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THE ANALYSIS OF VALUE AT RISK FOR PRECIOUS METAL RETURNS BY APPLYING EXTREME VALUE THEORY, COPULA MODEL AND GARCH MODEL

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Abstract: *This paper examines Value at Risk by applying GARCH-EVT-Copula model and finds the optimal portfolio for the precious metal. The 4,077 precious metal price observations are collected from 3rd January 2000 to 18th August 2015, traded in the London Metal Exchange, and all prices are traded in US dollars per troy ounce. First, we estimate the coefficients of the ARMA-GARCH equations based on the student t distribution. Second, we extract the filtered residuals from such estimation and then apply the extreme value distribution (EVT) for fitting the residual tails in order to model marginal residual distributions. Third, we use multivariate Student t-copula to construct the precious metal portfolio risk dependence structure. Finally, we simulate 10,000 portfolios and estimate value at risk (VaR) and Expected shortfall (ES). The empirical results displayed the VaR and ES values for an equally weighted portfolio of four precious metals. In addition, we found that the optimal investment focuses on the gold and silver investment due to high investment proportion, whereas palladium and platinum have little investment proportion.*

Keywords: *Value at Risk, Precious metal price, GARCH-EVT-Copula, Portfolio Optimization*

JEL Classification: *C22, G11, G17*

1. INTRODUCTION

Precious metals has been attracted great attention since it has never been valueless like gold. It could provide the investor a "safe haven" in terms of economic uncertainty and financial instability and also serve as a hedge against an unexpected inflation. Moreover, it is described as a proven asset diversifier when they included in an investment portfolio, they can reduce the overall risk of investment portfolio.

Even though precious metal is a good asset to hedge from many poor situations, precious metal price still has uncertainty situation in 2014. Precious metals prices can be shown in figure 1, representing the average percent change of each precious metal price in 2014. This figure illustrates that silver prices lost more than 19%,

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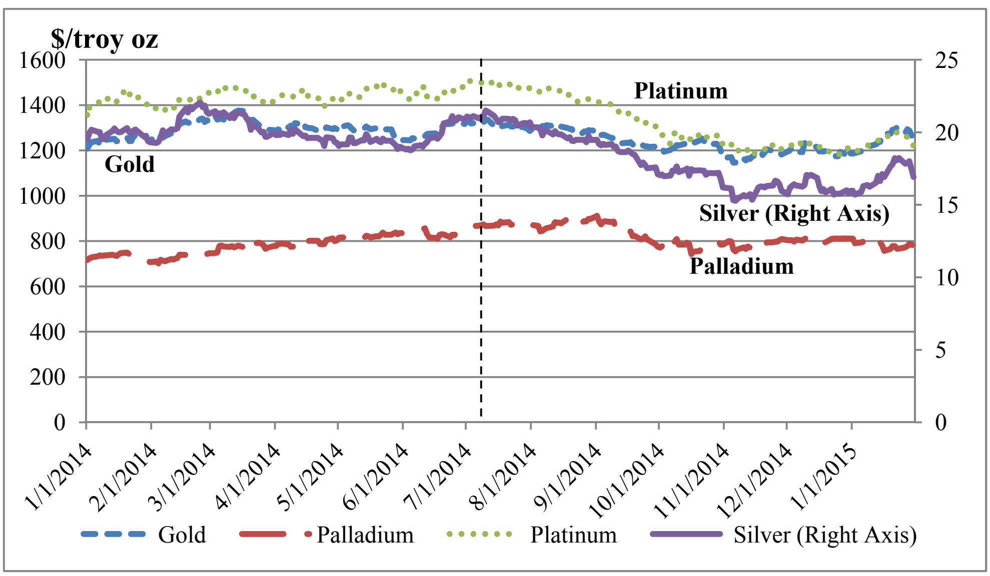
platinum prices decreased by more than 11%, palladium prices increased by around 11% and gold prices lost only 1.4% compared with the previous year. This precious metal prices demonstrate the prices from the first half through the second half of the year. For the first half of 2014, the figure shows strength of precious metal prices with increased rate of 7% to 17% due to a neutral US dollar and the stability of demand for precious metals. In the second half of 2014, precious metal lost between 5% and 25%, of palladium and silver respectively. This had a huge impact throughout the year. The main reason for this weakness in the second half of 2014 was US dollar strength. However, silver, gold and platinum had a strong start to 2015.

