I J C T A, 7(1) June 2014, pp. 35-47

Qualitative Analysis and Control of an Eleven-Term Novel 4-D Hyperchaotic System with Two Quadratic Nonlinearities

Sundarapandian Vaidyanathan*

ABSTRACT

First, thispaperannounces an eleven-term novel4-D hyperchaotic system and discusses its qualitative properties. The proposed 4-D system is an eleven-term novel polynomial hyperchaotic system with only two quadratic nonlinearities. The novel hyperchaotic system has a saddle-point equilibrium at the origin, which is unstable.The Lyapunov exponents of the novel hyperchaotic system are obtained as $L_1 = 1.39805$, $L_2 = 0.23933$, $L_3 = 0$ and $L_4 = -17.65085$. The maximal Lyapunov exponent (MLE) for the novelhyperchaotic system isobtained as $L_1 = 1.39805$ and Lyapunov dimension as $D_L = 3.09277$. Next, we derive a new result for the adaptive controller to globally stabilize the novel hyperchaotic system with unknown parameters. The adaptive control result has been established using adaptive control theory and Lyapunov stability theory. Numerical simulations with MATLAB have been shown to illustrate the phase portraits of the novel 4-D hyperchaotic system and the adaptive control results for the hyperchaotic system.

Keywords: Chaos, hyperchaos, hyperchaotic systems, adaptive control, Lyapunov stability theory.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions [1].A chaotic system is also defined as a dynamical system having at least one positive Lyapunov exponent.

In the last four decades, many chaotic systems have been found such as Lorenz system [2], Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], Zhou system [12],Sundarapandian system [13], Sundarapandian-Pehlivan system [14], Vaidyanathan systems [15-19], Vaidyanathan-Madhavan system [20], Pehlivan-Moroz-Vaidyanathan system [20], etc.

A *hyperchaotic system* is a chaotic system having more than one Lyapunov exponent. For continuous-time dynamical systems, the minimal dimension for a hyperchaotic system is four. The first hyperchaotic system was found by Rössler [21]. This was followed by the finding of many hyperchaotic systems such as hyperchaotic Lorenz system [22], hyperchaotic Lü system [23], hyperchaotic Chen system [24], hyperchaotic Wang system [25], hyperchaotic Vaidyanathan system [26], etc.

Hyperchaotic systems have attractive features like high security, high capacity and highefficiency and they find miscellaneous applications in several areas like neural networks[27-29],oscillators [30-31], circuits [32-35], secure communication [36-37], encryption [38], synchronization [39-56], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [57-63].

* Research and Development Centre, Vel Tech University, Avadi, Chennai-600062, India

In this paper, we have announced aneleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities. We establish that the novel hyperchaotic system has a saddle-point equilibrium at the origin, which is unstable. The Lyapunov exponents of the novel hyperchaotic system are obtained as $L_1 = 1.39805$, $L_2 = 0.23933$, $L_3 = 0$ and $L_4 = -17.65085$. The maximal Lyapunov exponent (MLE) for the novel hyperchaotic system is obtained as $L_1 = 1.39805$ and Lyapunov dimension as $D_L = 3.09277$. Next, we derive a new result for the adaptive controller to globally stabilize the novel hyperchaotic system with unknown parameters. The adaptive control result has been established using adaptive control theory and Lyapunov stability theory.

The rest of this paper is organized as follows. Section 2 contains the description of the eleven-term novel 4-D hyperchaotic system proposed in this paper. Section 3 contains the qualitative properties of the novel hyperchaotic system. Section 4 contains the adaptive control results for the novel hyperchaotic system with unknown parameters. MATLAB simulations have been provided to illustrate the phase portraits and the adaptive control results obtained in this paper.

2. ANELEVEN-TERM NOVEL 4-DHYPERCHAOTIC SYSTEM

In this section, we describe aneleven-term novel 4-D hyperchaotic system with only two quadratic nonlinearities.

The novel 4-D hyperchaotic system is modeled by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_3 + x_4 \\ \dot{x}_2 &= cx_1 - x_1x_3 + x_4 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= -d(x_1 + x_2) \end{aligned} \tag{1}$$

where x_1, x_2, x_3, x_4 are the state variables and a, b, c, d are constant, positive, parameters of the system.

The system (1) exhibits a *strange hyperchaotic attractor* when the constant parameter values are chosen as

$$a = 12, \quad b = 4, \quad c = 100, \quad d = 5 \tag{2}$$

For numerical simulations, we take the initial values as

$$x_1(0) = 1.5, x_2(0) = 0.6, \ x_3(0) = 1.8, \ x_4(0) = 2.5 \tag{3}$$

Figures 1-4 give the 3-D view of the strange hyperchaotic attractor in (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

3. PROPERTIES OF THE NOVELHYPERCHAOTIC SYSTEM

(A) Invariance

The x_3- axis $(x_1 = 0, x_2 = 0, x_4 = 0)$ is invariant for the system (1). Hence, all orbits of the system (1) starting on the x_3-axis stay in the x_3-axis for all values of time.

