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Analysis, Control, Synchronization and LabVIEW Implementation of a Seven-Term Novel Chaotic System

Sundarapandian Vaidyanathan* and Karthikeyan Rajagopal**

Abstract: First, this paper announces a seven-term novel 3-D chaotic system with a cubic nonlinearity and two quadratic nonlinearities. The phase portraits of the novel 3-D chaotic system are displayed and the mathematical properties are discussed. The proposed novel 3-D chaotic system has three equilibrium points, which are all unstable. We shall show that the equilibrium point at the origin is a saddle point, while the other two equilibrium points are saddle-foci. The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 3.20885$, $L_2 = 0$ and $L_3 = -23.63597$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel 3-D chaotic system is obtained as $L_1 = 3.20885$. Also, the Kaplan-Yorke dimension of the novel 3-D chaotic system is derived as $D_{KY} = 2.13576$. Since the sum of the Lyapunov exponents of the novel chaotic system is negative, it follows that the novel chaotic system is dissipative. Next, an adaptive controller is designed to globally stabilize the novel 3-D chaotic system with unknown parameters. Moreover, an adaptive controller is also designed to achieve global and exponential synchronization of the identical novel 3-D chaotic systems with unknown parameters. The main adaptive results for stabilization and synchronization are established using Lyapunov stability theory. MATLAB simulations are depicted to illustrate all the main results derived in this work. Finally, a circuit design of the novel 3-D chaotic system is implemented in LabVIEW to validate the theoretical chaotic model.

Keywords: Chaos, chaotic systems, dissipative systems, chaos control, chaos synchronization, circuit simulation, LabVIEW implementation.

1. INTRODUCTION

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1]. Chaos theory has applications in several areas in Science and Engineering.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler found a 3-D chaotic system [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

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Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [39-40], hyperchaotic Vaidyanathan-Azar system [41], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [42].

Chaos theory has several applications in a variety of fields such as oscillators [43-44], chemical reactors [45-58], biology [59-80], ecology [81-82], neural networks [83-84], robotics [85-86], memristors [87-89], fuzzy systems [90-91], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [92-93]. Some popular methods for chaos control are active control [94-98], adaptive control [99-100], sliding mode control [101-103], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [104-107], cryptosystems [108-109], encryption [110-111], *e*

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [112-113] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [114-132], adaptive control method [133-149], sampled-data feedback control method [150-151], time-delay feedback approach [152], backstepping method [153-164], sliding mode control method [165-173], etc.

In this paper, we announce a novel 3-D chaotic system with a cubic nonlinearity and two quadratic nonlinearities. We discuss the qualitative properties of the novel 3-D chaotic system and display the phase portraits of the novel 3-D chaotic system. The proposed novel chaotic system has three equilibrium points, which are all unstable.

The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 3.20885$, $L_2 = 0$ and $L_3 = -23.63597$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel 3-D chaotic system is obtained as $L_1 = 3.20885$. Also, the Kaplan-Yorke dimension of the novel 3-D chaotic system is derived as $D_{KY} = 2.13576$. Since the sum of the Lyapunov exponents of the novel chaotic system is negative, it follows that the novel chaotic system is dissipative.

Next, this paper derives an adaptive control law that stabilizes the novel 3-D chaotic system with unknown system parameters. This paper also derives an adaptive control law that achieves global chaos synchronization of identical 3-D chaotic systems with unknown parameters.

In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called*master* or *drive* system, and another chaotic system is called *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

This paper is organized as follows. In Section 2, we describe the seven-term novel 3-D chaotic system. In Section 3, we describe the qualitative properties of the novel 3-D chaotic system. In Section 4, we detail the adaptive control design for the global chaos stabilization of the novel 3-D chaotic system with unknown parameters. In Section 5, we detail the adaptive control design for the global and exponential synchronization of the identical novel 3-D chaotic systems. In Section 6, we give the circuit implementation of the novel chaotic system in LabVIEW, which validates the theoretical chaotic model. In Section 7, we give a summary of the main results derived in this work.