Figure 1: Precious metal prices



Source: Thomson Reuter Data Stream, 2015.

Value at Risk (VaR) estimation is a way to measure risk. It is a financial market instrument for measurement and evaluation of the portfolio market risk associated with financial asset and commodity price movements. It represents in the form of the expected worst loss of a portfolio over a given time horizon at a given confidence level. There are three approaches that are used to estimate portfolio VaR, such as Historical Simulation (HS), Variance Covariance (VC), and Monte Carlo Simulation approaches.

There are many examinations about VaR which have a difference way in the applying to estimate VaR and in each of these work, there are advantages and disadvantages of different. For the literature review, we show the works that apply VaR with other model. Hammoudeh et al. (2011) examines the volatility and correlation dynamics in price returns of gold, silver, platinum and palladium using Value at Risk estimation. The results are useful for participants in the global financial markets that are needed for investment in precious metals remains high volatility. Demiralay and Ulusoy (2014) predict the Value at Risk of four major precious metals (gold, silver, platinum, and palladium) with FIGARCH, FIAPARCH and HYGARCH or long memory volatility models, under normal and student-*t* innovations' distributions. The results showed that these models perform well in forecasting a one-day-ahead VaR and have potential implications for portfolio managers, producers, and policy makers. Chen and Giles (2014) analyze the risk of investment in gold, silver, and platinum by applying Extreme Value Theory and adopted Value at Risk and Expected Shortfall. These measures are obtained by fitting the Generalized Pareto Distribution, using the Peaks-Over-Threshold method, to the extreme daily price changes. The results show that silver is the most risky metal among the three considered and platinum is riskier than gold.

Bob (2013) uses VaR to estimate portfolio applying an approach combining Copula functions, Extreme Value Theory (EVT), and GARCH models. The result in this application has a better performance than a general estimation. Besides, Ghorbel and Trabelsi (2009) used ARMA-GARCH-EVT Copula approach to estimate VaR in multivariate financial data. They found that their approach can provide a better dependence structure in the multivariate data and obtain accurate VaR estimates. Leonard (2007), otherwise stated that the risk measure of VaR is not an identity method in estimation because its accuracy depends on the ability to analyze the true portfolio loss distribution, and the models of estimation such as Historical Simulation or Variance-Covariance cannot give the accurate estimates with high confidence level. Although Monte Carlo simulation has the advantages in modelling the loss distribution and potential in accurate estimates, it is hard to compute for portfolios that have a high number of risk factors.

Further, Artzner et al. (1997) states that VaR estimate cannot tell anything about the potential size of the loss and it has some failing. They propose "Expected Shortfall (ES)" to measure the expected loss given which the loss exceeds VaR. This ES has a closely relationship with VaR. Yamai and Yamai (2002) illustrated that ES is easily decomposed and optimized while VaR is not and they also showed that ES requires a larger sample size than VaR for the same level of accuracy. Therefore, this paper studies the risk that occurs in the investing precious metals which focus on gold, silver, platinum, and palladium using VaR estimate and Expected Shortfall (ES). This paper also estimates VaR and ES by applying with ARMA-GARCH models, Extreme Value Theory, and Copula model for more efficiency and precision. Moreover, there are also find portfolio optimization based on ES, and compare portfolio to choose optimal portfolio because of commercial banks and individual investors, one of the major concerns is to minimize the risk of the investment portfolio.

