = 2 and *g* = 4 the value $\theta = 23.398^0$.

Example 3: For given $\mu/\mu_0 = 4$, *a* = 1.5 and *r* = 9 Minimum angle corresponds to *c* = 2 and *g* = 1 the value $\theta = 32.9079^0$.

Example 4: For given $\mu/\mu_0 = 4$, *a* =.7 and *r* = 6 Minimum angle corresponds to *c* = 2 and *g* = 7 the value $\theta = 17.70131^0$.

From the above values we come to know that Minimum angle plan of GASP under generalized exponential distribution is given by (4, 9, 2, 4) corresponding to (μ/μ_0, *r, c*, *g*). Thus we design the Group sampling plan with generalized exponential distribution for the given values termination ratio *a* and the number of testers *r* corresponding to the groups using minimum angle method. Moreover, the operating characteristics function increases disproportionately when the Group sampling plan can be used to test multiple number of items, which would be beneficial in terms of test time and test cost.

8. CONCLUSION

The procedure and necessary tables for the selection of GASP under exponential distribution is presented for the given acceptable quality. According to Srinivasa Rao for given mean ratio $\mu/\mu_0 = 4$, *a* = .7 and *r* = 6 the producer's risk is .0098 and consumer's risk is .05 is obtained for *c* = 2 and *g* = 6. Whereas when we apply minimum angle method we can see that for given mean ratio $\mu/\mu_0 = 4$, $a = .7$ and $r = 6$ the producer's risk is .01051 and consumer's risk is .015 is obtained for $c = 2$ and $g = 11$ which increases the number of groups. Therefore this plan reduces time cost and cost of the tester. This criterion minimizes simultaneously the consumer's and producer's risk. This minimum angle plan provides better discrimination of accepting good lots among minimum number of groups.

This ability of the plan can discriminate between good and bad quality. Moreover, the operating characteristics function increases disproportionately when the quality improves. By this plan GASP would be beneficial in terms of test time and test cost.

Table 1
Minimum Angle GASP Under Exponential Distribution For $r = 6$, $c = 2$, $\delta = 2$

a	μ/μ_0	g	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
0.7	4	7	0.993299	0.068629	0.319166	17.70131
		8	0.992345	0.046806	0.312121	17.33425
		10	0.990441	0.021771	0.304668	16.94431
		11	0.98949	0.014848	0.302801	16.84638
	6	8	0.999132	0.046806	0.330453	18.2863
		7	0.99924	0.068629	0.338164	18.68369
		9	0.999023	0.031922	0.325404	18.02512
		10	0.998915	0.021771	0.32206	17.85168
	8	8	0.999825	0.046806	0.338015	18.67602
		6	0.999869	0.100629	0.35823	19.70906
		7	0.999847	0.068629	0.345929	19.08197
		9	0.999804	0.031922	0.332825	18.40871
	10	7	0.999957	0.068629	0.349747	19.27713
		10	0.999939	0.021771	0.332999	18.41771
		8	0.999951	0.046806	0.341741	18.86741
		9	0.999945	0.031922	0.336489	18.59751
	12	10	0.999978	0.021771	0.335036	18.52269
		11	0.999976	0.014848	0.332682	18.40136
		12	0.999974	0.010126	0.331096	18.3195
		9	0.999981	0.031922	0.338548	18.70344
0.8	4	8	0.984798	0.009995	0.352798	19.43275
		6	0.988577	0.031611	0.359374	19.76713
		5	0.990472	0.056217	0.36811	20.20918
		9	0.982914	0.00562	0.351899	19.38692