2. A SEVEN-TERM NOVEL 3-D CHAOTIC SYSTEM

In this section, we describe aneight-term novel 3-D chaotic system with a cubic nonlinearity and two quadratic nonlinearities, which is modeled by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + p x_2 x_3 \\ \dot{x}_2 = b x_2 - x_1 x_3^2 \\ \dot{x}_3 = -c x_3 + x_1 x_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b, c, p are constant, positive, parameters of the system.

The system (1) is *chaotic* when we take the parameter values as

$$a = 30, \quad b = 14, \quad c = 4.5, \quad p = 14 \tag{2}$$

For numerical simulations, we take the initial conditions of the state as

$$x_1(0) = 1.2, \quad x_2(0) = 0.8, \quad x_3(0) = 1.2 \tag{3}$$

The Lyapunov exponents of the 3-D chaotic system (1) for the parameter values (2) and the initial conditions (3) are numerically calculated as

$$L_1 = 3.20855, \quad L_2 = 0, \quad L_3 = -23.63597 \tag{4}$$

We note that the sum of the Lyapunov exponents of the chaotic system (1) is negative. Thus, the novel 3-D chaotic system (1) is dissipative.

The Kaplan-Yorke dimension of the 3-D novel chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.13576 \tag{5}$$

Figure 1 shows the 3-D phase portrait of the novel 3-D chaotic system (1). Figures 2-4 show the 2-D projection of the novel chaotic system (1) on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes, respectively.

