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$\mathbf{H}_{\!\scriptscriptstyle \infty}$ Loop Shaping Controller design for regulating Air Fuel Ratio and Speed of an SI Engine

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Abstract: This paper discuss about developing Multi Input Multi Output (MIMO) Robust H_{∞} controller for regulation Air fuel ratio (λ) and speed of an SI Engine based on H_{∞} Loop shaping approach in normal operating range. Mean Value Engine Model has been used for validating the controller performance. Developed H_{∞} controller is a 3x3 transfer function matrix of 13th order. Robustness of the system is verified by testing the closed loop system with load disturbance, Parameter variations and noise. Results show that under H_{∞} controller, SI engine performs better when subjected to load disturbance, Parameter variations and noise.

Keywords: SI Engine control, Robust control, Air fuel ratio, speed, H., Loop Shaping.

INTRODUCTION

SI engine is a highly nonlinear system which demands a good controller design for regulating its output against disturbances. Due to stringent emission norms of automotive standards it has become mandatory to maintain Air fuel ratio(AFR) at stiochiometric value and regulate speed at constant value. SI Engine works in idle operating mode and Normal operating Mode. This paper considers engine working in normal operating mode. Elbert Hendricks *et al.* developed Mean value SI Engine model for control studies [1]. Christopher H onder *et al.* [2] discuss about developing multivariable speed and Air fuel ratio control of SI Engine based on LQG/LQR method. Hendricks *et al.* [3] describes developing H_{∞} controller based on iterative process for controlling Air fuel ratio of an SI engine. Dingli Yu *et al.* [4] validate Robustness of an engine model that is developed using RBF Neural Networks.

After a short introduction this paper continues with section II describes governing equations of MIMO Engine Model. Section III discusses about development of H_{∞} Loop shaping controller. Section IV assess Robustness of SI engine with load disturbance parameter variation and noise.

ENGINE DYNAMICS

The Mean value Engine model of an SI engine consists of three sub systems. Fuelling sub system, crank shaft dynamics and Manifold Air flow system. The three subsystems are governed by the following equations.

I. Fuelling Sub system

$$\dot{m}_{ff} = \frac{1}{\tau_f} \left(-\dot{m}_{ff} + X\dot{m}_{fi} \right)$$

$$\dot{m}_{fv} = (1 - X)\dot{m}_{fi}$$

$$\dot{m}_f = \dot{m}_{ff} + \dot{m}_{fv}$$
(1)

II. Crank Shaft Dynamics

$$\dot{n} = \frac{1}{nI} (H_u \eta_i \dot{m}_f (t - \tau_d) - P_l - P_b)$$

$$P_l = n(k_1 + k_2 n + k_3 n^2) + n(-k_4 + k_5 n) p_i$$

$$P_b = k_b n^3$$

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where,

$$\begin{aligned} \eta_{i} &= \eta_{in} \eta_{ip} \eta_{i\lambda} \eta_{i\theta} \\ \eta_{in}(n) &= k_{6} (1 - k_{7} n^{-0.36} \\ \eta_{ip}(n) &= k_{8} + k_{9} p_{i} - k_{10} p i^{2} \\ \eta_{i\lambda} &= -k_{11} + k_{12\lambda} - k_{13\lambda^{2}} \\ \eta_{i\theta} &= e^{-\theta^{2}/2\theta_{mbt}^{2}} \\ \eta_{mbt} &= \min(\theta_{1}, \theta_{2}), 45) \\ \theta_{1} &= k_{14pi} + k_{15} + n_{47} \\ \theta_{2} &= k_{16pi} + k_{17} + n_{47} \\ n_{47} &= \begin{cases} 4.7n, & n < 4.8 \\ 4.7 \times 4.8, & otherwise, \end{cases} \end{aligned}$$

III. Manifold Airflow System

$$\begin{split} \dot{p_i} &= \frac{RT_i}{V_i} (-\dot{m}_{ap} + \dot{m}_{at}) \\ \dot{m}_{ap} &= \frac{RT_i}{V_i} (k_{18}p_i + k_{19})n \\ \dot{m}_{at} &= m_{at1} \frac{p_a}{\sqrt{(T_a)}} \beta_2(p_r) \beta_1(a) = k_{20} \beta_2(p_r) \beta_1(a) \end{split}$$

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$$\beta_{1}(a) = 1 - \cos(a - a_{0})$$

$$\beta_{2}(p_{r}) = \begin{cases} \frac{1}{p_{n}} \sqrt{p_{r}^{p_{1}} - p_{r}^{p_{2}}} & \text{if } p_{r} \ge p_{c} \\ 1 & \text{otherwise} \end{cases}$$