In this paper is the examining of risk in precious metal portfolio, which has the analysis in key situation in the past along with the literature review of the model used and the circumstances. For section 2 is the review of each methodology. Section 3 is the explaining of data, descriptive statistics, and unit root tests. Section 4 describes the empirical result estimates about the risk; there are analysis in VaR and ES, optimal portfolio, implications for optimal hedge ratios, and optimal portfolio weights. Finally, Section 5 provides some concluding remarks.

2. METHODOLOGY

2.1 Marginal Distribution

Bollerslev (1986) proposes the Generalized Autoregressive Conditional Heteroskedasticity (GARCH), which in this paper we use GARCH model for precious metal price to obtain the marginal distribution. We use ARMA (*p,q*) with univariate standard GARCH (1,1) form which model is defined as follows:

$$r_t = \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} + \varepsilon_t \quad (1)$$

$$\varepsilon_t = \sigma_t z_t \quad (2)$$

$$\sigma_t^2 = \varpi + \sum_{i=1}^{k} \alpha_i \varepsilon_{i-t}^2 + \sum_{i=1}^{l} \beta_i \sigma_{t-i}^2 \quad (3)$$

where ε_t is the innovation at time t. ν_t is a sequence of *i.i.d.* random variables with mean 0 and variance 1. $\varpi > 0$, α_i, $\beta_i > 0$ and $\sum_{i=1}^{k} \alpha_i + \sum_{i=1}^{l} \beta_i \leq 1$. In this case, z_t is assumed to be t distribution because financial data usually have a heavy tail distribution. The α_i and β_i are known as ARCH and GARCH parameters, respectively.

In order to better estimate the tails of the distribution, we applied EVT to the residuals that we obtained from the ARMA-GARCH model. We use the generalized Pareto distribution (GPD) estimate to model residuals for the upper and lower tails and Gaussian kernel estimate for the remaining part. The CDF of generalized Pareto distribution given by:

$$F_{\xi,\beta}(z_t) = \begin{cases} \frac{N_{m_L}}{N}\left(\left(1+\frac{\xi_L\left(m_L - z_t\right)}{\beta_L}\right)^{-1/\xi_L}\right), & z_t < m_L, \\ \varphi(z_t), & m_L < z_t < m_R, \\ 1-\frac{N_{m_R}}{N}\left(\left(1+\frac{\xi_R\left(m_R - z_t\right)}{\beta_R}\right)^{-1/\xi_R}\right), & z_t < m_R, \end{cases} \tag{4}$$

where ξ is the shape parameter, β is the scale parameter, and m_L (m_R) is the lower (upper) threshold. We have to choose one approach between high precision and low variance, which the critical step is the choice of the optimal threshold. Following DuMouchel (1983), we chose the exceedances to be the 10th percentile of the sample.

2.2 Copula Model

Copula is the function that we use to separate the marginal distribution of two or more than two random variables. So if we can separate the marginal distribution of that random variables => we can generate the parameter of that copula. That parameter is very important for model estimation and also important to find the measurement of dependency.

If we know copula parameter => we can generate Kendaul Tau, Spearman Rho, Low Tail, Upper Tail depend on which copula function we use.

Two copula families:

1. Elliptical Copula = Gaussian and Student-t (used for symmetric tail dependency)
2. Archimedean Copula = Frank, Gumbel and Clayton (used for asymmetric tail dependency)

The Copula model is a way to construct a joint distribution function, following Sklar's Theorem (1973), which is the most important theorem about copula functions because it is used in many practical applications.