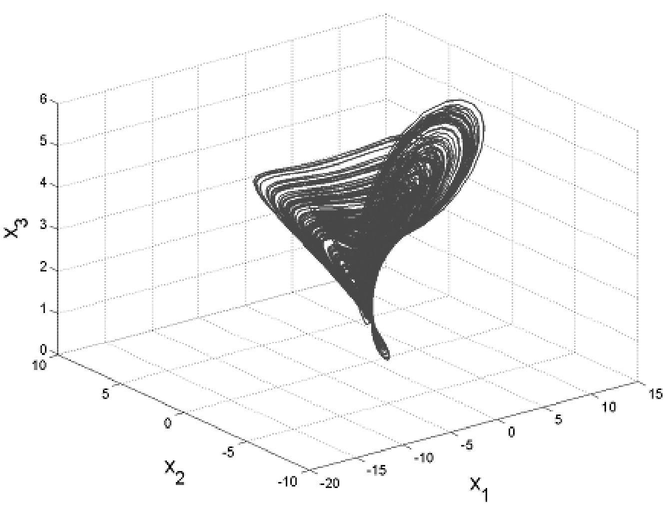


Figure 1: Phase portrait of the novel 3-D chaoticsystem

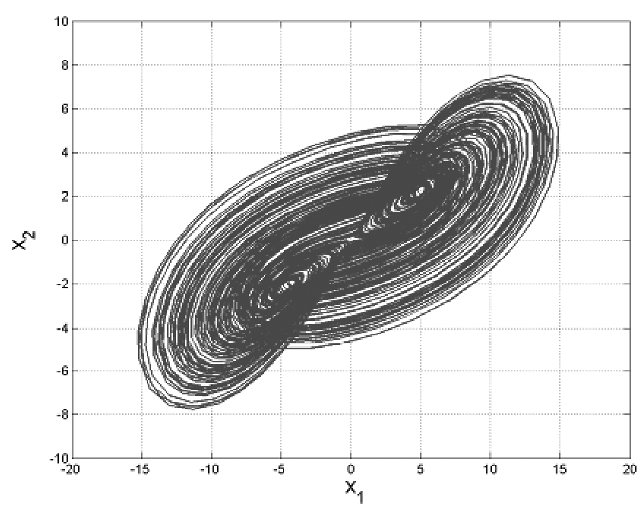


Figure 2: 2-D projection of the novel chaotic system on the (x_1, x_2) plane

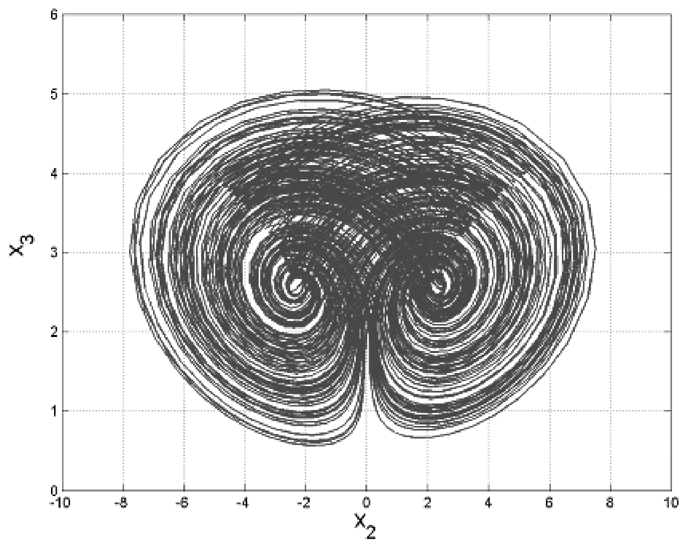


Figure 3: 2-D projection of the novel chaotic system on the (x_2, x_3) plane

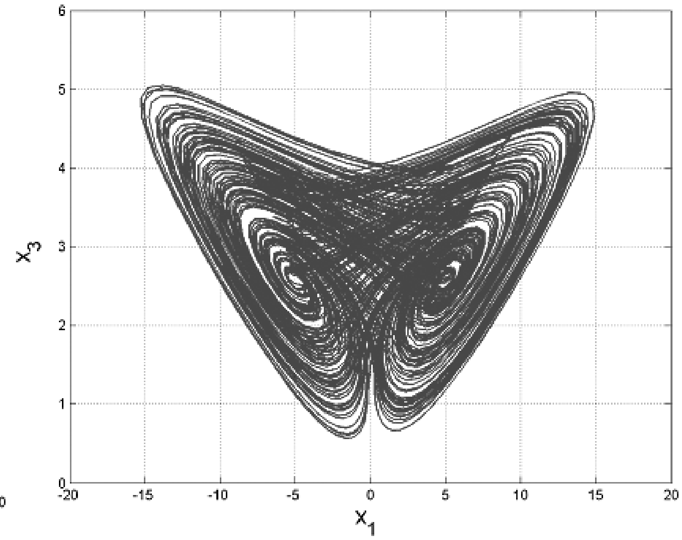


Figure 4: 2-D projection of the novel chaotic system on the (x_1, x_3) plane

3. PROPERTIES OF THE NOVEL3-D CHAOTIC SYSTEM

In this section, we discuss the qualitative properties of the novel 3-D chaotic system (1) introduced in Section 2. We suppose that the parameter values of the system (1) are as in the chaotic case (2), i.e. $a = 30$, $b = 14$, $c = 4.5$ and $p = 14$.

3.1. Dissipativity

In vector notation, we may express the system (1) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{6}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) + p x_2 x_3 \\ f_2(x_1, x_2, x_3) = b x_2 - x_1 x_3^2 \\ f_3(x_1, x_2, x_3) = -c x_3 + x_1 x_2 \end{cases} \tag{7}$$

Let Ω be any region in R^3 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector field f Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\dot{V} = \int\limits_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3 \tag{8}$$

The divergence of the novel chaotic system (1) is easily found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a + b - c = -\mu \tag{9}$$

where $\mu = a - b + c = 20.5 > 0$.

Substituting (9) into (8), we obtain the first order ODE

$$\dot{V} = -\mu V \tag{10}$$

Integrating (10), we obtain the unique solution as

$$V(t) = \exp(-\mu t)\; V(0) \text{ for all } t \geq 0 \tag{11}$$

Since $\mu > 0$, it follows that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the 3-D novel chaotic system (1) is dissipative. Thus, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2. Symmetry

It is easy to see that the system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) = (-x_1, -x_2, -x_3) \tag{12}$$

Thus, the system (1) exhibits *point reflection symmetry* about the origin in R^3.

