I J C T A, 9(25), 2016, pp. 295-300 ISSN: 0974-5572

On Totally bT^{μ}–Connected Functions in Supra Topological Spaces

K. Krishnaveni* and M. Vigneshwaran**

ABSTRACT

In this paper, we came out with the concept of totally bT^{μ} - connected function.

Also, we have obtained some of the separation axioms using totally bT^{i} - connected function in supra topological space.

Keywords: totally bT^{μ} - connected function; co bT^{μ} - T_1 space; co bT^{μ} - T_0 space; co bT^{μ} – Hausdorff space.

2010 MSC Subject Classification: 54C05, 54D10, 54D15

1. INTRODUCTION

In the field of Pure Mathematics, functions play an major role. The latest growth of simulating and universality of continuous function, separation axioms etc are used up by many experts. The concept of supra topology were introduced by Mashhour.et.al [10] and also studied S - continuous maps and S*- continuous maps in Supra topological spaces. Also Mashhour.et.al [10] discussed that many results of topological spaces, whereas some becomes false. Also the authors remarked that the intersection of two supra open sets need not be supra open and also the intersection of an open set and supra open set need not be supra open. They were also introduced S-T_0, S-T_1, S-T_2,S-T_2 spaces and discussed their relationship with the topological spaces T_0,T_1,T_2, spaces.

In 2008, R. Devi, S.Sampath kumar and M. Caldas [3] introduced supraα-open sets and Sα - continuous functions and investigate some of the basic properties of this function. Sayed and Takashi Noiri [11] studied the approach of b-open set and supra b -continuity in supra topological space in 2010 and also discussed about the relation between supra b-continuous maps and supra b-open maps. In 2013, Krishnaveni and Vigneshwaran [7] came out with the concept of supra bT closed set and studied some of its properties. Krishnaveni and Vigneshwaran [8] came out with the theory of supra bT connected function in supra topological spaces, and also came out with co bT^{μ} -T_1 space, co bT^{μ} - T_0 space and co bT^{μ} - Hausdorff space. Erdal Ekici and Takashi Noiri [4] introduced and studied the concept of generalization of normal, almost normal and mildly normal spaces and obtain the characterizations and the relationships of each normal space by using δ - pre closed functions and also obtained the preservation theorems.

Supra b-compact and supra b-Lindelof spaces were introduced by Jamal M. Mustafa.et.al [6] and also discussed about the compactness and Lindelof spaces. Ganes M. Pandya, C. Janaki and I. Arockiarani [5] introduced a new class of set connected functions called π - set connected and also investigated the relationship between π - set connected function, separation axioms and covering Properties.

* Research Scholar, Department of Mathematics, Kongunadu Arts and Science College, Coimbatore-641 029, Tamil Nadu, India, *Email: krishnavenikaliswami@gmail.com*

** Assistant Professor, Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, 641 029, Tamil Nadu, India, *Email: vignesh.mat@gmail.com*

The purpose of this paper is to bring out with the concept of totally bT^{I} - connected functions and also to bring out some separation axioms related to totally bT^{i} – connected functions in supra topological spaces.

2. PRELIMINARIES

Definition 2.1 [11] A subfamily of μ of X is said to be a supra topology on X, if

(i) $X, \phi \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.

The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition 2.2 [10, 11]

(i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is denoted as $cl^{\mu}(A) = \cap\{B: B$ is a supra closed set and $A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int^{\mu}(A)$ and is denoted as $int^{\mu}(A) = \cap\{B: B$ is a supra open set and $A \supseteq B\}$.

Definition 2.3 [10, 11] Let (X, τ) be a topological spaces and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition 2.4 [7] A subset A of a supra topological space (X, μ) is called bT^{μ}-closed set, if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is T^{μ} - open in (X, μ).

