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Military Spending as a Factor of Economic Growth

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ABSTRACT

In this paper a simple Keynesian type model of a closed economy is developed in order to investigate the effect of military spending on economic growth. A second order system of difference equations with constant coefficients is derived and is solved with respect to real income and real private investments. The dynamic characteristics of model's solution are investigated and it is shown that as time tends to infinity, the growth rate of real income's equilibrium value converges to the biggest in value growth rate among military and non – military spending variable. Moreover, the home country's reaction function with respect to changes in foreign growth rate of military spending is determined. The paper's theoretical conclusions are confirmed after the performance of a simulation analysis.

Keywords: *Growth, military expenditure, reaction function.*

J.E.L. Classification: *E12, E32, E61, (H56;)*

1. INTRODUCTION

The effects of military spending on economic activity were first examined by the Marxist economist Michael Kalecki (1943), the founder of Military Keynesianism, who investigated the link between the successful rise of the Nazism in Germany after the Great Depression and the positive effect of state's military spending on effective demand and employment. Kalecki argued that public investments on armaments are more preferable than other forms of government investment (on schools, hospitals, highways etc.) on behalf of private capital, since military spendings promote private profits without being competitive to other private economic activities in more conventional economic markets.

The question of the affection of economic growth by the military expenditures has been in the center of interest, especially after the seminal work of Benoit (1973, 1978), who found a positive effect of military spending on economic growth. Benoit's findings triggered since then the production of a large number of empirical studies, the findings of which concerning the question under consideration were quite contradicting [Rati Ram (1995), table 1].

In some studies the economic growth was found to be positively affected by military spending [Atesoglu (2002), Halicioglou (2004), Kollias *et al.* (2007)]. In other studies, the empirical findings could not establish the promotion of economic activity by the government spending on armaments [Ram (1994), Kollias (1997), Smith & Tuttle (2008),]. Finally, there are studies that resulted to the specification of a negative impact of military spending on economic activity [Faini *et al.* (1984), Lebovic & Ishaq (1987), Antonakis & Apostolou (2003)].

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The diversity of empirical results, regarding the effect of military spending on economic growth, could be attributed to methodological issues [Ram (1994), Dunne *et al.* (2001) & (2004)] concerning, firstly, the theoretical foundation of the used econometric model, secondly, the realization of misspecification errors in the context of the used theoretical models, and thirdly, the nature of the statistical data used (cross or individual country data) in the context of the performed econometric analysis.

As far as the first methodological issue is concerned, that is, the theoretical models used for econometric analysis, the types of models that are most frequently encountered in the literature regarding the nexus between military spending and economic growth are the following [Dunne *et al.* (2004)]: (i) the supply-side Neoclassical growth models such as (a) the *Feder – Ram model*, (b) the *Augmented Solow model* and (c) the *Barro model*, (ii) the single or simultaneous equation demand-side Keynesian models, such as the *Deger – type model* and the ones proposed by Atesoglu (2002) and Smith (1980).

In the present article a Keynesian type model of simultaneous equations (S.E.M.) is developed, in the context of which the impact of military spending on the dynamic equilibrium values and the diachronic evolution of income and private investments is investigated. Moreover, the functional form of the nexus between the growth rates of income and private investments on one hand, and military & non – military spending on the other, is derived when the economy is at the state of equilibrium. Finally, the country's reaction function to changes in the size of foreign growth rate of military spending is determined, so as the effects on the domestic dynamic equilibrium values of income and private investments are cancelled.

The following analysis is organized in six sections: section 2 involves the description of model's structural form. In section 3, the general solution of the model is derived and the inferences, concerning the dynamic characteristics of the solution, are stated. In section 4, the functional forms of country's reaction functions and the linkage between the growth rates of income, private investments, military and civilian spending are derived. In section 5, the theoretical findings are tested via a simulation analysis. The sixth and final section is devoted to the conclusions of the presented analysis.

2. THE STRUCTURAL FORM OF THE MODEL

The model under consideration is referred to a closed economy in the context of which the level of prices is assumed to be constant. The structural form of the model is comprised by the following equations and identities:

$$Y_t = \min\left(a\, A_t L_t, \beta\, K_t\right) = \beta\, K_t \tag{1}$$

$$A_t L_t \approx A_0 L_0 \left(1 + g + n\right)^t \tag{2}$$

$$C_t = c_0 + c_1 Y_t^d \tag{3}$$

$$Y_t^d = Y_t - T_t \tag{4}$$

$$I_t = \overline{I} + v\, \Delta Y_t^e + \delta\, K_{t-1} \tag{5}$$

$$\Delta Y_t^e - \Delta Y_{t-1}^e = \theta\left(\Delta Y_{t-1} - \Delta Y_{t-1}^e\right) \tag{6}$$

$$G_t = G_t^m + G_t^{nm} \tag{7}$$

$$G_t^m = G_0^m \tilde{\gamma}_m^t \tag{8}$$

$$G_t^{nm} = G_0^{nm} \tilde{\gamma}_{nm}^t \tag{9}$$

$$G_t^{m^*} = G_0^{m^*} \tilde{\gamma}_{m^*}^t. \tag{10}$$

$$G_t = T_t \tag{11}$$

$$Y_t = C_t + I_t + G_t \tag{12}$$

where: $a \succ 0$ $(\beta \succ 0)$: the average and marginal product of the effective labor (A_tL_t) [physical capital (K_t)], $A_0L_0 \succ 0$, $g(n) \in (0,1)$: the diachronically constant growth rate of technology (A_t) [labor (L_t)], $c_0\left(\overline{I}\right) \geq 0$: the level of autonomous private consumption (investments), $c_1 \in (0,1)$: the marginal propensity to consume, $v = 1/\beta \succ 0$: the accelerator of investments, $\delta \in (0,1)$: the depreciation rate of physical capital, $\theta \in (0,1)$: the adjustment coefficient of the adaptive expectations mechanism, $\tilde{\gamma}_i = 1 + \gamma_i \in (1,2)$ with $\gamma_m\left(\gamma_{m^*}\right) \in (0,1)$: the constant growth rate of the level of the home (foreign) military spending & $\gamma_{nm} \in (0,1)$: the constant growth rate of the domestic non-military spending, $G_0^i \succ 0$, $i = m, nm, m^*$ and $t = 0, 1, \ldots$: the time index.