(B) Dissipativity

We write the system (1) in vector notation as

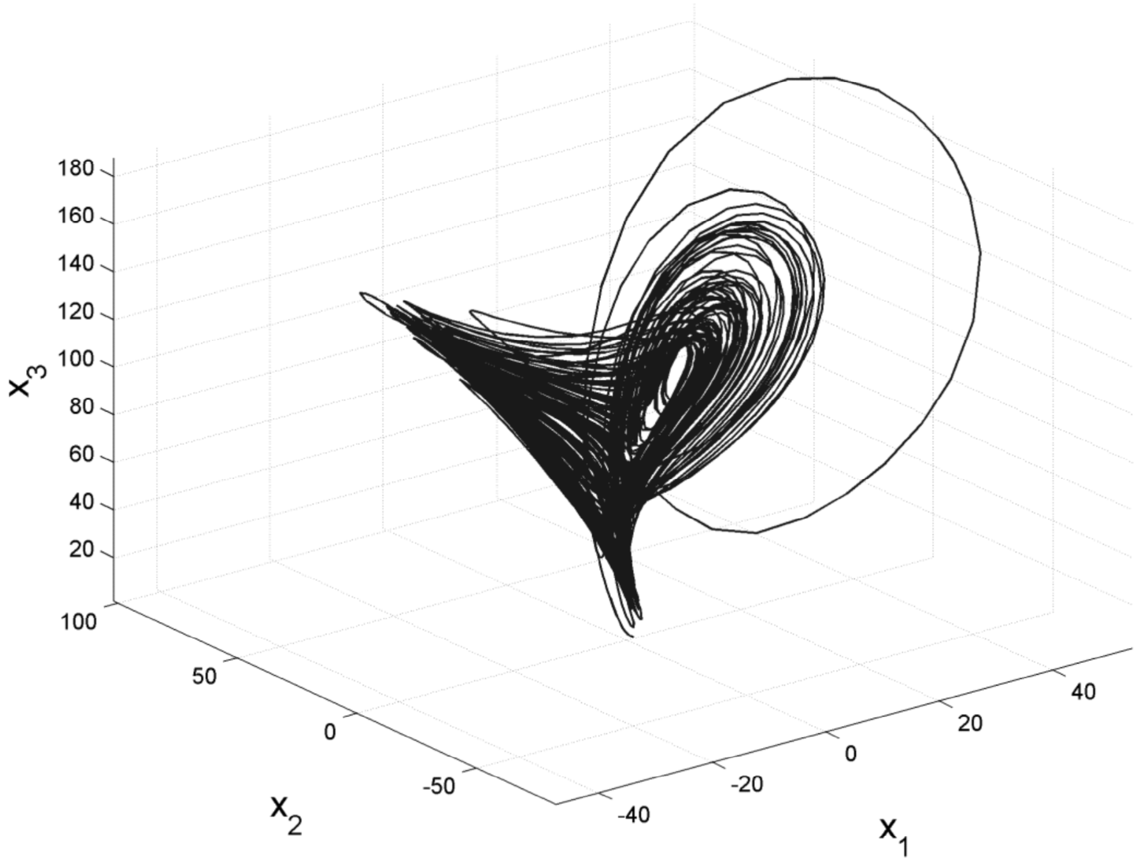

Figure 1. 3-D View of the Novel Chaotic System in (x_1, x_2, x_3) Space

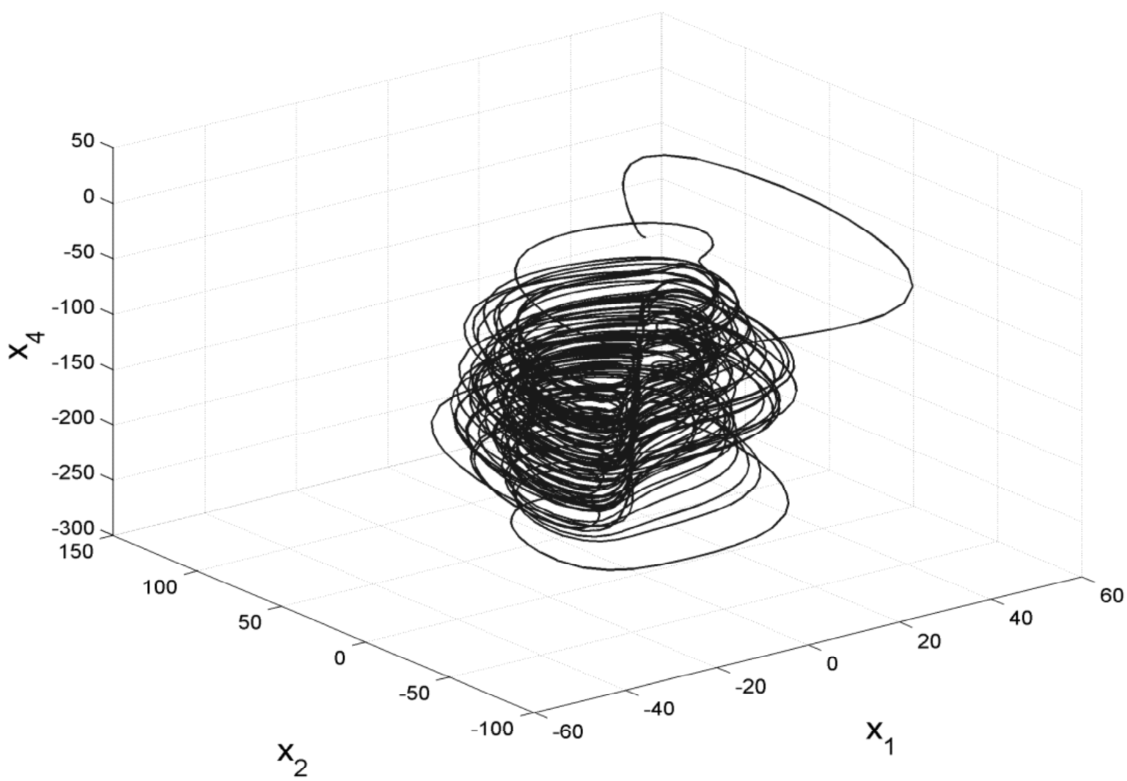

Figure 2. 3-D View of the Novel Chaotic System in (x_1, x_2, x_4) Space

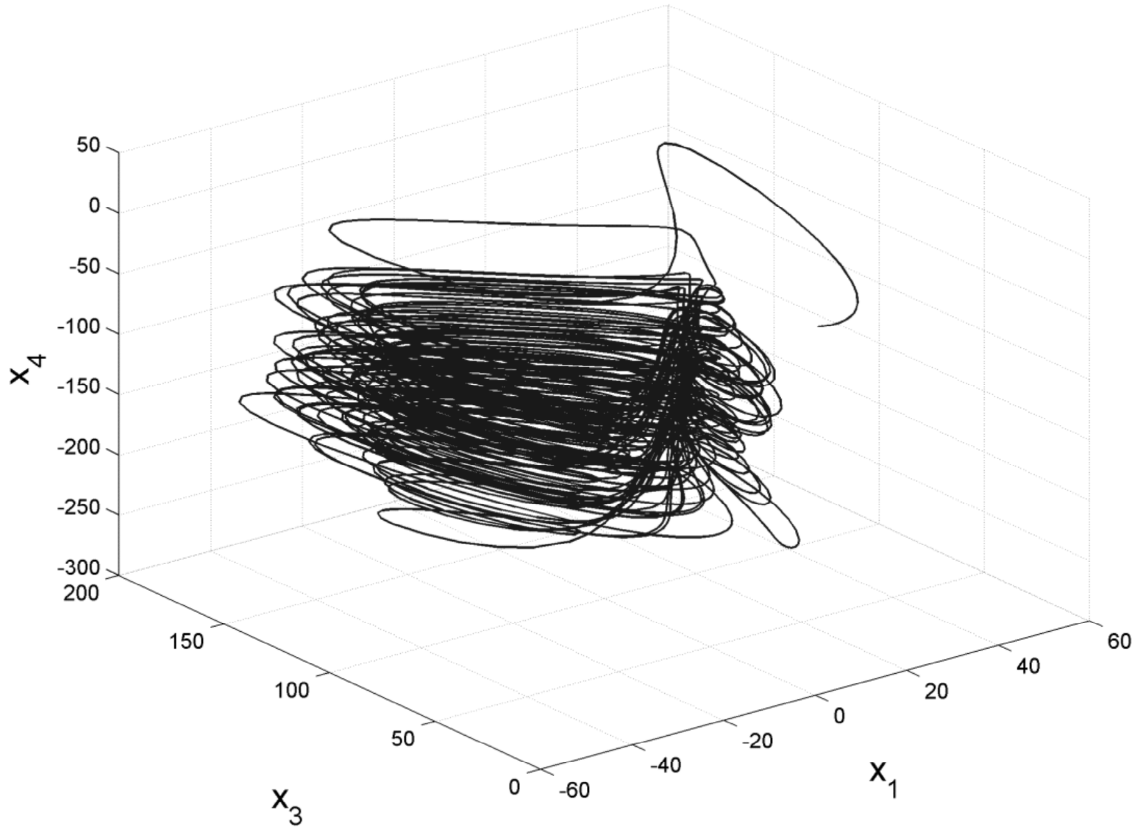

Figure 3. 3-D View of the Novel Chaotic System in (x_1, x_3, x_4) Space

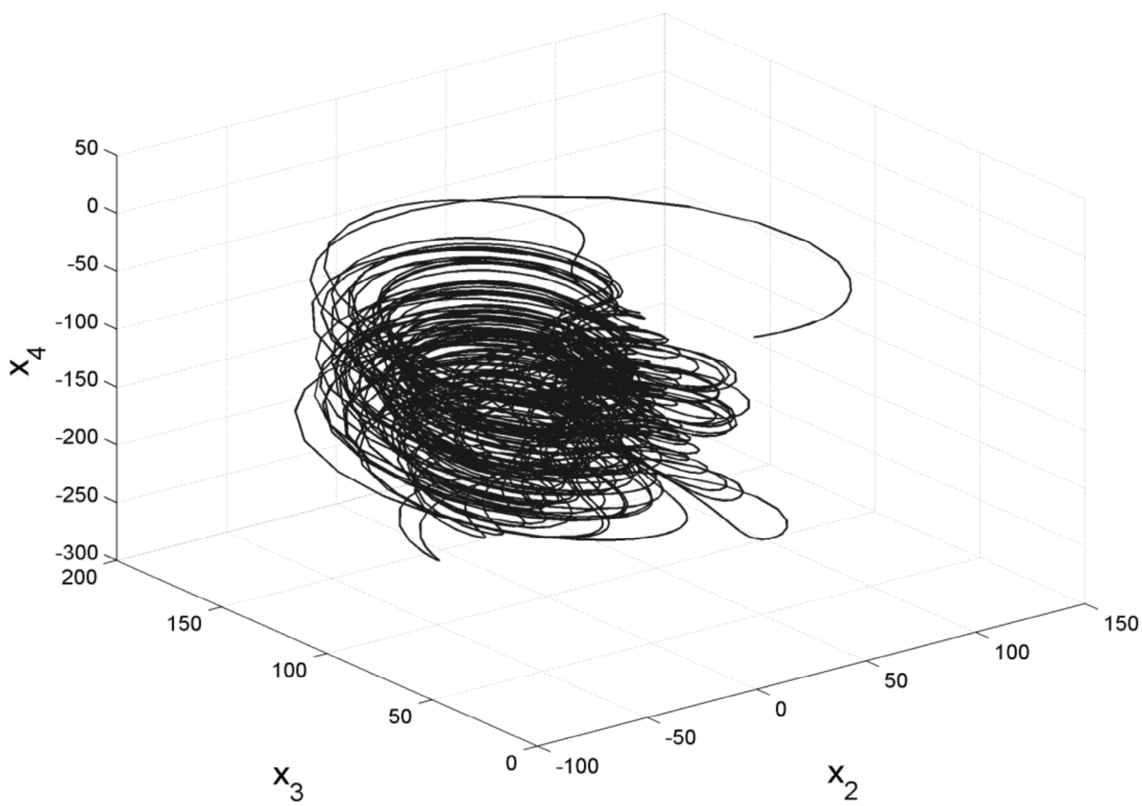

Figure 4. 3-D View of the Novel Chaotic System in (x_2, x_3, x_4) Space

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} f_1(x) &= a(x_2 - x_1) + x_3 + x_4 \\ f_2(x) &= cx_1 - x_1x_3 + x_4 \\ f_3(x) &= -bx_3 + x_1x_2 \\ f_4(x) &= -d(x_1 + x_2) \end{aligned} \tag{5}$$

The divergence of the vector field f on R^4 is obtained as

$$\operatorname{div} f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = -a - b = -\mu, \tag{6}$$

where

$$\mu = a + b > 0 \tag{7}$$

because a, b, c and d are assumed to be positive constants.

Let Ω be any region in R^4 having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Let $V(t)$ denote the hypervolume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV(t)}{dt} = \int\limits_{\Omega(t)} (\operatorname{div} f)\, dx_1dx_2dx_3dx_4 = -\mu \int\limits_{\Omega(t)} dx_1dx_2dx_3dx_4 = -\mu V(t) \tag{8}$$