Definition 2.5 [8] A supra topological space X is said to be clopen bT^{μ} - T_1 space if for each pair of distinct points x and y of X, there exist bT^{μ} - clopen sets U and V containing x and y respectively such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

Definition 2.6 [8] A supra topological space X is said to be supra clopen T_1 space if for each pair of distinct points x and y of X, there exist supra clopen sets U and V containing x and y respectively such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

Definition 2.7 [8] A supra topological space X is said to be clopen bT^{μ}-T_2 if every two distinct points of X can be separated by disjoint bT^{μ} - clopen sets.

Definition 2.8 [8] A supra topological space X is said to be clopen bT^{μ}-T_0 if for each pair of distinct points in X, there exist a bT^{μ} - clopen set containing one point but not the other.

Definition 2.9 [8] A supra topological space X is said to be supra ultra Hausdorff or UT_2 if every two distinct points of X can be separated by disjoint supra clopen sets.

Definition 2.10 [8] A supra topological space X said to be supra ultra regular if for each supra closed set F of X and each $x \notin F$, there exist disjoint supra clopen sets U and V such that $F \subset U$ and $x \in V$.

Definition 2.11 [9] A supra topological space X is said to be supra ultra normal if each pair of non-empty disjoint supra closed sets can be separated by disjoint supra clopen sets.

Definition 2.12 [5] A function $f: (X, \tau) \to (Y, \sigma)$ is said to be set connected if $f^{-1}(V)$ is clopen in X for every $V \in CO(Y)$.

3. CHARACTERIZATIONS OF TOTALLY BT^{μ} - CONNECTED FUNCTIONS

Definition 3.1 A function $f:(X, \tau) \to (Y, \sigma)$ is called supra set connected function if $f^{1}(V)$ is supra clopen in (X, τ) for every $V \in$ supra clopen (Y, σ).

Definition 3.2 A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be supra totally continuous function if the inverse image of every supra open subset of Y is supra clopen in X.

Definition 3.3 A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be totally bT^{μ} - connected functions if the inverse image of every bT^{μ} - clopen subset of Y is supra clopen in X.

Example 3.4 Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {c}, {a, c}, {a, b}, {b, c}}, τ^c = {X, ϕ, {a, {b}, {c}, {a, c}, {a, b}} σ = {Y, ϕ, {b}, {b, c},{a, c}}. Supra clopen sets in X are {X, ϕ, {b}, {c},{a, c}, {a,b}}. Supra bT - closed sets in Y are {Y, ϕ, {a}, {b}, {c},{a, c}, {a, b}}. Supra bT-open sets in Y are {Y, ϕ, {b}, {c}, {a, c}, {a, b}, {b, c}}. Supra bT clopen sets in Y are {Y,ϕ, {b}, {c}, {a, c}, {a, b}}. The function f: $(X, \tau) \rightarrow (Y, \sigma)$ is defined by f (a) = a, f (b) = b and f(c) = c. Here f is totally bT^{μ} - connected function.

Theorem 3.5 Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a function, where X and Y are supra topological spaces.

Then following are equivalent

(i) f is totally bT^{μ} - connected functions.

(ii) For each $x \in X$ and each bT^{μ} -clopen set V in Y with $f(x) \in V$, there is a supra clopen set U in X such that $x \in U$ and $f(U) \subset V$.

Proof

(i) $\rightarrow$ (ii) Suppose f is totally bT^{μ} -connected and V be any bT^{μ} -clopen set in Y containing f(x) so that $x \in f^{-1}(V)$. Since f is totally bT^{μ} -connected, $f^{-1}(V)$ is supra clopen in X. Let $U = f^{-1}(V)$ then U is supra clopen set in X and $x \in U$. Also $f(U) = f(f^{1}(V)) \subset V$. This implies $f(U) \subset V$.