According to equation (1) the economy's level of production is defined by a Leontief production function, assuming that the technical progress is Harrod neutral and that capital is the limiting factor of production. As a result of the latter assumption, the input of capital is fully utilized while labor is partially utilized in the context of the production process. The full employment of capital and the underemployment of labor imply that the structural form of the model is descriptive of a developing economy. At this point it has to be noted that according to relation (2), the magnitude of effective labor is increasing diachronically at a constant rate equal to $(g + n)$.

Relation (3) states that real private consumption[1] (C_t) is a positive function of disposable income $\left(Y_t^d\right)$. In the context of relation (4) disposable income is defined as the difference between the current income (Y_t) and the net tax revenues (T_t), with the latter being equal to the difference between total tax revenues and transfer payments.

On the basis of relation (5) gross private investments (I_t) are equal to the sum of net private investments and the depreciation of physical capital $\left(\delta K_{t-1}\right)$, with the former being defined[2] as a positive function of the expected variation of income between periods t and t–1 $(\Delta Y_t^e = Y_t^e - Y_{t-1}^e)$.

In the context of relation (6) the economic agents are forming their expectations with regard to the change of real income's magnitude after using the adaptive expectations mechanism.

The magnitude of real government spending (G_t) is described by relation (7) as the sum of the government's military $\left(G_t^m\right)$ and non – military $\left(G_t^{nm}\right)$ spending. As indicted by relations (8) and (9) both G_t^m and G_t^{nm} are growing diachronically at a constant growth rate γ_m and γ_{nm} respectively, the magnitudes of which are determined exogenously by the government. On the other hand, the level of the military spending of the home country's main rival country $\left(G_t^{m^*}\right)$ is assumed, via relation (10), to grow diachronically at a constant rate γ_{m^*}, the magnitude of which is exogenously set by the government of the rival country.

On the basis of relation (11), the budgetary constraint faced by the government of the home country is assumed to be satisfied at all times. That is, the magnitude of total government spending (G_t) is assumed equal to the magnitude of net tax revenues (T_t) at each point in time.

Finally, in the context of relation (12) the goods market of the home country is in equilibrium when the country's aggregate supply (Y_t) equals the level of the home aggregate demand ($C_t + I_t + G_t$).

3. THE GENERAL SOLUTION OF THE MODEL

The process of extracting the model's solution begins with the appropriate combination of the relations that are present in its structural form. More specifically, the appropriate combination of relations (3), (4), (7), (8), (9), (11) & (12) on one hand and (1), (5) & (6) on the other hand results to the specification of a two system equations, which in matrix notation has the following form:

$$\left|\boldsymbol{P(F)}\right| \boldsymbol{y_t} = \boldsymbol{\tilde{H}_t} \tag{13}$$

where F: the forward operator[3],

$$\boldsymbol{P(F)} = \begin{bmatrix} 1 & -\dfrac{1}{1-c_1} \\ P_{21}(F) & P_{22}(F) \end{bmatrix} \tag{14}$$

$$\left|\boldsymbol{P(F)}\right| = P_{22}(F) + \frac{P_{21}(F)}{1-c_1} \neq 0 \tag{15}$$

$$P_{21}(F) = (1 - F)\,(\delta + \theta) - \delta\theta \tag{16}$$

$$P_{22}(F) = \beta\left[F^2 - (1-\theta)F\right] \tag{17}$$

$$\boldsymbol{y_t} = \begin{bmatrix} Y_t & I_t \end{bmatrix}' \tag{18}$$

$$\tilde{\boldsymbol{H}}_t = \begin{bmatrix} \dfrac{\beta\,\theta\left(c_0 + \bar{I}\right)}{1-c_1} + \beta\left(\gamma_m + \theta\right)G_0^m\ \tilde{\gamma}_m^{t+1} + \beta\left(\gamma_{nm} + \theta\right)G_0^{nm}\ \tilde{\gamma}_{nm}^{t+1} \\ \\ \dfrac{\theta\left[\left(1-c_1\right)\beta\,\bar{I} + \delta\,c_0\right]}{1-c_1} + \sum_{j=m}^{nm}\left\{G_0^j\left[\left(\delta+\theta\right)\gamma_j + \delta\,\theta\right]\tilde{\gamma}_j^t\right\} \end{bmatrix} \tag{19}$$

The general solution ($\boldsymbol{y}_t$) of the system of difference equations (13) comprises the sum of the system's complementary solution $\left(\boldsymbol{y}_t^c\right)$ with its partial solution $\left(\boldsymbol{y}_t^p\right)$:

$$\boldsymbol{y}_t = \boldsymbol{y}_t^p + \boldsymbol{y}_t^c \tag{20}$$

The complementary solution $\left(\boldsymbol{y}_t^c\right)$ specifies the dynamic characteristics of the diachronic movement of the variables included in vector $\boldsymbol{y}_t$, that is of income (Y_t) and gross investments (I_t). The specific functional form of $\boldsymbol{y}_t^c$ depends on the sign of the determinant (D) of the system's characteristic equation, which results after the substitution of relations (16) & (17) in (15):

$$\left|\boldsymbol{P(F)}\right|_{F=\lambda} = 0 \Rightarrow P(\lambda) = \lambda^2 + z_1\lambda + z_2 = 0 \tag{21}$$

where:

$$z_1 = -\left[\left(1-\theta\right) + \frac{\delta+\theta}{\beta\left(1-c_1\right)}\right] \prec 0 \tag{22}$$

$$z_2 = \frac{\left(1-\theta\right)\delta + \theta}{\beta\left(1-c_1\right)} \succ 0 \tag{23}$$

The magnitude of D is given in the context of the following relation:

$$D = \left(\frac{\delta+\theta}{\beta}\right)^2 \times \prod_{i=1}^{2}\left(\chi - \chi_i\right) \times \left[\chi^2 + 2\ \frac{\sqrt{\beta\left[\theta + \left(1-\theta\right)\delta\right]}}{\delta+\theta}\ \chi + \frac{\left(1-\theta\right)\beta}{\delta+\theta}\right] \tag{24}$$

where: $\chi = 1\big/\sqrt{s_1} \succ 1$ with $s_1 = 1 - c_1 \in \left(0,1\right)$: the marginal propensity to save and

$$\chi_i = \frac{\sqrt{\beta}}{\delta+\theta}\ \left[\sqrt{\theta + \left(1-\theta\right)\delta} \pm \theta\right]\ ,\ i = 1,\ 2 \quad \text{with} \quad \chi_1 \prec \chi_2 \tag{25}$$