For the theorem, let F be an n-dimensional c.d.f. with continuous margins F_1, F_2,…F_n. Then it has the following unique copula representation

$$F(x_1, x_2, ..., x_n) = C(F_1(x_1), F_2(x_2), ..., F_n(x_n)) \tag{5}$$

From Sklar's Theorem, we see that, for continuous multivariate distribution, the univariate marginal distribution and the multivariate dependence structure can be separated. Assuming that the dependence structure does not change with time, we thus use t copula to estimate the joint distribution. So in this section, we applied t copula

For the n-variable t Copula, let $t_{\nu,\Sigma}$ be the standardized multivariate t distribution with correlation matrix Σ and ν degrees of freedom. The t Copula can be defined as follows:

$$C^{St}(u_{1t}, u_{2t}, ..., u_{nt}) = t_{\nu,\Sigma}(t'_\nu(u_{1t}), t'_\nu(u_{2t}), ..., t'_\nu(u_{nt})) \tag{6}$$

where t'_ν denotes the inverse of the Student's t cumulative distribution function.

From equation (4) we know that the marginal distribution is $F_i(z)$. Based on the historic data $\{z_{1t}, z_{2t},…,z_{nt}\}$, $t = 1, 2, …, T$ and the given degree of freedom ν, we set:

$$\begin{aligned} u_t &= (u_{1t}, u_{2t}, ..., u_{nt}) = (F_1(z_{1t}), F_2(z_{2t}), ..., F_n(z_{nt})) \\ \zeta_t &= (t'_\nu(u_{1t}), t'_\nu(u_{2t}), ..., t'_\nu(u_{nt})) \end{aligned} \tag{7}$$

Therefore we have $C^{st}(u_t) = t_{\nu,\Sigma}(\xi t)$. The parameter matrix Σ is also estimated using the MLE method which is the same as the Gaussian Copula, and it can be calculated as follows:

Step1: The initial matrix is the correlation coefficient matrix of multivariate normal Copula function estimated from $\hat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T} \xi_t \xi'_t$ (Gaussian Copula).

Step2: We can get the correlation coefficient matrix $\hat{\Sigma}_{n+1}$ of multivariate t Copula through the following iterative calculation method:

$$\hat{\Sigma}_{k+1} = \frac{1}{T}\left(\frac{\nu + n}{\nu}\right) \cdot \sum_{t=1}^{T} \frac{\zeta_t \zeta_t'}{1 + \frac{1}{\nu}\zeta_t' \hat{\Sigma}_k \zeta_t} \quad k = 1, 2, ... \tag{8}$$

Step 3: Repeat the above process until $\hat{\Sigma}_{n+1} = \hat{\Sigma}_n$, so using MLE we can get the correlation coefficient matrix Σ of t Copula which can be $\hat{\Sigma} = \hat{\Sigma}_n$.

2.3. Simulation Algorithm and Portfolio Risk Analysis

2.3.1 Simulating returns via t Copula

In order to simulate return via estimated t copula, *step1*: for $\hat{\Sigma}$ derived from equation (8) we use Cholesky decomposition to get $\hat{\Sigma} = A'A$;

Step 2: Generate m random vectors which are independently and identically distributed, such as $x = (x_1, x_2, \ldots, x_n)'$, $x_i\, N(0,1)$. Let $y = A'x$, then we produce random vector s, subject to Chi-square distribution and independent of x. Then we set: $z = (F_1^{-1}(t(y_1/\sqrt{s/\nu})), \ (F_2^{-1}(\phi(t(y_2/\sqrt{s/\nu}))), \ldots, (F_i^{-1}(\phi(y_i/\sqrt{s/\nu})))$ where F_i^{-1}, $i = 1,2,\ldots, n$ is the inverse of distribution of F_i in equation (4);

Step 3: Repeat the above step M times, we can get the vector $(z_{1m}, z_{2m}, \ldots z_{nm})'$, $m = 1,2\ldots M$. Then restoring it into equation (1), (2), and (3) we can get M returns at time t+1. The returns residuals' joint distribution is this t Copula. The returns can be defined by $r_{T+1} = (r_{1m}, r_{2m}, \ldots r_{nm})' = (\mu_1 + z_{1m}\sigma_{1,T+1}, \mu_2 + z_{2m}\sigma_{2,T+1}, \ldots, \mu_n + z_{nm}\sigma_{n,T+1})'$;