3.3. Equilibrium Points

The equilibrium points of the system (1) are obtained by solving the system of equations

$$\begin{cases} a(x_2 - x_1) + p x_2 x_3 = 0 \\ b x_2 - x_1 x_3^2 = 0 \\ -c x_3 + x_1 x_2 = 0 \end{cases} \tag{13}$$

Solving the system (13) with the values of the parameters as given in (2), we obtain three equilibrium points

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\; E_1 = \begin{bmatrix} 4.9872 \\ 2.2855 \\ 2.5330 \end{bmatrix},\; E_2 = \begin{bmatrix} -4.9872 \\ -2.2855 \\ 2.5330 \end{bmatrix} \tag{14}$$

The Jacobian of the system (1) at any point $x \in R^3$ is given by

$$J(x) = \begin{bmatrix} -a & a + x_3 & x_2 \\ -x_3^2 & b & -2x_1x_3 \\ x_2 & x_1 & -c \end{bmatrix} = \begin{bmatrix} -30 & 30 + x_3 & x_2 \\ -x_3^2 & 14 & -2x_1x_3 \\ x_2 & x_1 & -4.5 \end{bmatrix} \tag{15}$$

We find that

$$J_0 = J(E_0) = \begin{bmatrix} -30 & 30 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & -4.5 \end{bmatrix} \tag{16}$$

Since J_0 is triangular, its eigenvalues are given by the diagonal entries, viz.

$$\lambda_1 = -30,\;\; \lambda_2 = 14,\;\; \lambda_3 = -4.5 \tag{17}$$

This shows that the equilibrium E_0 is a saddle-point, which is unstable.

Next, we find that

$$J_1 = J(E_1) = \begin{bmatrix} -30.0000 & 32.5330 & 2.2855 \\ -6.4161 & 14.0000 & -25.2652 \\ 2.2855 & 4.9872 & -4.5000 \end{bmatrix} \tag{18}$$

The eigenvalues of J_1 are numerically determined as

$$\lambda_1 = -27.5611, \quad \lambda_{2,3} = 3.5306 \pm 12.7930i \tag{19}$$

This shows that the equilibrium point E_1 is a saddle-focus, which is unstable.

We also find that

$$J_2 = J(E_2) = \begin{bmatrix} -30.0000 & 32.5330 & -2.2855 \\ -6.4161 & 14.0000 & 25.2652 \\ -2.2855 & -4.9872 & -4.5000 \end{bmatrix} \tag{20}$$

The eigenvalues of J_2 are numerically determined as

$$\lambda_1 = -27.5611, \quad \lambda_{2,3} = 3.5306 \pm 12.7930i \tag{21}$$

This shows that the equilibrium point E_2 is a saddle-focus, which is unstable.

3.4. Lyapunov Exponents and Kaplan-yorke Dimension

We take the parameter values of the novel system (1) as in the chaotic case (2), i.e. $a = 30$, $b = 14$, $c = 4.5$ and $p = 14$.

We choose the initial values of the state as $x_1(0) = 1.2$, $x_2(0) = 0.8$ and $x_3(0) = 1.2$.

Then we obtain the Lyapunov exponents of the system (1) as

$$L_1 = 3.20885, \quad L_2 = 0, \quad L_3 = -23.63597. \tag{22}$$

Figure 5 shows the Lyapunov exponents of the system (1) as determined by MATLAB.

We note that the sum of the Lyapunov exponents of the system (1) is negative. This shows that the novel chaotic system (1) is dissipative.

Also, the Maximal Lyapunov Exponent of the system (1) is $L_1 = 3.20885$.

The Kaplan-Yorke dimension of the novel 3-D chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2 + \frac{3.20885 + 0}{23.63597} = 2.13576 \tag{23}$$

which is fractional.

4. ADAPTIVE CONTROL DESIGN FOR THE STABILIZATION OF THE NOVEL CHAOTIC SYSTEM

In this section, we use adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the novel 3-D chaotic system with unknown parameters.