Taking into account that $\chi \succ 1,\ \beta \succ 0\ \ \&\ \ 0 \prec \delta, \theta \prec 1$, the sign of the determinant D is designated by the sign of the polynomial $Q_1\left(\chi\right) = \prod_{i=1}^{2}\left(\chi - \chi_i\right)$. The alteration of the sign of

$Q_1(\chi)$ and D could be investigated with respect to the magnitude of the marginal propensity to save (s_1). For the domain of values of s_1 for which $Q_1(\chi) \geq 0\left[Q_1(\chi) \prec 0\right]$, the determinant will be non-negative (negative), that is $D \geq 0 \ (D \prec 0)$, and both Y_t and I_t will exhibit an exponential (sinusoidal) pattern of movement[4]. It is easy to show that if:

$$\begin{cases} s_1 \in \left(0, s_{1,2}\right] \cup \left[s_{1,1}, +\infty\right) \rightarrow Q_1(\chi),\ D \geq 0 \Rightarrow \text{ Exponential Evolution of } Y_t \ \& \ I_t \\ \\ s_{1,2} \prec s_1 \prec s_{1,1} \qquad \rightarrow Q_1(\chi),\ D \prec 0 \Rightarrow \text{ Sinusoidal Evolution of } Y_t \ \& \ I_t \end{cases} \tag{26}$$

where:

$$s_{1,i} = \frac{1}{\beta}\left[\frac{\delta+\theta}{\sqrt{1-(1-\theta)(1-\delta)} \pm \theta}\right]^2 \succ 0 \ , \ i = 1,\ 2 \quad \text{with} \quad s_{1,2} \prec s_{1,1} \tag{27}$$

Since in real economic terms the diachronic evolution of income is characterized by fluctuations around a deterministic or stochastic trend, the following analysis will be narrowed to the case where $D \prec 0$. In this case the characteristic equation (21) will have two conjugate complex roots (λ_i) and the functional form of the complementary solution will be as follows:

$$\boldsymbol{y}_t^c = R^t \left(\boldsymbol{c}_3 \cos\hat{\varphi}t + \boldsymbol{c}_4 \sin\hat{\varphi}t\right) \tag{28}$$

where $\boldsymbol{y}_t^c = \left(Y_t^c \quad I_t^c\right)'$, $R = \sqrt{m^2 + \ell^2}$: the absolute value of the complex roots, $m = -z_1/2 \succ 0$ and $\ell = \sqrt{-D}/2 \succ 0$: the real and imaginary part of the complex roots respectively, $\hat{\varphi} = arctan\left(\ell/m\right)$ and finally $\boldsymbol{c}_3 = \left(c_{31} \quad c_{32}\right)'$ & $\boldsymbol{c}_4 = \left(c_{41} \quad c_{42}\right)'$: two vectors of arbitrary constant coefficients the magnitude of which could be determined with the help of two initial conditions.

Writing the constant coefficients c_{3i} & c_{4i}, i = 1, 2, in polar coordinates as $c_{3i} = A_i \cos\varepsilon_i$ & $c_{4i} = A_i \sin\varepsilon_i$ respectively and substituting these into (28), the latter could take the equivalent form:

$$Y_t^c = A_1 R^t \cos\left(\hat{\varphi}\, t - \varepsilon_1\right) \tag{29}$$

$$I_t^c = A_2 R^t \cos\left(\hat{\varphi}\, t - \varepsilon_2\right) \tag{30}$$

where A_1 (A_2): the amplitude of oscillation of Y_t (I_t) and ε_1 (ε_2): the phase of income's (investment's) oscillation.

The functional forms of A_i, ε_i and Π, i = 1, 2, are given by the following relations:

$$A_i = \sqrt{c_{3i}^2 + c_{4i}^2} \tag{31}$$

$$\varepsilon_i = ArcTan\left(\frac{c_{4i}}{c_{3i}}\right) \tag{32}$$

$$\Pi = \frac{2\,\pi}{\hat{\varphi}} \text{ with } \pi = 3.14 \tag{33}$$

The partial solution $\left(\boldsymbol{y}_t^p\right)$ describes the dynamic equilibrium of the system (13), that is the values towards which the magnitudes of real income and gross investments will conditionally converge as $t \to +\infty$. After making use of the operational method, the resultant functional form of $\boldsymbol{y}_t^p = \left(Y_t^p \quad I_t^p\right)'$ is as follows:

$$Y_t^p = B_0 + B_1\,\tilde{\gamma}_m^t + B_2\,\tilde{\gamma}_{nm}^t \tag{34}$$

$$I_t^p = \Gamma_0 + \Gamma_1\,\tilde{\gamma}_m^t + \Gamma_2\,\tilde{\gamma}_{nm}^t \tag{35}$$

where:

$$B_0 = \frac{\beta\left(c_0 + \overline{I}\right)}{\beta\left(1 - c_1\right) - \delta} \tag{36.1}$$

$$B_1 = \frac{\tilde{\gamma}_m\left(\gamma_m + \theta\right)}{P\left(\tilde{\gamma}_m\right)} G_0^m \tag{36.2}$$

$$B_2 = \frac{\tilde{\gamma}_{nm}\left(\gamma_{nm} + \theta\right)}{P\left(\tilde{\gamma}_{nm}\right)} G_0^{nm} \tag{36.3}$$

$$\Gamma_0 = \frac{\delta\,c_0 + \left(1 - c_1\right)\beta\,\overline{I}}{\beta\left(1 - c_1\right) - \delta} \tag{37.1}$$

$$\Gamma_1 = \frac{\left(\delta + \theta\right)\tilde{\gamma}_m - \left[1 - \left(1 - \theta\right)\left(1 - \delta\right)\right]}{\beta\,P\left(\tilde{\gamma}_m\right)} G_0^m \tag{37.2}$$

$$\Gamma_2 = \frac{\left(\delta + \theta\right)\tilde{\gamma}_{nm} - \left[1 - \left(1 - \theta\right)\left(1 - \delta\right)\right]}{\beta\,P\left(\tilde{\gamma}_{nm}\right)} G_0^{nm} \tag{37.3}$$

$$P\left(\tilde{\gamma}_j\right) = \tilde{\gamma}_j^2 + z_1\,\tilde{\gamma}_j + z_2 \quad , \; j = m \; , \; nm \tag{38}$$