2.3.2 Value at Risk and Expected Shortfall

In order to analyze the risk, we calculate the empirical VaR and ES of an eqully weighted portfolio with 4 assets. The equations are follows:

$$\text{Min } ES = E[r \mid r \le r_\alpha]$$

$$\text{Subject to } \quad r = w[r_{(1,t+1)} + r_{(2,t+1)} + r_{(3,t+1)} + r_{(4,t+1)}]$$

$$w_1 = w_2 = w_3 = w_4 = \frac{1}{4}$$

$$0 \le w_i \le 1,\ i = 1,2,3,4$$

where r_α is the lower α- quantile and $r_{i,t}$+1 is the return on individual asset at time $t + 1$

2.3.3 Portfolio Optimization with Minimum Risk

From the above section, we can estimate the VaR and ES (or CVaR) of equally weighted portfolio. In this part, we use the Monte Carlo simulation with estimated multivariate t copula to generate N sample size. The optimal portfolios weights of the selected assets then are constructed under minimize expected shortfalls with respect to maximize returns, which can be given by:

$$\text{Min } ES = E[r \mid r \le r_\alpha]$$

$$\text{Subject to } r = w_1 r_{(1,t+1)} + w_2 r_{(2,t+1)} + w_3 r_{(3,t+1)} + w_4 r_{(4,t+1)}$$

$$w_1 + w_2 + w_3 + w_4 + 1$$

$$0 \le w_i \le 1,\ i = 1,2,3,4$$

where r_α is the lower α- quantile and $r_{i,t}$+1 is the return on individual asset at time $t + 1$.

3. DATA

In the first state, we use the precious metal prices (secondary data) to calculate the natural log returns which are defined as $r_{i,j} = \ln (P_{i,j}/P_{i,j-1})$ where $P_{i,j}$ is the i^{th} metal price at time j, $r_{i,j}$ is the i^{th} log return of metal price at time j, and i indicated the i^{th} precious metal price. The descriptive statistics is shown in table 1

In table 1, it is clear that mean of each precious metal variable is positive by the highest mean returns is gold (0.00033), the lowest mean return is palladium (0.000067), and the standard deviations in palladium is highest (0.0215) and in gold is lowest (0.0113). For the value of skewness and kurtosis in all of the precious metal return is not equal to zero and have excess kurtosis, respectively. So these imply that the distribution of metal returns are fatter tail instead of normal distribution. Moreover, Jaque-Bera test, which is the normal distribution test of return series, rejects the null hypothesis, thus the return series of precious metal price is non-normal distribution. The Augmented Dickey-Fuller (ADF), Phillips and Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are applied to check unit roots in the series. Unit root test for each variable have a statistical significance level of 0.01. This means that all of precious metal returns is stationary characteristics. Therefore, these variables can be used to estimate ARMA-GARCH model in the next step.

4. EMPIRICAL RESULT

4.1 MARGINAL MODEL RESULT

Table 2 illustrates the coefficient for the ARMA(p,q)-GARCH(1,1) with normal and student t distribution for each precious metal price return series. The optimum lag lenght for ARMA(p,q)-GARCH(1,1) is selected by the minimum value of Akaike information criteria (AIC) and Bayesian information criterion (BIC) information. The estimated equation of gold and palladium are ARMA(3,2)-GARCH(1,1), while platinum and silver is ARMA(1,1)-GARCH(1,1) and ARMA(5,3)-GARCH(1,1), respectively. In addition, the coefficient of t distribution for each equation is statistically significant at 1% in most cases, meaning that the assumption of t distribution for ARMA-GARCH estimation is reasonable.