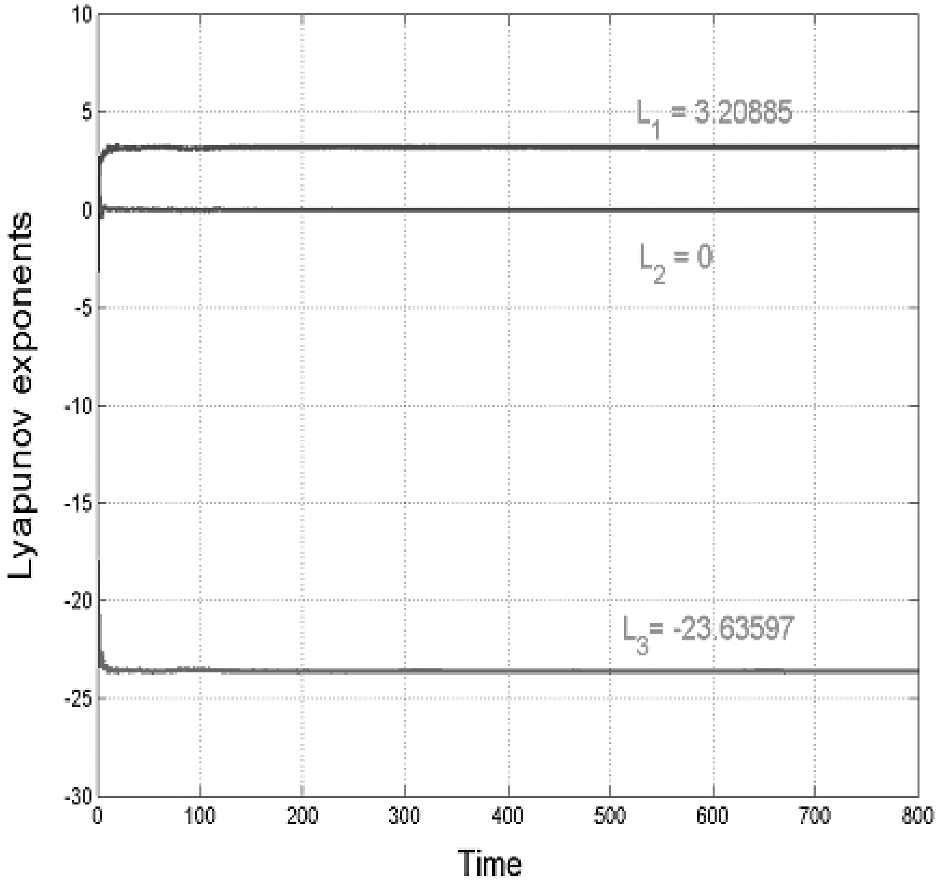


Figure 5: Lyapunov exponents of the novel chaotic system

Thus, we consider the novel 3-D chaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + p x_2 x_3 + u_1 \\ \dot{x}_2 = b x_2 - x_1 x_3^2 + u_2 \\ \dot{x}_3 = -c x_3 + x_1 x_2 + u_3 \end{cases} \tag{24}$$

In (24), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ and $\hat{p}(t)$ for the unknown parameters a, b, c and p, respectively.

We consider the adaptive control law defined by

$$\begin{cases} u_1 = -\hat{a}(t)(x_2 - x_1) - \hat{p}(t) x_2 x_3 - k_1 x_1 \\ u_2 = -\hat{b}(t) x_2 + x_1 x_3^2 - k_2 x_2 \\ u_3 = \hat{c}(t) x_3 - x_1 x_2 - k_3 x_3 \end{cases} \tag{25}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (25) into (24), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = [a - \hat{a}(t)](x_2 - x_1) + [p - \hat{p}(t)] x_2 x_3 - k_1 x_1 \\ \dot{x}_2 = [b - \hat{b}(t)] x_2 - k_2 x_2 \\ \dot{x}_3 = -[c - \hat{c}(t)] x_3 - k_3 x_3 \end{cases} \tag{26}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \end{cases} \tag{27}$$

Using (27), we can simplify the plant dynamics (26) as

$$\begin{cases} \dot{x}_1 = e_a(x_2 - x_1) + e_p x_2 x_3 - k_1 x_1 \\ \dot{x}_2 = e_b x_2 - k_2 x_2 \\ \dot{x}_3 = -e_c x_3 - k_3 x_3 \end{cases} \tag{28}$$

Differentiating (27) with respect to we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \end{cases} \tag{29}$$

We use adaptive control theory to find an update law for the parameter estimates.