a	μ/μ_0	g	$L(p_1)$	$L(p_2)$	tan θ	θ
	6	4	0.999099	0.099975	0.409862	22.28685
		5	0.998874	0.056217	0.390934	21.3522
		7	0.998424	0.017775	0.375788	20.59564
		9	0.997974	0.00562	0.371356	20.37278
	8	8	0.999631	0.009995	0.38193	20.90337
		6	0.999723	0.031611	0.390421	21.32674
		5	0.999769	0.056217	0.400584	21.83023
		4	0.999815	0.099975	0.420043	22.78451
	10	6	0.999921	0.031611	0.39509	21.55849
		7	0.999908	0.017775	0.38953	21.28239
		5	0.999934	0.056217	0.405386	22.06695
		8	0.999895	0.009995	0.386473	21.13019
	12	5	0.999977	0.056217	0.408097	22.20023
		9	0.999958	0.00562	0.387339	21.17333
		8	0.999963	0.009995	0.389049	21.25846
		6	0.999972	0.031611	0.39773	21.68918
0.9	4	8	0.972634	0.001592	0.400657	21.83386
		6	0.979404	0.007969	0.400495	21.82585
		5	0.982807	0.017831	0.403176	21.95809
		9	0.969266	0.000711	0.401686	21.88464
	6	4	0.998301	0.039898	0.437217	23.61576
		5	0.997877	0.017831	0.427562	23.14971
		7	0.997029	0.003562	0.421786	22.86932
		9	0.996182	0.000711	0.420937	22.82803
	8	8	0.999289	0.001592	0.431675	23.34864
		6	0.999467	0.007969	0.434374	23.47888
		5	0.999556	0.017831	0.438698	23.68698
		4	0.999645	0.039898	0.448744	24.16788
	10	6	0.999846	0.007969	0.439955	23.74734
		7	0.999821	0.003562	0.43802	23.65438
		5	0.999872	0.017831	0.444362	23.95854
		8	0.999795	0.001592	0.437167	23.61336
	12	5	0.999954	0.017831	0.447589	24.11275
		9	0.999918	0.000711	0.439936	23.74644
		8	0.999927	0.001592	0.44032	23.76487
		6	0.999945	0.007969	0.443143	23.90019

a	μ/μ_0	g	$L(p_1)$	$L(p_2)$	tan θ	θ
1.2	4	3	0.959364	0.006565	0.524031	27.65594
		2	0.972722	0.035063	0.532492	28.03493
		5	0.933194	0.00023	0.535172	28.15443
		4	0.946189	0.001229	0.528378	27.85101
	6	4	0.99237	0.001229	0.551719	28.88634
		5	0.990472	0.00023	0.55222	28.90835
		2	0.996178	0.035063	0.568955	29.63791
		3	0.994272	0.006565	0.553637	28.97053
	8	2	0.99915	0.035063	0.58703	30.41421
		2	0.99915	0.035063	0.58703	30.41421
		5	0.997877	0.00023	0.567283	29.56551
		4	0.998301	0.001229	0.56761	29.57969
	10	3	0.99962	0.006565	0.5795	30.09229
		2	0.999746	0.035063	0.596542	30.81787
		5	0.999366	0.00023	0.575972	29.94076
		2	0.999746	0.035063	0.596542	30.81787
	12	5	0.999769	0.00023	0.581162	30.16351
		2	0.999908	0.035063	0.602059	31.05042
		3	0.999861	0.006565	0.584814	30.3197
		4	0.999815	0.001229	0.581716	30.18726
1.5	4	3	0.894042	0.000265	0.639886	32.6146
		2	0.928051	0.004127	0.619006	31.75776
		3	0.894042	0.000265	0.639886	32.6146
		1	0.963354	0.064239	0.636086	32.45988
	6	1	0.994192	0.064239	0.686283	34.46116
		3	0.982676	0.000265	0.649638	33.00928
		2	0.988417	0.004127	0.648397	32.95927
		3	0.982676	0.000265	0.649638	33.00928
	8	2	0.997254	0.004127	0.670397	33.83779
		1	0.998626	0.064239	0.712542	35.47145
		1	0.998626	0.064239	0.712542	35.47145
		2	0.997254	0.004127	0.670397	33.83779
	10	3	0.998726	0.000265	0.680833	34.24832
		2	0.99915	0.004127	0.683185	34.34028
		1	0.999575	0.064239	0.726781	36.00894
		2	0.99915	0.004127	0.683185	34.34028
	12	3	0.999524	0.000265	0.688346	34.54142
		3	0.999524	0.000265	0.688346	34.54142
		2	0.999683	0.004127	0.690906	34.64083
		2	0.999683	0.004127	0.690906	34.64083

Table 2
Minimum Angle GASP Under Exponential Distribution For $r = 9$, $c = 2$, $\delta = 2$