Integrating the linear differential equation (10), we get the solution as

$$V(t) = V(0)\exp(-\mu t) \tag{9}$$

From Eq. (9), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel hyperchaotic system (1) is dissipative. Hence, the asymptotic motion of the system (1) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

(C) Equilibrium Points

The equilibrium points of the novel hyperchaotic system (1) are obtained by solving the nonlinear equations

$$\begin{aligned} f_1(x) &= a(x_2 - x_1) + x_3 + x_4 = 0 \\ f_2(x) &= cx_1 - x_1x_3 + x_4 = 0 \\ f_3(x) &= -bx_3 + x_1x_2 = 0 \\ f_4(x) &= -d(x_1 + x_2) = 0 \end{aligned} \tag{10}$$

From the last equation in (10), since $d > 0$, we must have

$$x_1 = -x_2 \tag{11}$$

Substituting (11) into (10), we obtain the simplified system of equations as

$$\begin{aligned} 2ax_2 + x_3 + x_4 &= 0 \\ -cx_2 + x_2x_3 + x_4 &= 0 \\ bx_3 + x_2^2 &= 0 \end{aligned} \tag{12}$$

We suppose that the parameter values are taken as in the chaotic case, i.e.

$$a = 12, \quad b = 4, \quad c = 100, \quad d = 5 \tag{13}$$

Then, it is easy to show that the system (12) has only the trivial solution $x_2 = 0,\ x_3 = 0,$ and $x_4 = 0.$

Using (11), we conclude that the system (10) has only the trivial solution $x = 0.$

Thus, $x = 0$ is the only equilibrium point of the novel hyperchaotic system (1).