The relationship between Lambda and states of the state space model is given y equation 2.

$$\lambda = \frac{\dot{m}_{ap}}{14.67\dot{m}_f} = \frac{1}{14.67} \frac{V_d}{120RT_i \dot{m}_f} (k_{18} p_i + k_{19}) n \tag{2}$$

H LOOP SHAPING CONTROLLER DESIGN

Controller Design

 H_{∞} loop shaping controller is well known for its effective methodology in designing robust controllers. In this method, the plant is shaped by using a pre and post compensators namely W_1 and W_2 (i.e., $G_s = W_1 G W_2$), respectively. Once the desired loop shape is achieved, a stabilizing controller (K_{∞}) is achieved by minimizing the ∞ -norm of the closed loop system ($\|T_{zw}\|_{\infty}$) from the disturbances d_1 and d_2 to the outputs z_1 and z_2 . The ∞ -norm of the closed loop system is given in the equation (3).

$$\gamma = \frac{1}{\varepsilon} = \|\mathbf{T}_{zw}\|_{\infty} = \left\| \begin{bmatrix} \mathbf{I} \\ \mathbf{K}_{\infty} \end{bmatrix} (\mathbf{I} - \mathbf{G}_{s}\mathbf{K}_{\infty})^{-1} \widetilde{M}_{s}^{-1} \right\|_{\infty}$$
(3)

where, ε represents the stability margin of the closed loop system, G_s is the shaped system, M_s is the co-prime factor of the shaped system. Then, the final controller is constructed by combining the stabilizing controller (K_n) with W₁ and W₂ as in the equation (4).

$$\mathbf{K} = \mathbf{W}_1 \mathbf{K}_{\infty} \mathbf{W}_2 \tag{4}$$

The objective of the controller is to maintain Air fuel ratio and Speed. This is done by regulating mass fuel flow rate, speed and manifold pressure. For the stationary point [.00170, 2.3899, 0.83548], the Lambda value to be regulated is 1 and crank speed 2.3899. To achieve this, inputs fuel injection, spark timing and throttle angle of plant are manipulated by controller. Fig. 1 shows the overall structure of control system.



Figure 1: Block Diagram of Closed Loop System

Operating Point

SI Engine has three states $X = [\dot{m}_f, n, p_i]$, three inputs $u = [\dot{m}_{fi}, \delta, \theta]$ and three outputs $y = [\dot{m}_f, n, p_i]$. The operating point are chosen for linearization X = [0.002, 2.5, 0.9]. λ value of the engine is a function of Hence the \ddot{e} value of is maintained at constant value of 1.0 by regulating $[\dot{m}_f, n, p_i]$. The linearised state space model is given by.

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 573800 & -912.2 & -336.9 \\ 0 & -14.31 & -133.8 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2528 & 0 \\ 0 & 0 & 321.6 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Conformity of the state space model to non linear equations was verified. This is done by giving incremental input to the model and adding the obtained output with operating point. The same incremental input is given to the non linear equations. Both the outputs were compared and found to be nearly same.

Weighting function selection:

 W_1 and W_2 have been selected in such a way that the resulting loop shape provides load disturbance rejection in the lower frequency region, robust stability and noise rejection at higher frequency region.

$$W_1 = \begin{bmatrix} \frac{100}{S+.001} & 0 & 0\\ 0 & \frac{100}{S+.001} & 0\\ 0 & 0 & \frac{100}{S+.001} \end{bmatrix} \text{ and } W_2 = I_{3X3}$$

Fig. 2 shows sigma plot of the Nominal plant and shaped plant. From the figure it is inferred that sigma plot of the shaped plant has higher gain at low frequencies and lower gain at higher frequencies.



Figure 2: Sigma plot of Nominal plant and Shaped Plant

The developed $\mathbf{H}_{\mathbf{x}}$ controller resulted in an epsilon value of .4565(\geq .25) which guarantees robust stability.

Fig. 3. Shows Simulink Diagram of the Closed Loop System. Fig 4, Fig 5 and Fig 6 shows that the H_acontroller performs well in maintaining states. Fig. 7 shows that lambda value is also regulated well.

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Figure 3: Simulink Diagram of closed loop system

Response of Closed Loop system

Step input for regulating Mass fuel flow rate, speed and Manifold pressure is given for the closed loop system and its responses for mass fuel flow rate, speed, Manifold pressure and Lambda are given below.



Figure 4: Mass fuel flow rate



Robustness Verification

(a) Load Disturbance

A Saw tooth wave form of 1v and 5 Hz has been given as load variation. Fig 8. Shows Load disturbance signal. From Fig. 9, Fig. 10, Fig. 11, Fig. 12 it is verified that the effect of disturbance is less than 0.1 in the mass fuel flow rate, speed and manifold pressure signal.