The relations describing the diachronic evolution of the real magnitudes of income and gross investments, that is, the general solution of the system of difference equations (13), result after the summation of relations (29) & (34) and (30) & (35):

$$Y_t = Y_t^p + Y_t^c = B_0 + B_1\,\tilde{\gamma}_m^t + B_2\,\tilde{\gamma}_{nm}^t + A_1 R^t \cos\left(\hat{\varphi}\,t - \varepsilon_1\right) \tag{39}$$

$$I_t = I_t^p + I_t^c = \Gamma_0 + \Gamma_1\,\tilde{\gamma}_m^t + \Gamma_2\,\tilde{\gamma}_{nm}^t + A_2 R^t \cos\left(\hat{\varphi}\,t - \varepsilon_2\right) \tag{40}$$

Using the Routhian theorem of stability, the general solution will be stable if the parameters $z_1 \prec 0$ and $z_2 \succ 0$ of the characteristic equation satisfy the following set of conditions:

$$1 + z_1 + z_2 \succ 0 \tag{41.1}$$

$$1 - z_1 + z_2 \succ 0 \tag{41.2}$$

$$1 - z_2 \succ 0 \tag{41.3}$$

Given that $z_1 \prec 0$ and $z_2 \succ 0$ the stability condition (41.2) is satisfied. After the substitution of relations (22) and (23) in (41.1) and (41.3), the first and third stability condition will be respectively satisfied if the following inequalities hold:

$$s_1 \succ s_{1,3} = \frac{\delta}{\beta} \succ 0 \tag{42.1}$$

$$s_1 \succ s_{1,4} = \frac{1 - (1-\theta)(1-\delta)}{\beta} \succ 0 \tag{42.2}$$

It can be proved[5] that $0 \prec s_{1,3} \prec s_{1,2} \prec s_{1,4} \prec s_{1,1}$. This implies that relation (42.2) describes the model's only stability condition, since its satisfaction guarantees the satisfaction of the stability condition (42.1).

At this point it has to be noted that relations (34) & (35) will produce acceptable equilibrium values for real income and gross investments if $Y_t^p \succ 0$ & $I_t^p \succ 0\ \forall\, t \in \mathbb{N}$. Since $\beta \succ 0,\ \bar{I} \succ 0,\ c_0 \succ 0,\ 1 - c_1 \in (0,1),\ \delta \in (0,1),\ G_0^j \succ 0$ and $\gamma_j \in (0,1),\ j = m, nm,$ it can be proved[6] that if $s_{1,4} \prec s_1 \prec s_{1,1}$ then $B_i \succ 0$ & $\Gamma_i \succ 0,\ i = 0,\ 1,\ 2.$ As a result for $s_1 \in (s_{1,4}, s_{1,1})$ we will have that $Y_t^p \succ 0$ & $I_t^p \succ 0\ \forall\, t \in \mathbb{N}$ and, additionally, real income and real gross investments will converge through oscillations towards their dynamic equilibrium values.

Table 1

Effects on Y_t^p and I_t^p as a Result of an Infinitesimal Change in the Magnitude of their Constituent Parameters

x	β	c_0	c_1	γ_{nm}	γ_m	δ	θ	$\bar{I}$	t
$\partial Y_t^p / \partial x$	−	+	+	Indeter.	Indeter.	+	+	+	+
$\partial I_t^p / \partial x$	−	+	+	Indeter.	Indeter.	+	+	+	+

In table 1 the effects of an infinitesimal change in the magnitude of only one of the parameters β, c_0, c_1, γ_{nm}, γ_m, δ, θ, $\bar{I}$ and t on the dynamic equilibrium values Y_t^p and I_t^p are presented. As we see the government can induce the rise (fall) of Y_t^p & I_t^p by increasing (decreasing) the magnitude of the rate of the depreciation of physical capital (δ). The effects of a change in γ_{nm} or γ_m on the equilibrium values of income and gross investments are indeterminable and their functional forms are as follows:

$$\frac{\partial Y_t^p}{\partial \gamma_j} = B_j\, \tilde{\gamma}_j^{t-1} \left(E_{\tilde{\gamma}B}^j + t \right) \;\&\; \frac{\partial I_t^p}{\partial \gamma_j} = \Gamma_j\, \tilde{\gamma}_j^{t-1} \left(E_{\tilde{\gamma}\Gamma}^j + t \right) \;,\; j = m,\; nm \tag{43}$$

where: $E_{\tilde{\gamma}B}^j = \dfrac{\partial B_j}{\partial \gamma_j} \dfrac{\tilde{\gamma}_j}{B_j}$, $E_{\tilde{\gamma}\Gamma}^j = \dfrac{\partial \Gamma_j}{\partial \gamma_j} \dfrac{\tilde{\gamma}_j}{\Gamma_j}$, $j = m, nm$, $B_{nm} = B_1$, $B_m = B_2$, $\Gamma_{nm} = \Gamma_1$, $\Gamma_m = \Gamma_2$.

The change of the sign of the partial derivatives $\partial Y_t^p / \partial \gamma_j$ & $\partial I_t^p / \partial \gamma_j$, $j = m, nm$ can be presented in the context of the following relation:

$$\begin{cases} \dfrac{\partial Y_t^p}{\partial \gamma_j} \succ 0 \;\&\; \dfrac{\partial I_t^p}{\partial \gamma_j} \succ 0 \;\; \forall \; t \;\&\; 0 \prec \gamma_j \leq \gamma_{j,2} \\ \\ \dfrac{\partial Y_t^p}{\partial \gamma_j} \lesseqqgtr 0 \;\&\; \dfrac{\partial I_t^p}{\partial \gamma_j} \lesseqqgtr 0 \;\; \forall \; t \lesseqqgtr -E_{\tilde{\gamma}B}^{nm} \;\&\; \gamma_{nm,2} \prec \gamma_{nm} \prec 1 \end{cases} ,\; j = m,\; nm \tag{44}$$

where:

$$\gamma_{m,2} = \gamma_{nm,2} = \frac{\theta}{\delta + \theta} \left[\sqrt{(1-\theta)\,\delta + \theta} - \delta \right] \succ 0 \tag{45}$$

It is quite evident from the above cited analysis that the government of the home country may affect (a) the dynamic characteristics of the diachronic evolution of income and gross investments and (b) the dynamic equilibrium values Y_t^p & I_t^p , through the appropriate changes of the tools of economic policy, namely, the rate of depreciation of physical capital (δ) and the growth rates of military (γ_m) and civilian (γ_{nm}) spending.