From the estimate of ARMA-GARCH result, the standardize residual of each precious metal return series are obtained. EVT is then applied to those residuals which GPD specially is for tail estimation. We choose the exceedances to be the 10th percentile of the sample for the upper and lower tail of the residual distribution

Table 1
Descriptive Statistics on Precious Metal Returns

	Gold		*Palladium*		*Platinum*		*Silver*	
Mean	0.000330		6.76E-05		0.000196		0.000258	
Median	0.000138		0.000000		0.000000		0.000000	
Maximum	0.068653		0.158406		0.084278		0.182786	
Minimum	-0.101624		-0.178590		-0.172773		-0.186926	
Std. Dev.	0.011337		0.021529		0.014610		0.020787	
Skewness	-0.424856		-0.243452		-0.874030		-0.584971	
Kurtosis	8.852380		8.943808		13.53889		13.39984	
Jarque-Bera	5939.474		6040.281		19382.04		18601.08	
Probability	0.000000		0.000000		0.000000		0.000000	
ADF-test	Statistic	CV	Statistic	CV	Statistic	CV	Statistic	CV
None	-64.73945***	-2.565530	-61.86531***	-2.565530	-62.70875***	-2.565530	-69.43875***	-2.565530
Intercept	-64.78992***	-3.431770	-61.85842***	-3.431770	-62.71292***	-3.431770	-69.44198***	-3.431770
Trend and Intercept	-64.79927***	-3.960291	-61.85469***	-3.960291	-62.75094***	-3.960291	-69.44159***	-3.960291
PP-test	Statistic	CV	Statistic	CV	Statistic	CV	Statistic	CV
None	-64.73945***	-2.565530	-61.86531***	-2.565530	-62.70875***	-2.565530	-69.43875***	-2.565530
Intercept	-64.78992***	-3.431770	-61.85842***	-3.431770	-62.71292***	-3.431770	-69.44198***	-3.431770
Trend and Intercept	-64.79927***	-3.960291	-61.85469***	-3.960291	-62.75094***	-3.960291	-69.44159***	-3.960291
KPSS-test	Statistic	CV	Statistic	CV	Statistic	CV	Statistic	CV
Intercept	0.237014	0.739000	0.150169	0.739000	0.317234	0.739000	0.145725	0.739000
Trend and Intercept	0.144097	0.216000	0.097710	0.216000	0.049785	0.216000	0.100360	0.216000

Note: 1. All statistics are daily returns from January 3, 2000 to August 18, 2015, yielding 4,076 observations.
2. CV is Critical Value.

Table 2
Estimate of ARMA (q, p) GARCH (1, 1) Result

Variable	*Gold*		*Palladium*		*Platinum*		*Silver*	
	Normal	*Student' t*	*Normal*	*Student' t*	*Normal*	*Student' t*	*Normal*	*Student' t*
μ	0.00033**	0.00045***	0.00032	0.00025	0.00034**	0.00048***	2.86E-05	0.00016
AR(1)	-0.51899***	-0.92697***	1.31834***	1.59550***	-0.42876***	0.48307***	0.29524***	-0.43807**
AR(2)	0.18949***	-0.87544***	-0.84417***	-0.95972***	0.32139***	-	-0.95764***	-0.87085***
AR(3)	-0.18807**	-0.04219***	0.04819***	0.06228***	0.88635***	-	-0.08248***	-0.61334***
AR(4)	-	-	-	-	-0.03179*	-	-	-0.04536*
AR(5)	-	-	-	-	-	-	-	0.03243**
MA(1)	0.52034***	0.89843***	-1.27014***	-1.55109***	0.45884***	-0.49029***	-0.38140***	0.33674*
MA(2)	-0.19460***	0.85029***	0.75766***	0.86380***	-0.31686***	-	0.99461***	0.84606***
MA(3)	0.19153**	-	-	-	-0.90817***	-	-	0.55277***
ω	2.25E-06***	8.63E-07***	1.18E-05***	8.81E-06***	3.31E-06***	4.14E-06***	1.53E-06***	1.08E-06***
α	0.06133***	0.04726***	0.08477***	0.12073***	0.07744***	0.07828***	0.05940***	0.04023***
β	0.92162***	0.95031***	0.89166***	0.87989***	0.90702***	0.90340***	0.94088***	0.95921***
K (t-coefficeint)	–	3.97211***	-	3.68664***	-	4.77615***	-	4.55871***
AIC	-6.304234	-6.423668	-5.046726	-5.197842	-5.888064	-5.964881	-5.237183	-5.342067
BIC	-6.288736	-6.408171	-5.032778	-5.182344	-5.871013	-5.954037	-5.223235	-5.321912