(ii) $\rightarrow$ (i) Let V be bT^{μ} - clopen in Y. Let $x \in f^{-1}(V)$ be any arbitrary point. This implies $f(X) \in V$. Therefore by (ii) there is a supra clopen set $f(A_x) \subset X$ containing x such that $f(A_x) \subset V$, which implies $A_x \subset f^{1}(V)$ is supra clopen neighborhood of x. Since x is arbitrary, it implies $f^{-1}(V)$ is supra clopen neighborhood of each of its points. Hence it is supra clopen set in X. Therefore f is totally bT^{μ} -connected.

Definition 3.6 A supra topological space X is said to be supra locally indiscrete if every supra open set of X is supra closed in X.

Theorem 3.7 For a function f: $(X, \tau) \rightarrow (Y, \sigma)$ is supra totally continuous and X is locally indiscrete then f is totally bT^{μ} - connected.

Proof Let V be supra clopen in Y. We know that every supra clopen is bT^{μ} -clopen.

Therefore V is bT^{μ} -clopen in Y. Since f is supra totally continuous and X is supra locally indiscrete, $f^{-1}(V)$ is supra open and supra closed in X. Hence $f^{-1}(V)$ is supra clopen in X. Therefore f is totally bT^{μ} - connected.

Theorem 3.8 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally bT^{μ} -connected injection on Y is clopen bT^{μ} -T_1 space, then X is supra clopen T_1 space.

Proof Let x and y be any two distinct points in X. Since f is injective, we have f(x) and f(y) belongs to Y such that $f(x) \neq f(y)$. Since Y clopen bT^{μ} -T_1, there exist a bT^{μ} - clopen sets U and V in Y such that $f(x) \in U$, $f(y) \notin U$, $f(y) \in V$ and $f(x) \notin V$. Therefore we have $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$, $y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are supra clopen subsets of X because f is totally bT^{μ} - connected. This shows that X is supra clopen T_1 space.

Theorem 3.9 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally bT^{μ} -connected injection on Y is clopen bT^{μ} -T_0 space, then X is clopen bT^{μ} -Hausdroff space.

Proof Let x and y be any two distinct points in X and f is injective. Then, $f(x) \neq f(y)$ in Y. Since Y clopen bT^{μ} -T_0, there exist a bT^{μ} -clopen sets containing f(x) but not f(y).

Then we have $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$. Since f is totally bT^{μ}-connected, $f^{-1}(U)$ is supra clopen in X. Also $x \in f^{-1}(U)$ and $y \in X - f^{-1}(U)$. This implies every pair of distinct points of X can be separated by disjoint supra clopen set in X. We know that every supra clopen set is bT^{μ}-clopen set. Therefore every pair of distinct points of X can be separated by disjoint bT^{μ}-clopen sets in X. Therefore X is clopen bT^{μ}-Hausdorff.

Theorem 3.10 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally bT^{μ}- connected injection on Y is clopen bT^{μ}-T_0 space, then X is supra ultra Hausdroff space.

Proof Let a and b be any two distinct points in X and f is injective. Then, $f(a) \neq f(b)$ in Y. Since Y clopen bT^{μ}-T_0, there exist a bT^{μ}- clopen sets containing f (a) but not f (b).

Then we have $a \in f^{-1}(U)$ and $b \notin f^{-1}(U)$. Since f is totally bT^{μ}- connected, $f^{-1}(U)$ is supra clopen in X. Also $a \in f^{-1}(U)$ and $b \in X - f^{-1}(U)$. This implies every pair of distinct points of X can be separated by disjoint supra clopen set in X . Therefore every pair of distinct points of X can be separated by disjoint supra clopen sets in X. Therefore X is supra ultra Hausdorff.

Theorem 3.11 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally bT^{μ}- connected injection on Y is clopen bT^{μ}-T_2 space, then X is supra ultra Hausdroff space.