The Jacobian matrix of the novel hyperchaotic system (1) at $x = 0$ is obtained as

$$J = \begin{bmatrix} -a & a & 1 & 1 \\ c & 0 & 0 & 1 \\ 0 & 0 & -b & 0 \\ -d & -d & 0 & 0 \end{bmatrix} = \begin{bmatrix} -12 & 12 & 1 & 1 \\ 100 & 0 & 0 & 1 \\ 0 & 0 & -4 & 0 \\ -5 & -5 & 0 & 0 \end{bmatrix}. \tag{14}$$

The matrix J has the eigenvalues

$$\lambda_1 = -41.2284, \quad \lambda_2 = -4, \quad \lambda_3 = 28.7045, \quad \lambda_4 = 0.5239 \tag{15}$$

This shows that the equilibrium $x = 0$ is a saddle-point, which is unstable.

Hence, the novel 4-D hyperchaotic system (1) has a unique equilibrium point at $x = 0$, which is unstable.

(D) Lyapunov Exponents

We take the parameter values of the system (1) as

$$a = 12, \quad b = 4, \quad c = 100, \quad d = 5 \tag{16}$$

We take the initial state as

$$x_1(0) = 1.5,\ x_3(0) = 0.6, \ \ x_3(0) = 1.8, \ \ x_4(0) = 2.5 \tag{17}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$L_1 = 1.39805, \quad L_2 = 0.23933, \quad L_3 = 0, \quad L_4 = -17.65085 \tag{18}$$

Eq. (18) shows that the system (1) is hyperchaotic, since it has two positive Lyapunov exponents. Since the sum of the Lyapunov exponents is negative, the system (1) is a dissipative hyperchaotic system.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1 = 1.39805.$

The dynamics of the Lyapunov exponents is depicted in Figure 5.