It is easy to show that the range of the interval ($s_{1,4}$, $s_{1,1}$), that is the value interval of the marginal propensity to save, for which both Y_t & I_t converge through oscillations towards their dynamic equilibrium values, is positively affected by changes in δ on behalf of the government.

4. ECONOMIC GROWTH & THE MILITARY SPENDING REACTION FUNCTION

As it was stated earlier, the government can affect the dynamic properties of the solution and the dynamic equilibrium values of income and gross investments through changes of the magnitudes of δ, γ_m and/or γ_{nm}. In order to investigate the government's ability to affect the growth of the economy under consideration, it will be assumed that $s_1 \in (s_{1,4}, s_{1,1})$. As a result of this assumption, the analysis will be focused on the growth rates of the dynamic equilibrium values of real income $\left(g_{Y_t^p}\right)$ and real gross investments $\left(g_{I_t^p}\right)$.

In order to facilitate the mathematical presentation of the realized analysis, the functional forms of Y_t^p & I_t^p, as these are described by relations (34) & (35) respectively, are modified and presented equivalently as:

$$Y_t^p = B_0 + B_1\, e^{\gamma_m t} + B_2\, e^{\gamma_{nm} t} \tag{34.1}$$

$$I_t^p = \Gamma_0 + \Gamma_1\, e^{\gamma_m t} + \Gamma_2\, e^{\gamma_{nm} t} \tag{35.1}$$

The functional form of the growth rates $g_{Y_t^p} = \dfrac{1}{Y_t^p}\dfrac{\partial Y_t^p}{\partial t}$ & $g_{I_t^p} = \dfrac{1}{I_t^p}\dfrac{\partial I_t^p}{\partial t}$ that ensue after the use of the above stated relations is as follows:

$$g_{Y_t^p} = \frac{\gamma_m\, B_1\, e^{\gamma_m t} + \gamma_{nm}\, B_2\, e^{\gamma_{nm} t}}{B_0 + B_1\, e^{\gamma_m t} + B_2\, e^{\gamma_{nm} t}} = \begin{cases} \dfrac{\gamma_m\, B_1\, e^{(\gamma_m - \gamma_{nm})t} + \gamma_{nm}\, B_2}{B_0\, e^{-\gamma_{nm} t} + B_1\, e^{(\gamma_m - \gamma_{nm})t} + B_2} & \text{if} \quad \gamma_m \prec \gamma_{nm} \\[2ex] \dfrac{\gamma_m\left(B_1 + B_2\right)}{B_0\, e^{-\gamma_m t} + B_1 + B_2} & \text{if} \quad \gamma_m = \gamma_{nm} \\[2ex] \dfrac{\gamma_m\, B_1 + \gamma_{nm}\, B_2\, e^{(\gamma_{nm} - \gamma_m)t}}{B_0\, e^{-\gamma_m t} + B_1 + B_2\, e^{(\gamma_{nm} - \gamma_m)t}} & \text{if} \quad \gamma_m \succ \gamma_{nm} \end{cases} \tag{46}$$

$$g_{I_t^p} = \frac{\gamma_m\, \Gamma_1\, e^{\gamma_m t} + \gamma_{nm}\, \Gamma_2\, e^{\gamma_{nm} t}}{\Gamma_0 + \Gamma_1\, e^{\gamma_m t} + \Gamma_2\, e^{\gamma_{nm} t}} = \begin{cases} \dfrac{\gamma_m\, \Gamma_1\, e^{(\gamma_m - \gamma_{nm})t} + \gamma_{nm}\, \Gamma_2}{\Gamma_0\, e^{-\gamma_{nm} t} + \Gamma_1\, e^{(\gamma_m - \gamma_{nm})t} + \Gamma_2} & \text{if} \quad \gamma_m \prec \gamma_{nm} \\[2ex] \dfrac{\gamma_m\left(\Gamma_1 + \Gamma_2\right)}{\Gamma_0\, e^{-\gamma_m t} + \Gamma_1 + \Gamma_2} & \text{if} \quad \gamma_m = \gamma_{nm} \\[2ex] \dfrac{\gamma_m\, \Gamma_1 + \gamma_{nm}\, \Gamma_2\, e^{(\gamma_{nm} - \gamma_m)t}}{\Gamma_0\, e^{-\gamma_m t} + \Gamma_1 + \Gamma_2\, e^{(\gamma_{nm} - \gamma_m)t}} & \text{if} \quad \gamma_m \succ \gamma_{nm} \end{cases} \tag{47}$$

where $e \approx 2.718$: the base of the natural logarithms.

Taking the limits of relations (46) & (47) as $t \to +\infty$ we have that:

$$\lim_{t\to+\infty} g_{Y_t^p} = \lim_{t\to+\infty} g_{I_t^p} = \begin{cases} \gamma_{nm} & \text{if} \quad \gamma_m \prec \gamma_{nm} \\ \gamma_m = \gamma_{nm} & \text{if} \quad \gamma_m = \gamma_{nm} \\ \gamma_m & \text{if} \quad \gamma_m \succ \gamma_{nm} \end{cases} \tag{48}$$

According to relation (48) the magnitudes of the growth rates $g_{Y_t^p}$ and $g_{I_t^p}$ will converge, on the long run, to the greater in value growth rate among γ_m and γ_{nm} set by the government. In other words, in periods of war (peace), where normally the home government sets the growth rate of military spending at a higher (lower) level than the growth rate of non – military spending, that is $\gamma_m \succ \gamma_{nm}$ $(\gamma_m \prec \gamma_{nm})$, the growth rates of both Y_t^p & I_t^p will converge to the greater in magnitude growth rate $\gamma_m(\gamma_{nm})$ of the (non) military spending. In the special case where $\gamma_m = \gamma_{nm}$, the growth rates $g_{Y_t^p}$ and $g_{I_t^p}$ will converge on the long run to this specific common rate of growth.