Proof Let $a, b \in X$, $a \neq b$. Since f is injective, $f(a) \neq f(b)$ in Y. Since Y clopen bT^{μ}-T_2, there exist U and V subset of bT^{μ}- clopen set of Y such that $f(a) \in U$ and $f(b) \in V$ and $U \cap V = \phi$. This implies $a \in f^{-1}(U)$ and $b \in f^{-1}(V)$. Since f is totally bT^{μ}- connected $f^{-1}(U)$ and $f^{-1}(V)$ are supra clopen set in X . Also $f^{1}(U) \cap f(V) = f^{-1}(U \cap V) = \phi$. Thus every two distinct points of Y can be separated by disjoint supra clopen set. Therefore X is supra ultra Hausdroff space.

Definition 3.12 [8] A supra topological space X is said to be co bT^{μ}- normal if for each pair of disjoint supra clopen set A and B of X, there exist two disjoint bT^{μ}- clopen set U and V such that $A \subset U$ and $B \subset V$.

Definition 3.13 A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called clopen bT^{μ}- map if the image f(A) is bT^{μ}-clopen in Y for each supra clopen set A in X.

Theorem 3.14 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally bT^{μ}-connected, clopen bT^{μ}- map injection and Y is co bT^{μ} - normal, then X is supra ultra normal.

Proof Let A and B are disjoint supra open subset of X. Since f is clopen bT^{μ}- map and injection, f(A) and f(B) are disjoint bT^{μ}- clopen subsets of Y. Since Y is co bT^{μ}- normal f (A) and f(B) are separated by disjoint bT^{μ}- clopen sets U and V. Therefore we obtain $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Since f is totally bT^{μ}- connected functions, $f^{1}(U)$ and $f^{1}(V)$ are supra clopen sets in X. Also $f^{1}(U) \cap f^{1}(V) = f^{-1}(U \cap V) = \phi$. Thus each pair of non empty disjoint supra open sets in X can be separated by disjoint supra clopen set in X. Therefore X is supra ultra normal.

Definition 3.15 [8] A supra topological space X is said to be co bT^{μ}- regular if for each bT^{μ}- clopen set F and each point $x \neq F$, there exist disjoint supra clopen sets U and V such that $F \subset U$ and $x \in V$.

Theorem 3.16 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally bT^{μ}- connected injection clopen bT^{μ}- map from a supra ultra regular space X onto a space Y, then Y is co bT^{μ}- regular.

Proof Let F be a bT^{μ}- clopen set in Y and $y \notin F$. Then $y = f(x)$, since F is totally bT^{μ}- connected, $f^{-1}(F)$ is supra clopen set in X. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is supra ultra regular space, there exist disjoint supra clopen U and V, such that $G \subset U$ and $x \in V$. We obtain that $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$ such that f(U) and f(V) are disjoint supra clopen sets. This shows that Y is co bT^{μ}- regular space.

4 CO BT^{μ} - COMPACT SPACES

Definition 4.1 A space X is said to be supra co compact space if every supra clopen cover of X has a finite subcover.

Definition 4.2 A space X is said to be co bT^{μ}-compact space if every bT^{μ} - clopen cover of X has a finite subcover.

Definition 4.3 A subset A of a space X is said to be co bT^{μ} - compact if the subspace A is co bT^{μ} - compact.

Theorem 4.4 A totally bT^{μ} - connected image of a supra co compact space is co bT^{μ}-compact.

Proof Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a totally bT^{μ}-connected map for a co bT^{μ} - compact space X onto a supra topological space Y. Let $\{A_i : i \in \wedge\}$ be an bT^{μ} - clopen cover of Y. Then $\{f^{-1} (A_i : i \in \wedge\}$ is a supra clopen cover of X. Since X is supra co compact, it has a finite subcover say $f^{-1} (A_1), f^{-1} (A_2), \ldots, f^{-1} (A_n)\}$. Since f is onto $\{A_1, A_2, \ldots, A_n\}$ is a cover of Y which is finite. Therefore Y is co bT^{μ} - compact.

Definition 4.5 A space X is said to be countably co bT^{μ} - compact if every countable bT^{μ} - clopen cover of X has a finite subcover.

Definition 4.6 A space X is said to be co bT^{μ}-Lindelof if every bT^{μ} - clopen cover of X has a countable subcover.