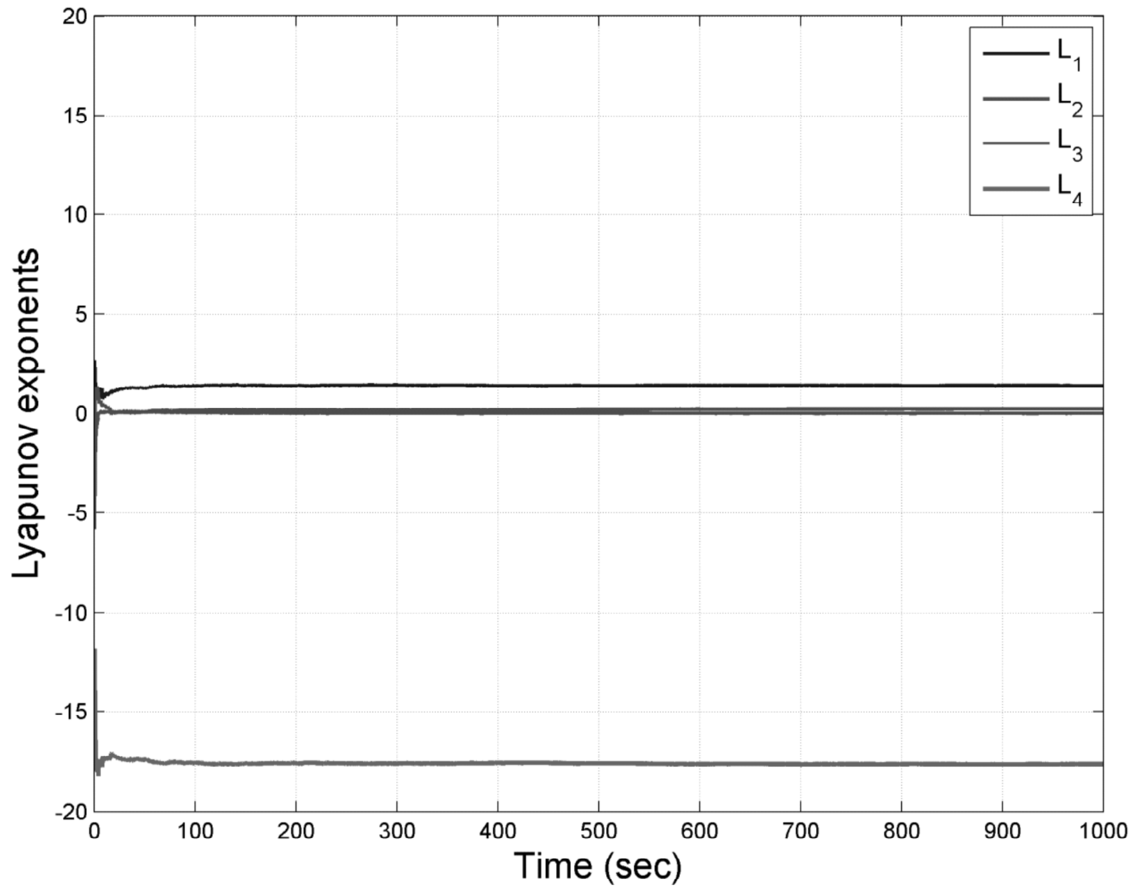

Figure 5: Dynamics of the Lyapunov Exponents

(E) Lyapunov Dimension

The Lyapunov dimension of the hyperchaotic system (1) is determined as

$$D_L = j + \frac{\sum_{i=1}^{j} L_i}{|L_{j+1}|} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.09277, \tag{19}$$

which is fractional. Thus, the ten-term 4-D system (1) is a dissipative hyperchaotic system with fractional Lyapunov dimension.

4. ADAPTIVE CONTROL OF THE NOVEL HYPERCHAOTIC SYSTEM

In this section, we derive new results for the adaptive controller to stabilize the unstable novel chaotic system with unknown parameters for all initial conditions.

Thus, we consider the controlled novel 4-D hyperchaotic system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_3 + x_4 + u_1 \\ \dot{x}_2 &= cx_1 - x_1 x_3 + x_4 + u_2 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 + u_3 \\ \dot{x}_4 &= -d(x_1 + x_2) + u_4 \end{aligned} \tag{20}$$

where x_1, x_2, x_3, x_4 are state variables, a, b, c, d are constant, unknown, parameters of the system and u_1, u_2, u_3, u_4 are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$\begin{aligned} u_1 &= -A(t)(x_2 - x_1) - x_3 - x_4 - k_1 x_1 \\ u_2 &= -C(t)x_1 + x_1 x_3 - x_4 - k_2 x_2 \\ u_3 &= B(t)x_3 - x_1 x_2 - k_3 x_3 \\ u_4 &= D(t)(x_1 + x_2) - k_4 x_4 \end{aligned} \tag{21}$$

where $A(t), B(t), C(t), D(t)$ are estimates for the unknown parameters a, b, c, d, respectively, and k_1, k_2, k_3 are positive gain constants.

The closed-loop system is obtained by substituting (21) into (20) as

$$\begin{aligned} \dot{x}_1 &= (a - A(t))(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= (c - C(t))x_1 - k_2 x_2 \\ \dot{x}_3 &= -(b - B(t))x_3 - k_3 x_3 \\ \dot{x}_4 &= -(d - D(t))(x_1 + x_2) - k_4 x_4 \end{aligned} \tag{22}$$

To simplify (22), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \\ e_d(t) &= d - D(t) \end{aligned} \tag{23}$$

Substituting (23) into (22), we obtain

$$\begin{aligned} \dot{x}_1 &= e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= e_c x_1 - k_2 x_2 \\ \dot{x}_3 &= -e_b x_3 - k_3 x_3 \\ \dot{x}_4 &= -e_d(x_1 + x_2) - k_4 x_4 \end{aligned} \tag{24}$$

Differentiating the parameter estimation error (23) with respect to t, we get

$$\begin{aligned} \dot{e}_a(t) &= -\dot{A}(t) \\ \dot{e}_b(t) &= -\dot{B}(t) \\ \dot{e}_c(t) &= -\dot{C}(t) \\ \dot{e}_d(t) &= -\dot{D}(t) \end{aligned} \tag{25}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, x_4, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{26}$$

which is positive definite on R^8.