a	μ/μ_0	g	$L(p_1)$	$L(p_2)$	tan θ	θ
0.7	4	7	0.974352	0.001127	0.303242	16.86953
		8	0.970742	0.000427	0.304152	16.91723
		4	0.985262	0.020676	0.305958	17.01191
		3	0.988926	0.054526	0.315842	17.5283
	6	8	0.996501	0.000427	0.31594	17.5334
		7	0.996937	0.001127	0.316023	17.53774
		9	0.996064	0.000162	0.315994	17.53622
		5	0.997812	0.00784	0.317887	17.63479
	8	8	0.999284	0.000427	0.322504	17.87471
		6	0.999463	0.002973	0.32327	17.91445
		7	0.999373	0.001127	0.322701	17.88494
		9	0.999194	0.000162	0.322447	17.87176
	10	7	0.999822	0.001127	0.326155	18.06398
		4	0.999898	0.020676	0.33264	18.39921
		8	0.999797	0.000427	0.325935	18.05258
		9	0.999771	0.000162	0.325856	18.04853
	12	3	0.999973	0.054526	0.346645	19.11861
		4	0.999964	0.020676	0.334666	18.50365
		5	0.999955	0.00784	0.330339	18.28042
		9	0.999919	0.000162	0.327814	18.14988
0.8	4	3	0.978509	0.015338	0.357059	19.64956
		6	0.957481	0.000235	0.359269	19.7618
		5	0.96444	0.000947	0.35694	19.6435
		4	0.971449	0.003811	0.355411	19.56575
	6	4	0.99641	0.003811	0.371264	20.36816
		5	0.995514	0.000947	0.370529	20.33115
		7	0.993725	0.000001	0.370844	20.34698
		6	0.994619	0.000235	0.370598	20.33459
	8	4	0.999247	0.003811	0.379705	20.792
		6	0.998871	0.000235	0.378488	20.73105
		5	0.999059	0.000947	0.378687	20.741
		3	0.999435	0.015338	0.38408	21.01077
	10	6	0.999675	0.000235	0.382784	20.94606
		3	0.999838	0.015338	0.388593	21.2358
		5	0.999729	0.000947	0.383036	20.95865
		4	0.999783	0.003811	0.384117	21.01263
	12	5	0.999904	0.000947	0.385548	21.08404
		3	0.999942	0.015338	0.391168	21.36386
		2	0.999961	0.061733	0.410503	22.31831
		6	0.999884	0.000235	0.385281	21.07072

a	μ/μ_0	g	$L(p_1)$	$L(p_2)$	tan θ	θ
0.9	4	4	0.950008	0.000552	0.409766	22.28216
		2	0.974684	0.023504	0.409024	22.24572
		3	0.962267	0.003603	0.405831	22.08882
		5	0.937905	0.000001	0.414813	22.5293
	6	4	0.99332	0.000552	0.422083	22.88379
		3	0.994986	0.003603	0.422673	22.91246
		5	0.991657	0.000001	0.422556	22.90679
		2	0.996654	0.023504	0.430592	23.29631
	8	3	0.998922	0.003603	0.432707	23.39845
		2	0.999281	0.023504	0.441372	23.81533
		5	0.998204	0.00001	0.43146	23.33827
		4	0.998563	0.000552	0.431539	23.3421
	10	4	0.999581	0.000552	0.436806	23.596
		3	0.999685	0.003603	0.438098	23.65813
		2	0.99979	0.023504	0.446981	24.08374
		5	0.999476	0.00001	0.436615	23.58679
	12	4	0.99985	0.000552	0.439896	23.74453
		3	0.999887	0.003603	0.441227	23.80837
		2	0.999925	0.023504	0.450203	24.23741
		5	0.999812	0.00001	0.439674	23.73386
1.2	4	2	0.909283	0.000742	0.549559	28.79137
		1	0.953563	0.027246	0.539012	28.3252
		3	0.867058	0.000001	0.575851	29.93554
	6	1	0.992785	0.027246	0.566348	29.52496
		3	0.978509	0.0001	0.558898	29.20073
		2	0.985621	0.000742	0.555227	29.0402
	8	2	0.996654	0.000742	0.568271	29.60833
		1	0.998326	0.027246	0.582803	30.23376
		3	0.994986	0.0001	0.568858	29.63371
	10	1	0.99949	0.027246	0.591904	30.62145
		2	0.99898	0.000742	0.576491	29.96306
		3	0.99847	0.00001	0.576363	29.95754
	12	1	0.999812	0.027246	0.59728	30.84901
		2	0.999624	0.000742	0.581544	30.1799
		3	0.999435	0.0001	0.58128	30.16858
1.5	4	1	0.887391	0.003612	0.647125	32.9079
		2	0.787462	0.000001	0.726277	35.99005
	6	1	0.979191	0.003612	0.654187	33.1922
		2	0.958814	0.000001	0.665626	33.64877
	8	1	0.994758	0.003612	0.671738	33.89076
		2	0.989543	0.000001	0.672827	33.93372
	10	1	0.998326	0.003612	0.683397	34.3486
		2	0.996654	0.00001	0.682074	34.29686
	12	1	0.999364	0.003612	0.690771	34.63557
		2	0.998728	0.000001	0.688713	34.55568

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