Theorem 4.7 Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a totally bT^{μ} - connected surjective function then the following statements hold

(i) if X is supra co Lindelof then Y is co bT^{μ} - Lindelof.

(ii) if X is countably supra co compact then Y is countably bT^{μ} - compact.

Proof

(i) Let $\{A_i : i \in I\}$ be an supra clopen cover of Y. We know that every supra clopen set is bT^{μ}-clopen set. Therefore $\{A_i : i \in I\}$ be an bT^{μ} - clopen of Y. Since f is totally bT^{μ} - connected functions, then $\{f^{-1}(A_i) : i \in I\}$ is a supra clopen cover of X. Therefore $\{f^{-1} (A_i) : i \in I\}$ is a bT^{μ} - clopen cover of X. Since X is supra co Lindelof, there exist a countable subset I_0 of I such that $X = \cup\{f^{-1} (A_i) : i \in I_0\}$. Thus $Y = \cup \{A_i : i \in I_0\}$ and hence Y is co bT^{μ} - Lindelof.

(ii) Let $\{A_i : i \in I\}$ be a countable supra clopen cover of (Y, σ). Since f is totally bT^{μ} - connected, $\{f^{-1} (A_i) : i \in I\}$ is a countable supra clopen cover of (X, τ). Therefore $f^{-1} (A_i) : i \in I\}$ is a countable bT^{μ} - clopen cover of (X, τ). Again, since (X, τ) is countably supra co compact, the countable supra clopen cover $\{ f^{-1} (A_i) : i \in I\}$ of (X, τ) has a finite subcover say $\{f^{-1} (A_i) : i = 1, 2,\ldots, n\}$. Therefore $X = \bigcup_{i=1}^{n} \{f^{-1}(A_i)\}$, which implies $f(X) = \bigcup_{i=1}^{n} \{(A_i)\}$ so that $Y = \bigcup_{i=1}^{n} \{(A_i)\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is countably co bT^{μ}-compact.

REFERENCES

[1] D. Andrijevic,On b- open sets, mat. Vesnik, 48(1996), no.1-2, 59-64.

[2] Arya S.P and Nour T.M 1990, characterizations of s-normal spaces, Indian Journal of Pure Applied Mathematics, 21(8), 717-719.

[3] R. Devi, S. Sampath kumar and M. Caldas, On supra a-open set and Sa-continuous functions, General Mathematics-16(2008),no.2,77-84.

[4] Erdal Ekici and Takashi Noiri, On a generalization of Normal, Almost normal and Mildly Normal spaces-I, Mathematica Moravica - 10(2006), 9-20.

[5] Ganes M. Pandya, C. Janaki and I. Arockiarani, p- set connected functions in topological spaces, International Journal of Contempory Mathematical Sciences, 5(2010), no. 36, 1789-1798.

[6] Jamal M. Mustafa, Supra b-compact and supra b-Lindelof spaces, Journal of Mathematical Applications, 36(2013), 79-83.

[7] K. Krishnaveni and M.Vigneshwaran,On bT^m- closed sets in supra topological spaces, International Journal of Mathematical Archive-4(2),2013, 1-6.

[8] Krishnaveni. K and Vigneshwaran.M, Supra bT^m- connected functions in supra topological Spaces (communicated).

[9] Krishnaveni.K and Vigneshwaran.M, Some forms of bT^m- normal spaces in supra topological Spaces, International Journal of Multidisciplinary and Current Research, Vol-4, 2016, 435-440.

[10] A.S. Mashhour, A.A. Allam, F.S. Mohamoud and F.H. Khedr, On supra topological spaces, Indian Journal Pure and Applied Mathematics, No. 4, 14 (1983), 502-510.

[11] O.R. Sayed and Takashi Noiri, On b-open sets and supra b-continuity on topological spaces, European Journal of Pure and Applied Mathematics, 3(2) (2010), 295-302.