Differentiating V along the trajectories of (24) and (25), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a\left[x_1(x_2 - x_1) - \dot{A}\right] + e_b\left[-x_3^2 - \dot{B}\right] + e_c\left[x_1 x_2 - \dot{C}\right] + e_d\left[-x_4(x_1 + x_2) - \dot{D}\right] \tag{27}$$

In view of (27), we define an update law for the parameter estimates as

$$\begin{aligned} \dot{A} &= x_1(x_2 - x_1) \\ \dot{B} &= -x_3^2 \\ \dot{C} &= x_1 x_2 \\ \dot{D} &= -x_4(x_1 + x_2) \end{aligned} \tag{28}$$

Theorem 1. The novel 4-D hyperchaotic system (20) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (21) and the parameter update law (28), where k_i, $(i = 1, 2, 3, 4)$ are positive constants.

Proof. The result is proved using Lyapunov stability theory [64].

We consider the quadratic Lyapunov function V defined by (26), which is a positive definite function on R^8.

Substituting the parameter update law (28) into (27), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 \tag{29}$$

which is a negative semi-definite function on R^8.

Thus,it can be concluded that the state vector $x(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad e_a(t) \quad e_b(t) \quad e_c(t) \quad e_d(t)\right]^T \in L_\infty. \tag{30}$$

We define $k = \min\{k_1, k_2, k_3, k_4\}$.

Then it follows from (29) that

$$\dot{V} \le -k\|x\|^2 \text{ or } k\|x\|^2 \le -\dot{V}. \tag{31}$$

Integrating the inequality (31) from 0 to t, we get

$$k\int_o^t \|x(\tau)\|^2 \, d\tau \;\le\; -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{32}$$

From (32), it follows that $x(t) \in L_2$. Using (24), we can conclude that $\dot{x}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$. This completes the proof. n

Numerical Results

For the novelsystem (20), the parameter values are taken as in the hyperchaotic case, *viz*.

$$a = 12, \quad b = 4, \quad c = 100, \quad d = 5 \tag{33}$$

We take the feedback gains as $k_i = 5$ for $i = 1, 2, 3, 4$.

The initial values of the chaotic system (27) are taken as

$$x_1(0) = 4.8, \; x_2(0) = -2.9, \; x_3(0) = -4.7, \quad x_4(0) = 9.2 \tag{34}$$

The initial values of the parameter estimates are taken as

$$A(0) = 21, \; B(0) = 12, \; C(0) = 5, \quad D(0) = 22 \tag{35}$$

Figure 6 depicts the time-history of the controlled novel hyperchaotic system.