In peace time and for reasons of national security, the government of the home country may set the magnitude of the growth rate of the home military spending (γ_m) equal to its foreign correspondent (γ_{m^*}), that is $\gamma_m = \gamma_{m^*}$, so as to a retain a constant ratio of military spending $(G_t^m / G_t^{m^*})$. This policy rule gives to the government of the rival country the ability to affect the home country's dynamic equilibrium values Y_t^p & I_t^p through changes of $\gamma_{m^*}(=\gamma_m)$.

To cancel this ability without the policy rule $\gamma_m = \gamma_{m^*}$ being breached, the government of the home country will use relations (34) & (35) in order to derive the functional form of the appropriate reaction functions, which could be used in order to achieve her objective. More specifically, after taking the total derivative of relations (34) & (35) for given values of the parameters β, c_0, c_1, δ, θ, $\bar{I}$ & t, and by setting in the context of them where $\gamma_m = \gamma_{m^*}$ $(\Rightarrow d\gamma_m = d\gamma_{m^*})$ and $dY_t^p = dI_t^p = 0$, the derived functions have the following form:

$$\overset{o}{\gamma}_{nm} = -\,\mathcal{E}_{11}\,\frac{\gamma_{m^*}}{\gamma_{nm}}\,\overset{o}{\gamma}_{m^*} - \mathcal{E}_{12}\,\frac{\delta}{\gamma_{nm}}\,\overset{o}{\delta} \tag{49.1}$$

$$\overset{o}{\gamma}_{nm} = -\,\mathcal{E}_{21}\,\frac{\gamma_{m^*}}{\gamma_{nm}}\,\overset{o}{\gamma}_{m^*} - \mathcal{E}_{22}\,\frac{\delta}{\gamma_{nm}}\,\overset{o}{\delta} \tag{49.2}$$

where $\overset{o}{\gamma}_{nm} = d\gamma_{nm}/\gamma_{nm}$ & $\overset{o}{\gamma}_{m^*} = d\gamma_{m^*}/\gamma_{m^*}$: the percentage changes of the growth rates of the home non-military spending and the foreign military spending respectively, $\overset{o}{\delta} = d\delta/\delta$: the percentage change of the rate of depreciation of physical capital, $\mathcal{E}_{11} = \dfrac{\partial Y_t^p / \partial \gamma_m}{\partial Y_t^p / \partial \gamma_{nm}}$, $\mathcal{E}_{12} = \dfrac{\partial Y_t^p / \partial \delta}{\partial Y_t^p / \partial \gamma_{nm}}$, $\mathcal{E}_{21} = \dfrac{\partial I_t^p / \partial \gamma_m}{\partial I_t^p / \partial \gamma_{nm}}$ & $\mathcal{E}_{22} = \dfrac{\partial I_t^p / \partial \delta}{\partial I_t^p / \partial \gamma_{nm}}$.

Relations (49.1) & (49.2) represent the reactions functions of the home government to a change in the growth rate of the foreign military spending and form a linear system of equations that could be solved with respect to $\overset{o}{\gamma}_{nm}$ & $\overset{o}{\delta}$. The solution of this system is described by the following set of equations:

$$\overset{o}{\gamma}_{nm} = -\frac{\gamma_{m^*}}{\gamma_{nm}} \frac{\mathcal{E}_{11}\mathcal{E}_{22} - \mathcal{E}_{12}\mathcal{E}_{21}}{\mathcal{E}_{22} - \mathcal{E}_{12}} \overset{o}{\gamma}_{m^*} \tag{50.1}$$

$$\overset{o}{\delta} = -\frac{\gamma_{m^*}}{\delta} \frac{\mathcal{E}_{21} - \mathcal{E}_{11}}{\mathcal{E}_{22} - \mathcal{E}_{12}} \overset{o}{\gamma}_{m^*} \tag{50.2}$$

The above stated relations are used by the government of the home country after a change in γ_{m^*}, in order to determine the percentage changes of the growth rates of γ_{nm} and δ so as to cancel out the effects on the dynamic equilibrium values Y_t^p & I_t^p and retain the ratio of military spending unchanged. It has to be noted here that in the special case where $\mathcal{E}_{21} - \mathcal{E}_{11} = 0$, the home government does not have to alter the level of the rate of depreciation of physical capital in order to achieve her objectives. As a result the value interval ($s_{1,4}$, $s_{1,1}$) and, consequently, the dynamic characteristics of the model's solution are not affected by the actions of the home government.

5. SIMULATION ANALYSIS

In the present section the mathematical and the theoretical inferences of the paper will be tested by performing a simulation analysis. In its context the government of the home country is assumed to set for reasons of national security the size of the home growth rate of military spending equal to the growth rate of military spending of the rival country.

The arithmetic values of the model's parameters and the initial conditions that will be used in the simulation are as follows: $\alpha = 4$, $\beta = 4$, $\gamma_m = \gamma_{nm} = \gamma_{m*} = 0.01$, $\tilde{\gamma}_m = \tilde{\gamma}_{nm} = \tilde{\gamma}_{m^*} = 1.01$, $\delta = -1 + 2\sqrt{\frac{14}{5}} \approx 0.4967$, $\theta = \frac{69}{88} - \frac{19}{220}\sqrt{\frac{7}{2}} \approx 0.6225$, $c_0 = 50$, $c_1 = 0.75$, $s_1 = 0.25$, $v = 0.25$, $\bar{I} = 50$, $G_0^m = 50, G_0^{nm} = 150$, $\bar{Y}_0 = 1000, \bar{Y}_1 = 1050, \bar{I}_0 = 162.5, \bar{I}_1 = 174.625$ and $\tilde{k}_0 = 0.5$.