Note:*, **, and *** denote significant at 90%, 95%, and 99%, respectively of the residuals (see Dumouchel, 1983) because of the appropriateness to choose 10th percentile in the generalize Pareto model and the same biases occurred as in the stable law analysis. We assume that excess residuals over threshold follow the GPD and use the Gaussian kernel estimate is for the remaining part. The parameter estimation for each precious metal's residuals is demonstrated in Table 3.

Table 3
Parameter estimation for each precious metal's residual.

	Gold	*Palladium*	*Platinum*	*Silver*
Threshold m_L	-0.0170	-0.0325	-0.0211	-0.0297
ξ_L	0.1572	0.0747	0.1589	0.1644
β_L	0.0068	0.0160	0.0090	0.0126
Threshold m_R	0.0187	0.0334	0.0215	0.0302
ξ_R	0.1422	0.0900	0.2582	0.2027
β_R	0.0081	0.0176	0.0109	0.0171

4.2 ESTIMATE COPULA RESULT

After obtaining the parameters of GPD and the residuals z_{it}, $i = 1,2,3,4$, $t = 1,2,\ldots,T$, we substitute z_{it} into equation (4) and get the marginal distribution $u_i = f(z_i)$. According to the parameter estimation method in Section 2.1, we can get the parameters of Copulas that is the correlation matrix Σ of student *t* Copula and degree of freedom, ν, demonstrated in Table 4.

Table 4
Empirical t copulas parameters ($\hat{\rho}$)

Precious Metal	*GARCH-t EVT Copula*			
	Gold	*Palladium*	*Platinum*	*Silver*
Gold	1.00000	0.35901	0.45797	0.46487
Palladium	0.35901	1.00000	0.63867	0.46504
Platinum	0.45797	0.63867	1.00000	0.54121
Silver	0.46487	0.46504	0.54121	1.00000

$\hat{\nu} = 7.2777$

4.3 Portfolio Risk Analysis

In this paper, we focused on the student *t* copulas, and applying the simulation algorithm described in Section 2.3 above, we can simulate the returns at time *t*+1 based on correlation structure specified in student *t* Copula. The calculated VaR and ES of the portfolio with an equally weighted portfolio of four precious metals (Gold, Palladium, Platinum, and Silver), showed in Table 5.

In Table 5 shows the estimated VaR and ES at level of 1%, 5% and 10% under the equally weighted assumption. In period $t + 1$, the estimated ES are higher than VaR and converges to -2.50, -3.28 and 5.56 at 10%, 5% and 1% level, respectively.

Table 5
Value at risk equally weighted portfolios

Portfolio	*Expected Value (GARCH-t EVT Copula)*		
	1%	*5%*	*10%*
VaR	-3.902%	-2.087%	-1.449%
ES	-5.557%	-3.283%	-2.503%

Figure 2 shows the result of the efficient frontier of the portfolio under different expected return at given significant level of 5%, which come from the optimized portfolio based on mean-CVaR (ES) model. This result, we applied the Monte Carlo simulation to simulate a set of 10,000 samples and to estimate the expected shortfall of an optimal weighted portfolio.