We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{x}, e_a, e_b, e_c, e_p) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right) \tag{30}$$

Clearly, V is a positive definite function on R^7.

Differentiating V along the trajectories of (28) and (29), we obtain

$$\begin{aligned} \dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_2^2 - \dot{\hat{b}}\right] \\ + e_c\left[-x_3^2 - \dot{\hat{c}}\right] + e_p\left[x_1 x_2 x_3 - \dot{\hat{p}}\right] \end{aligned} \tag{31}$$

In view of (31), we take the parameter update law as follows:

$$\begin{cases} \dot{\hat{a}} = x_1(x_2 - x_1) \\ \dot{\hat{b}} = x_2^2 \\ \dot{\hat{c}} = -x_3^2 \\ \dot{\hat{p}} = x_1 x_2 x_3 \end{cases} \tag{32}$$

Theorem 1. The novel 3-D chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions $x(0) \in R^3$ by the adaptive control law (25) and the parameter update law (32), where k_1, k_2, k_3 are positive gain constants.

Proof. We prove this result by using Lyapunov stability theory [174].

We consider the quadratic Lyapunov function defined by (30), which is positive definite on R^7.

By substituting the parameter update law (32) into (31), we obtain the time derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{33}$$

From (33), it is clear that $\dot{V}$ is a negative semi-definite function on R^7.

Thus, we conclude that the state vector $x(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\begin{bmatrix} \boldsymbol{x}(t) & e_a(t) & e_b(t) & e_c(t) & e_p(t) \end{bmatrix}^T \in L_\infty$$

We define $k = \min\{k_1, k_2, k_3\}$. Thus, it follows from (33) that

$$\dot{V} \le -k \left\| \boldsymbol{x}(t) \right\|^2 \tag{34}$$

Thus, we have

$$k \left\| \boldsymbol{x}(t) \right\|^2 \le -\dot{V} \tag{35}$$

Integrating the inequality (35) from 0 to t, we get

$$k \int_0^t \left\| \boldsymbol{x}(\tau) \right\|^2 d\tau \le V(0) - V(t) \tag{36}$$

From (36), it follows that $x \in L_2$. Using (28), we can conclude that $\dot{x} \in L_\infty$.

Using Barbalat's lemma [174], we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (24) and (32), when the adaptive control law (25) is applied.

The parameter values of the novel chaotic system (24) are taken as in the chaotic case (2), i.e.

$$a = 30, \quad b = 14, \quad c = 4.5, \quad p = 14 \tag{37}$$

We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

Furthermore, as initial conditions of the novel chaotic system (24), we take

$$x_1(0) = -5.4, \quad x_2(0) = 12.7, \quad x_3(0) = -3.9 \tag{38}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 5.8, \quad \hat{b}(0) = 2.4, \quad \hat{c}(0) = 1.3, \quad \hat{p}(0) = 12.3 \tag{39}$$

Figure 6 shows the exponential convergence of the controlled state trajectories of the 3-D novel chaotic system (24).

5. ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL NOVEL CHAOTIC SYSTEMS

In this section, we use adaptive control method to derive an adaptive feedback control law for globally synchronizing identical 3-D novel chaotic systems with unknown parameters.