Figure 9: Mass fuel flow rate











(b) Noise Rejection

Noise signals of magnitude 1.0 at higher frequencies were added with feedback signals and its effect of influence on the output has been studied. Fig 13 shows the noise signal.Fig 14, Fig 15, Fig 16shows the effect on the output signal is less than 0.1v and this assures that developed controller ensures noise rejection.





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Figure 16: Manifold Pressure with load disturbance and noise

(c) Parameter Variations

The state space model obtained for variations in decrease in 5% volumetric efficiency, 1% decrease in inertia and increase of Engine displacement volume of 1% is given by

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 575600 & -919 & -336.8 \\ 0 & -15.62 & -141.5 \end{bmatrix} B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2439 & 0 & -77.36 \\ 0 & 0 & 328.1 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Fig. 17, Fig 18, Fig 19 shows the response of SI engine under parameter variations, Load Disturbance and Noise. H_{∞} Loop Shaping controller ensures that the variation is less than 0.1.



Figure 18: Speed with load disturbance, parameter variations and noise

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H_m LOOP SHAPING CONTROLLER

The developed H_a Loop Shaping controller resulted in 13nth order 3X3 controller. The transfer function matrix of the controller is given below.



Hardware in Loop Simulation and Rapid Controller Prototyping

Hardware in loop Simulation

Hardware in loop simulation of SI Engine was done using NI PXI system. The PXI system consists of NI PXIe-1082 chassis with NI PXIe-8820 controller and data acquisition cards such as NI PXIe-6341 and NI PXI 6704. NI PXIe-6341 X series Multifunction DAQ card can acquire analog inputs from 16 channels at the maximum rate of 500 KSa/s. It also has 2 Analog outputs and 4 counters. NI PXI 6704 Analog output card has 16 voltage output channels and 16 current output channels and 8 digital I/O lines. Analog inputs are obtained via SCB-68A terminal unit and Analog outputs are obtained via CB-68 LP terminal unit. Fig 21. Show the photographic view of the overall setup.



Figure 20: Hardware loop simulation of SI Engine using NI PXI system



Figure 21: Photographic view of the overall setup

Rapid Controller Prototyping

NI Compact RIO is one of the hardware platforms that combine the power of Processor and FPGA technology. While processors run computations in sequence FPGAs can run computations in parallel.cRIO consists of NI cRIO chassis 9014, Analog input Modules cRIO-9215, Analog current output Modules NI-9265 and Digital I/O Modules NI-9472.Rapid controller prototyping needs one time critical loop that implements controller running on real time target. The set point for regulating three output variables are given in cRIO.

Analog input such as Speed, Lambda and Manifold pressure are obtained using cRIO-9215. Output from PID Controller such as fuel injection, spark timing and throttle angleis sent through NI 9265. MIMO H infinity loop shaping PID Controller is implemented using cRIO as 3X3 matrix.

Parameter of Engine

The simulations are carried out for a 1275cc British Leyland engine for which data have been reported[24], [25]. The values of the parameters used in the simulations are:

$$\begin{split} I &= (2\pi/60)^2 \times 0.5 \text{ kg} \ 0\text{m}^2, \ \tau_j = 0.25, \\ X &= 0.22, \ V_i = 646 \times 10^{-6}, \ H_u = 4.3 \times 104, \ V_d = 1.275, \ R = 287 \times 10^{-5}, \ T_i = 308, \\ T_a &= 297, \ P_a = 1.013, \ k_1 = 1.673, \ k_2 = 0.272, \ k_3 = 0.0135, \ k_4 = 0.969, \ k_5 = 0.206, \ k_6 = 0.558, \ k_7 = 0.392, \ k_8 = 0.9301, \\ k_9 &= 0.2154, \ k_{10} = 0.1657, \ k_{11} = 1.025, \ k_{12} = 4.5, \ k_{13} = 2.5, \ k_{14} = 47.31, \ k_{15} = 2.6214, \ k_{16} = -56.55, \ k_{17} = 57.34, \ K_{18} = 0.952, \\ k_{19} &= 0.075, \ k_{20} = 7.32 Pa/\sqrt{Ta} \,. \end{split}$$

CONCLUSION

In this work H" Loop Shaping controller has been developed for regulating AFR and Speed of an SI Engine. Robustness of the controller is evaluated for Load disturbance, Parameter Variations and noise. The results show that the variation in the output is very less under the above influences. But the developed controller resulted in a 3X3 Transfer function matrix of order 13.In the future work it is proposed to develop fixed structure controller using H" Loop Shaping approach.

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