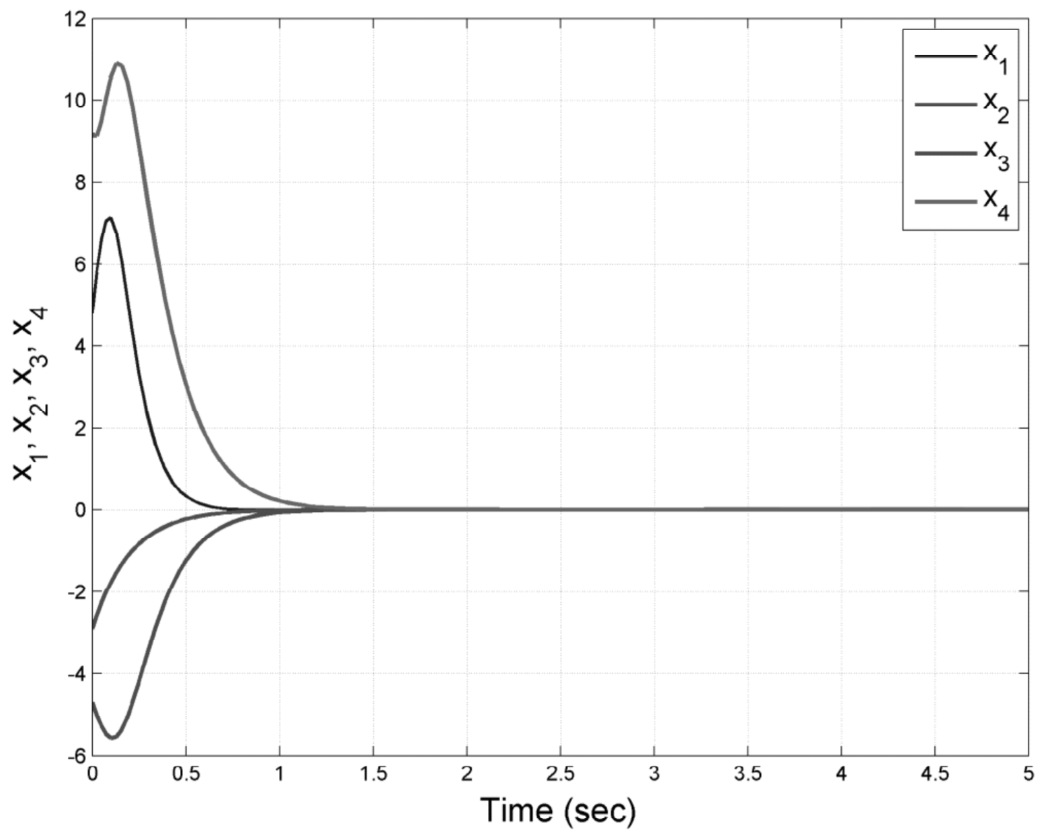

Figure 6: Time-History of the Controlled Novel Hyperchaotic System

5. CONCLUSIONS

In this paper, we have announced an eleven-term novel 4-D hyperchaotic system with only two quadratic nonlinearities. We have given a detailed qualitative analysis of the proposed system in this paper. The novel hyperchaotic system has an unstable equilibrium at the origin, which is a saddle point. The novel hyperchaotic system has the Lyapunov exponents given by $L_1 = 1.39805$, $L_2 = 0.23933$, $L_3 = 0$ and $L_4 = -17.65085$. Since the sum of the Lyapunov exponents is negative, the novel hyperchaotic system is a dissipative system. The maximal Lyapunov exponent (MLE) for the novel hyperchaotic system is obtained as $L_1 = 1.39805$ and Lyapunov dimension as $D_L = 3.09277$. In this paper, we have derived a new result for the adaptive controller to globally stabilize the novel hyperchaotic system with unknown parameters. MATLAB simulations have been shown to illustrate the phase portraits of the novel 4-D hyperchaotic system and the adaptive control results for the novel hyperchaotic system.

REFERENCES

[1] K.T. Alligood, T. Sauer and J.A. Yorke, *Chaos: An Introduction to Dynamical Systems*, Springer-Verlag: New York, USA, 1997.

[2] E.N. Lorenz, "Deterministic nonperiodic flow," *Journal of Atmospheric Sciences*, **20**, 130-141, 1963.

[3] O.E. Rössler, "An equation for continuous chaos," *Physics Letters*, **57A**, 397-398, 1976.

[4] T. Shimizu and N. Morioka, "On the bifurcation of a symmetric limit cycle to an assymetric one in a simple model," *Physics Letters A*, **76**, 201-204, 1980.

[5] R. Shaw, "Strange attractors, chaotic behavior and information flow," *Z. Naturforsch*, **36A**, 80-112, 1981.

[6] G. Chen and T. Ueta, "Yet another chaotic oscillator," *International Journal of Bifurcation and Chaos*, **9**, 1465-1466, 1999.

[7] J. Lü and G. Chen, "A new chaotic attractor coined," *International Journal of Bifurcation and Chaos*, **12**, 659-661, 2002.

[8] H.K. Chen and C.I. Lee, "Anti-control of chaos in rigid body motion," *Chaos, Solitons and Fractals*, **21**, 957-965, 2004.

[9] G. Cai and Z. Tan, "Chaos synchronization of a new chaotic system via nonlinear control," *Journal of Uncertain Systems*, **1**, 235-240, 2007.

[10] G. Tigan and D. Opris, "Analyis of a 3D chaotic system," *Chaos, Solitons and Fractals*, **36**, 1315-1319, 2008.

[11] D. Li, "A three-scroll chaotic attractor," *Physics Letters A*, **372**, 387-393, 2008.

[12] W. Zhou, Y. Xu, H. Lu and L. Pan, "On the dynamics of a new chaotic attractor," *Physics Letters A*, **372**, 5773-5778, 2008.

[13] V. Sundarapandian, "Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers," *Journal of Engineering Science and Technology Review*, **6**, 45-52, 2013.

[14] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system," *Mathematics and Computer Modelling*, **55**, 1904-1915, 2012.

[15] S. Vaidyanathan, "A new six-term 3-D chaotic system with an exponential nonlinearity," *Far East Journal of Mathematical Sciences*, **79**, 135-143, 2013.

[16] S. Vaidyanathan, "A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities," *Far East Journal of Mathematical Sciences*, **84**, 219-226, 2014.

[17] S. Vaidyanathan, "Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters," *Journal of Engineering Science and Technology Review*, **6**, 53-65, 2013.

[18] S. Vaidyanathan, "Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities," *European Physics Journal: Special Topics*, **223**, 1519-1529, 2014.

[19] S. Vaidyanathan, "Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities," *International Journal of Modelling, Identification and Control*, **22,** 41-53, 2014.

[20] S. Vaidyanathan and K. Madhavan, "Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system," *International Journal of Control Theory and Applications*, **6**, 121-137, 2013.

[21] O.E. Rössler, "An equation for hyperchaos," *Physics Letters A*, **71**, 155-157, 1979.

[22] Q. Jia, "Hyperchaos generated from the Lorenz chaotic system and its control," *Physics Letters A*, **366**, 217-222, 2007.

[23] A. Chen, J. Lu, J. Lü and S. Yu, "Generating hyperchaotic Lü attractor via state feedback control," *Physica A*, **364**, 103-110, 2006.

[24] X. Li, "Modified projective synchronization of a new hyperchaotic system via nonlinear control," *Commun. Theor. Physics*, **52**, 274-278, 2009.

[25] J. Wang and Z. Chen, "A novel hyperchaotic system and its complex dynamics," *International Journal of Bifurcation and Chaos*, **18**, 3309-3324, 2008.

[26] S. Vaidyanathan, "A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control," *International Journal of Control Theory and Applications*, **6**, 97-109, 2013.

[27] Q. Li, X.S. Yang and F. Yang, "Hyperchaos in Hopfield-type neural networks," *Neurocomputing*, **67**, 275-280, 2004.

[28] Y. Huang and X.S. Yang, "Hyperchaos and bifurcation in a new class of four-dimensional Hopfield neural networks," *Neurocomputing*, **69**, 1787-1795, 2006.

[29] P.C. Rech, "Chaos and hyperchaos in a Hopfield neural network," *Neurocomputing*, **74**, 3361-3364, 2011.

[30] K. Grygiel and P. Szlachetka, "Chaos and hyperchaos in coupled Kerr oscillators," *Optics Communications*, **177**, 425-431, 2000.

[31] Y.L. Zou, J. Zhu, G. Chen, X.S. Luo, "Synchronization of hyperchaotic oscillators via single unidirectional chaotic coupling," *Chaos, Solitons and Fractals*, **25**, 1245-1253, 2005.