The fact that $\tilde{k}_0 = 0.5 \prec 1 = \alpha/\beta$ implies that the physical capital is the limiting factor of production, with the level of production of the home country being determined via a production function of the form described by relation (1). In this case the linear system of difference equations that has to be solved in order to specify the functional form of the relations describing the diachronic evolution of real income and real gross investments, is given by relation (13) with:

$$|\boldsymbol{P(F)}| = P_{22}(F) + \frac{P_{21}(F)}{1-c_1} = 4\,F^2 - 5.9867\,F + 3.24 \tag{15.1}$$

$$\tilde{\boldsymbol{H}}_t = \begin{bmatrix} 996.0310 + 127.7689(1.01)^t + 255.5378(1.01)^t \\ \\ 186.3403 + 16.0187(1.01)^t + 32.0374(1.01)^t \end{bmatrix} \tag{19.1}$$

The characteristic equation of the system is described by relation (21) with: $z_1 = -\,2\,\sqrt{14}/5 \approx -\,1.4967 \prec 0$ and $z_2 = 81/100 = 0.81 \succ 0$. After the substitution of the parameter's arithmetic values in relations (27), (42.1) and (42.2) it is established that $0 \prec s_{1,3} \approx 0.1242 \prec s_{1,2} \approx 0.1351 \prec s_{1,4} = 0.2025 \prec s_{1,1} \approx 4.0670$. Since $s_1 = 0.25 \in (s_{1,4}\,,\, s_{1,1})$ we expect to have (*i*) that $D \prec 0$, that is the characteristic equation has two conjugate complex roots, (*ii*) that the stability condition is satisfied, that is $0 \prec R \prec 1$, and (*iii*) that $B_i \succ 0\ \ \&\ \ \Gamma_i \succ 0,\ i = 0,\ 1,\ 2.$

These expectations are confirmed since (*i*) $D = z_1^2 - 4\,z_2 = -1 \prec 0$, $\lambda_1 = m - \ell\ i$ & $\lambda_2 = m + \ell\ i$, with $m = \dfrac{\sqrt{14}}{5} \approx 0.7483,\ \ell = 1/2 = 0.5\ \&\ i = \sqrt{-1}$, (*ii*) $R = \sqrt{m^2 + \ell^2} = 0.9 \in (0,1)$ and (*iii*) the resulting in the context of relations (36.1) ~ (38) magnitude of constant parameters B_i and Γ_i, i = 0, 1, 2, have as follows: $B_0 \approx 794.6961$, $B_1(= B_m) \approx 100.2989$, $B_2(= B_{nm}) \approx 200.5978$, $\Gamma_0 \approx 148.6740$, $\Gamma_1(= \Gamma_m) \approx 12.5747$ and $\Gamma_2(= \Gamma_{nm}) \approx 25.1494$.

The model's general solution that results in the context of relations (39) & (40), and after making use of the initial conditions $Y_0 = \overline{Y}_0 = 1000$, $Y_1 = \overline{Y}_1 = 1050$, $I_0 = \overline{I}_0 = 162.5$ & $I_1 = \overline{I}_1 =$ 174.625, has the following form:

$$Y_t = 794.6961 + 100.2989(1.01)^t + 200.5978(1.01)^t + A_1(0.9)^t \cos(0.589\,t - \varepsilon_1) \tag{39.1}$$

$$I_t = 148.674 + 12.5747(1.01)^t + 25.1494(1.01)^t + A_2(0.9)^t \cos(0.589\,t - \varepsilon_2) \tag{40.1}$$

where $A_1 \approx -\,106.0270,\ A_2 \approx -\,26.5068,\ \varepsilon_1 \approx -\,0.4474\ \&\ \varepsilon_2 \approx -\,0.4474$.

The diachronic evolutions of real income and real gross investments, as these are described by relations (39.1) & (40.1) respectively, are presented graphically in the context of figure 1.

The analysis performed on the basis of relation (48) suggests that since in the context of the simulation analysis we have $\gamma_m = \gamma_{nm} = 0.01$ and the solution is stable, the growth rates of Y_t^p & I_t^p, and consequently of Y_t & I_t, must converge on the long run to 1%. This is graphically affirmed in the context of figure 2 where the diachronic evolution of the growth rates of

Figure 1: Diachronic Evolution of Income and Gross Investments

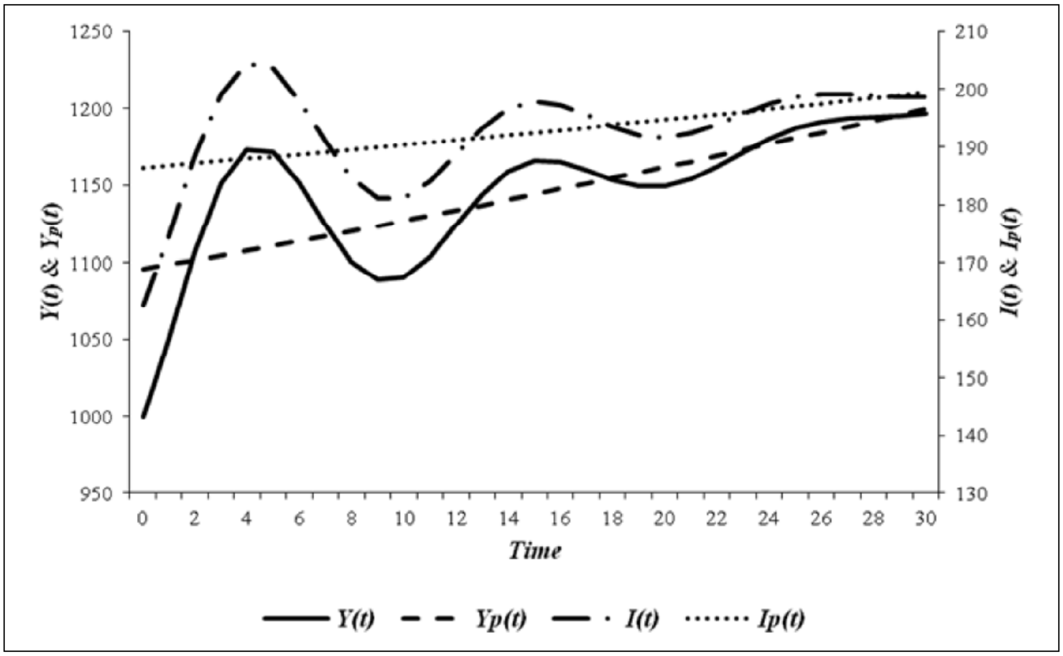
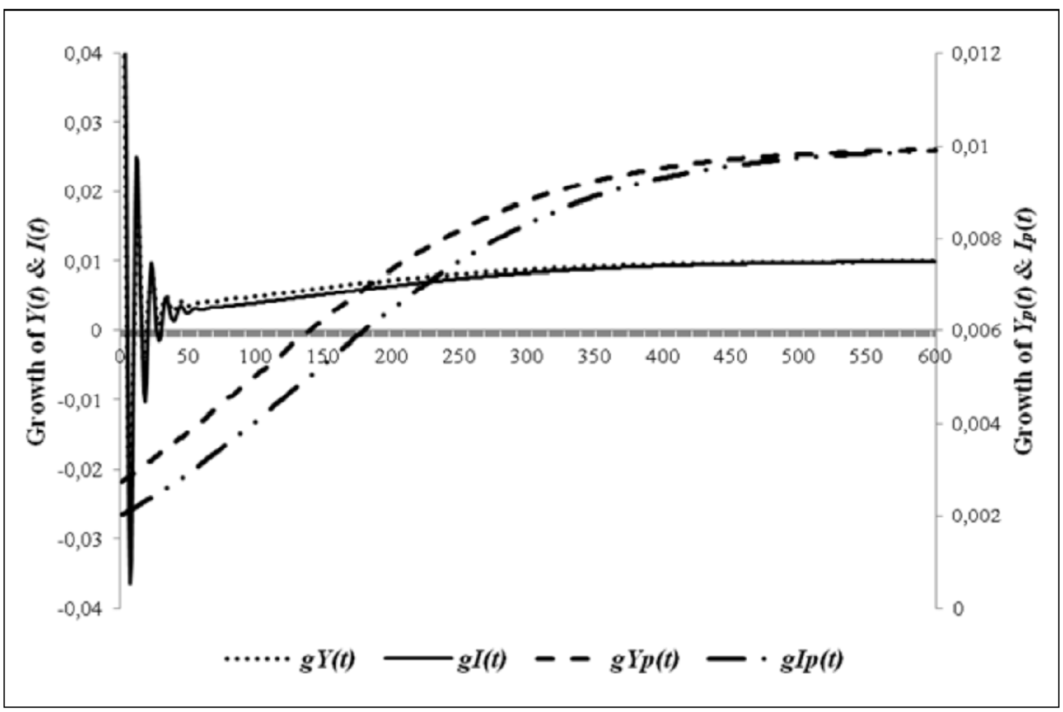