For the discussion above, we focused on estimating the VaR and CVaR of an equally weighted portfolio. However, for commercial banks and individual investors, one of the major concerns is to minimize the risk of the investment portfolio. In order to address this concern, we can find the optimal portfolio weight that minimizes the portfolio risk under minimize expected shortfall with respect to maximize returns. The result is shown in Table 6. This result illustrate that most of investment proportion is gold and silver, whereas palladium and platinum have little investment proportion.

As can be seen in table 6 that portfolio one to six focus on gold, while the portfolio seven to ten focus on silver which in each portfolio will provide the different return on each one. However, in general, portfolio 10 would provide the maximum return, which should be the one to invest on. In figure 2, each plot in this figure illustrates each portfolio. First plot show that low risk provide low return. It means that the more return, the more risky it will be. For the suggestion, for risk-averse investors, they better choose the low return with low risk while risk-lover investors suit for high risk, high return.

Figure 2: The efficient frontiers of CVaR under mean

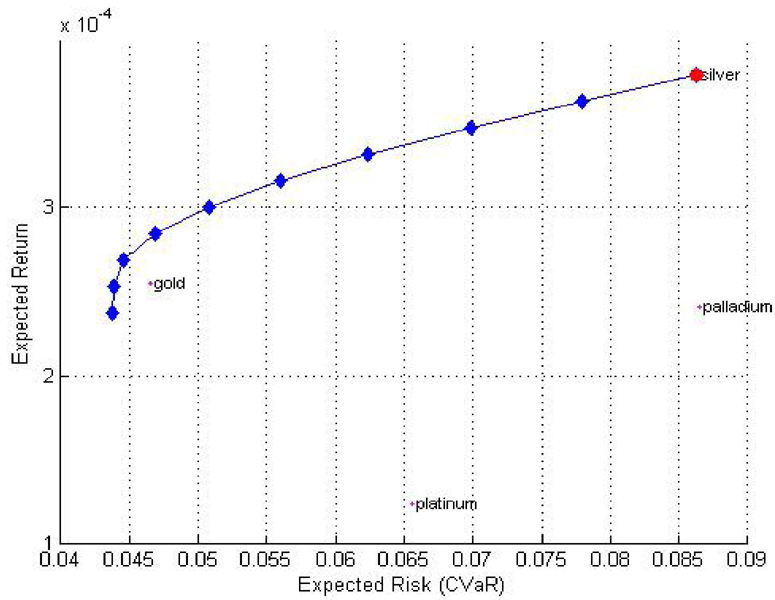


Table 6
Optimal investment proportion of precious metal portfolio with minimum risk (ES 5%)

Portfolios	*Investment proportion*				*Returns*
	Gold	*Palladium*	*Platinum*	*Silver*	
1	0.728	0.056	0.172	0.045	0.024%
2	0.772	0.090	0.069	0.068	0.025%
3	0.769	0.107	0.000	0.124	0.027%
4	0.698	0.057	0.000	0.245	0.028%
5	0.614	0.018	0.000	0.368	0.030%
6	0.507	0.000	0.000	0.493	0.032%
7	0.381	0.000	0.000	0.619	0.033%
8	0.254	0.000	0.000	0.746	0.035%
9	0.127	0.000	0.000	0.873	0.036%
10	0.000	0.000	0.000	1.000	0.038%

5. CONCLUSION

In this paper, we focus on the risk that occurs in the investing precious metals traded at London Metal Exchange. We examine Value at Risk and Expected Shortfall applying with GARCH-EVT-Copula model. Empirical results showed that the ARMA-GARCH with student-*t* distribution is appropriate. Then we can obtain the parameter estimation for each precious metal's residual to get the marginal distribution which has been used in multivariate Student *t*-copula to describe the precious metal portfolio risk dependence structure. The estimated VaR and ES (CVaR) are calculated based on 10%, 5%, 1% levels, respectively. Finally, the optimal portfolio weight illustrate gold and silver share most of investment proportion while little share is for palladium and platinum.

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