[32] A.S. Elwakil and M.P. Kennedy, "Inductorless hyperchaos generator," *Microeletronics Journal*, **30**, 739-743, 1999.

[33] N. Yujun, W. Xingyuan, W. Mingjun and Z. Huaguang, "A new hyperchaotic system and its circuit implementation," *Communications in Nonlinear Science and Numerical Simulation*, **15**, 3518-3524, 2010.

[34] K. Thamilmaran, M. Lakshmanan and A. Venkatesan, "Hyperchaos in a modified canonical Chua's circuit," *International Journal of Bifurcation and Chaos*, **14**, 221-243, 2004.

[35] T. Banerjee, D. Biswas and B.C. Sarkar, "Design of chaotic and hyperchaotic time-delayed electronic circuit," *Bonfring International Journal of Power Systems and Integrated Circuits*, **2**, 13-17, 2012.

[36] C. Li, X. Liao and K. Wong, "Lag synchronization of hyperchaos with application to secure communications," *Chaos, Solitons and Fractals*, **23**, 183-193, 2005.

[37] X. Wu, H. Wang and H. Lu, "Modified generalized projective synchronization of a new fractional-order hyperchaotic system and its application to secure communication," *Nonlinear Analysis: Real World Applications*, **13**, 1441-1450, 2012.

[38] Q. Zhang, L. Guo and X. Wei, "A novel image fusion encryption algorithm based on DNA sequence operation and hyper-chaotic system," *Optik-International Journal for Light and Electron Optics*, **124**, 3596-3600, 2013.

[39] M.T. Yassen, "On hyperchaos synchronization of a hyperchaotic Lü system," *Nonlinear Analysis: Theory, Methods and Applications*, **68**, 3592-3600, 2008.

[40] S. Vaidyanathan and S. Rasappan, "Hybrid synchronization of hyperchaotic Qi and Lü systems by nonlinear control," *Communications in Computer and Information Science*, **131**, 585-593, 2011.

[41] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control," *Communications in Computer and Information Science*, **198**, 10-17, 2011.

[42] S. Vaidyanathan and S. Sampath, "Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control," *Communications in Computer and Information Science*, **205**, pp. 156-164, 2011.

[43] P. Sarasu and V. Sundarapandian, "The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control,"*International Journal of Soft Computing*, **6**, 216-223, 2011.

[44] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinar control," *Communications in Computer and Information Science*, **204**, 84-93, 2011.

[45] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control," *International Journal of Systems Signal Control and Engineering Application*, **4**, 18-25, 2011.

[46] S. Vaidyanathan and K. Rajagopal, "Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control," *International Journal of Systems Signal Control and Engineering Application*, **4**, 55-61, 2011.

[47] V. Sundarapandian and R. Karthikeyan, "Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control," *International Journal of Soft Computing*, **7**, 28-37, 2012.

[48] S. Vaidyanathan and S. Pakiriswamy, "The design of active feedback controllers for the generalized projective synchronization of hyperchaotic Qi and hyperchaotic Lorenz systems," *Communications in Computer and Information Science*, **245**, 231-238, 2011.

[49] S. Pakiriswamy and S. Vaidyanathan, "Generalized projective synchronization of hyperchaotic Lü and hyperchaotic Cai systems via active control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 53-62, 2012.

[50] S. Rasappan and S. Vaidyanathan, "Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback," *Archives of Control Sciences*, **22**, 343-365, 2012.

[51] S. Vaidyanathan and S. Sampath, "Hybrid synchronization of hyperchaotic Chen systems via sliding mode control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **85**, 267-276, 2012.

[52] S. Rasappan and S. Vaidyanathan, "Synchronization of hyperchaotic Liu system via backstepping control with recursive feedback," *Communications in Computer and Information Science*, **305**, 212-221, 2012.

[53] V. Sundarapandian and R. Karthikeyan, "Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control," *Journal of Engineering and Applied Sciences*, **7**, 254-264, 2012.

[54] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control," *International Journal of Soft Computing*, **7**, 28-37, 2012.

[55] S. Vaidyanathan, “Analysis and synchronization of the hyperchaotic Yujun system via sliding mode control,” *Advances in Intelligent Systems and Computing*, **176**, 329-337, 2012.

[56] S. Vaidyanathan, “Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control,” *Advances in Intelligent Systems and Computing*, **177**, 1-10, 2013.

[57] E. Ott, C. Grebogi and J.A. Yorke, “Controlling chaos,” *Physical Review Letters*, **64**, 1196-1199, 1990.

[58] J. Wang, T. Zhang and Y. Che, “Chaos control and synchronization of two neurons exposed to ELF external electric field,” *Chaos, Solitons and Fractals*, **34**, 839-850, 2007.

[59] V. Sundarapandian, “Output regulation of the Lorenz attractor,” Far East Journal of Mathematical Sciences, 42, 289-299, 2010.

[60] S. Vaidyanathan, “Output regulation of Arneodo-Coullet chaotic system,” *Communications in Computer and Information Science*, **133**, 98-107, 2011.

[61] S. Vaidyantahan, “Output regulation of the unified chaotic system,” *Communications in Computer and Information Science*, **198**, 1-9, 2011.

[62] S. Vaidyanathan, “Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system,”*International Journal of Control Theory and Applications*, **5**, 15-20, 2012.

[63] S. Vaidyanathan, “Global chaos control of hyperchaotic Liu system via sliding mode control,” *International Journal of Control Theory and Applications*, **5**, 117-123, 2012.

[64] W. Hahn, *The Stability of Motion*, Springer-Verlag, New York, 1967.