Figure 2: Evolution of the Growth Rates of $Y(t)$, $I(t)$, $Y_p(t)$ & $I_p(t)$



real income, real gross investments and their dynamic equilibrium values are presented for $t = 0, \ldots, 600$.

Let us assume now that the government of the rival country rises at time $t = 20$ the growth rate of its military spending by 20% $\left(\overset{o}{\gamma}_{m^*} = 0.2\right)$ from $\gamma_{m^*} = 0.01$ to $\gamma'_{m^*} = 0.012$. Given the home country's policy to maintain a constant ratio of military spending, the home government will use (54.1) and (54.2) in order to achieve this goal and additionally to cancel out the effects on the dynamic equilibrium values of domestic income and gross investments.

The magnitudes of $\mathcal{E}_{ij}$, $i, j = 1, 2$, which are calculated[7] at $t = 20$ and for the given values of the model's parameters used in the simulation analysis, are as follows: $\mathcal{E}_{11} = \mathcal{E}_{21} = 0.5$, $\mathcal{E}_{12} \approx 0.3113$ & $\mathcal{E}_{22} = 0.5969$. For these values and for $\gamma_{nm} = \gamma_{m^*} = 0.01$, $\delta \approx 0.4967$ & $\overset{o}{\gamma}_{m^*} = 0.2$, the resulted from relations (54.1) & (54.2) magnitudes of the growth rates of home non-military spending and the rate depreciation are as follows: $\overset{o}{\gamma}_{nm} \approx -0.1$ & $\overset{o}{\delta} = 0$. In other words after the rise of γ_m from 1% to 1.2%, the home government will have to reduce the growth rate of the non-military spending from 1% to 0.9%.

6. CONCLUSIONS

In the present paper a Keynesian type model of a closed developing economy has been expounded. In the context of this model a second order system of difference equations with constant coefficients was developed and was solved with respect to the variables of (real) income and (real) gross investments.

As it was shown the government can affect the dynamic properties of the model's general solution, through changes of the rate of depreciation of physical capital (δ). More specifically, a rise (fall) in δ on behalf of the government results to the enlargement (shrinkage) of the specified value interval ($s_{1,4}$, $s_{1,1}$) of the marginal propensity to save, for which the real income and the real gross investments converge through oscillations towards their positively defined dynamic equilibrium values.

Moreover, the government can affect the dynamic equilibrium values of income and gross investments, through changes of the rate of depreciation (δ) and the growth rates of military (γ_m) & non-military (γ_{nm}) spending. The model's dynamic equilibrium is proved to be positively affected by changes in δ, while its affection by changes in the magnitude of the growth rates γ_m and γ_{nm} was proved to be indeterminable. As it was shown, when the model's stability condition is satisfied, the growth rates of the real income and the real gross investments tend, on the long run, towards the biggest in magnitude among the set by the government growth rates γ_m and γ_{nm}.

When the government of the home country sets the magnitude of γ_m equal to the growth rate of the military spending $\gamma_{m'}$ of a rival country, the equality $\gamma_m = \gamma_{m'}$ gives to the government of the rival country the opportunity to affect the dynamic equilibrium values of the home real income and gross investments, through changes of the magnitude of the growth rate γ_m. A set of reaction functions have been specified in the paper under the policy restriction $\gamma_m = \gamma_{m*}$, that could be used on behalf of the home government, so as to cancel out the effects of a change in γ_{m*} on the home dynamic equilibrium values of income and gross investments.

The mathematical and theoretical inferences of the paper have been tested and confirmed in the context of a simulation analysis.

Notes

1. The realized assumption that the level of prices is constant over time implies in the present analysis that real and nominal magnitudes are treated as equal.
2. The functional form of gross private investments, as described by relation (5), is a modification of the investment function used in the context of Hicks' (1950) business cycles model, which resulted after the replacement of ΔY_t by its expected counterpart as an explanatory variable in the gross investment function.
3. The multiplication of the forward operator F^n, $n = 0, 1, \ldots$, with the time variable X_t results to the forward shift of the time variable by n periods, that is, $F^n X_t = X_{t+n}$.
4. The possibility to have a sinusoidal way of movement as a result of the existence of negatively defined real characteristic root is ruled out.
5. The proof is available on request.
6. *Ibid*
7. The terms $\mathcal{E}_{ij}$ are in fact functions of time. This implies that after the use of relations (54.1) & (54.2), the growth rates γ_{nm} & δ are no longer constant over time. In order to avoid this "uncomfortable" development the terms $\mathcal{E}_{ij}$ are calculated at the time of the change of γ_{m^*}.

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