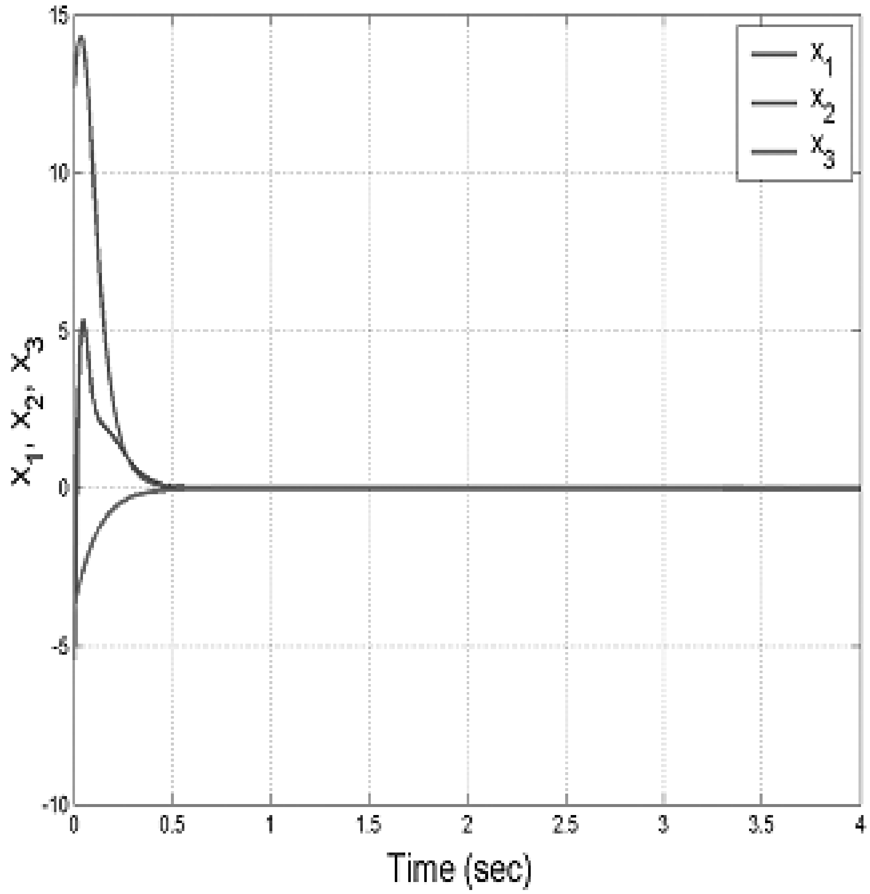


Figure 6: Time-history of the controlled state trajectories of the novel chaotic system

As the master system, we consider the novel chaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + p x_2 x_3 \\ \dot{x}_2 = b x_2 - x_1 x_3^2 \\ \dot{x}_3 = -c x_3 + x_1 x_2 \end{cases} \tag{40}$$

where x_1, x_2, x_3 are the states and a, b, c, p are unknown system parameters.

As the slave system, we consider the controlled novel chaotic system given by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + p y_2 y_3 + u_1 \\ \dot{y}_2 = b y_2 - y_1 y_3^2 + u_2 \\ \dot{y}_3 = -c y_3 + y_1 y_2 + u_3 \end{cases} \tag{41}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ for the unknown system parameters a, b, c, p, respectively.

The synchronization error between the novel chaotic systems (40) and (41) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{42}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + p(y_2 y_3 - x_2 x_3) + u_1 \\ \dot{e}_2 = be_2 - y_1 y_3^2 + x_1 x_3^2 + u_2 \\ \dot{e}_3 = -ce_3 + y_1 y_2 - x_1 x_2 + u_3 \end{cases} \tag{43}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) - \hat{p}(t)(y_2 y_3 - x_2 x_3) - k_1 e_1 \\ u_2 = -\hat{b}(t)e_2 + y_1 y_3^2 - x_1 x_3^2 - k_2 e_2 \\ u_3 = \hat{c}(t)e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{cases} \tag{44}$$

where k_1, k_2, k_3 are positive constants and $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{p}(t)$ are estimates of the unknown parameters a, b, c, p, respectively.

Substituting (44) into (43), we can simplify the error dynamics (43) as

$$\begin{cases} \dot{e}_1 = [a - \hat{a}(t)](e_2 - e_1) + [p - \hat{p}(t)](y_2 y_3 - x_2 x_3) - k_1 e_1 \\ \dot{e}_2 = [b - \hat{b}(t)]e_2 - k_2 e_2 \\ \dot{e}_3 = -[c - \hat{c}(t)]e_3 - k_3 e_3 \end{cases} \tag{45}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \\ e_c = c - \hat{c}(t) \\ e_p = p - \hat{p}(t) \end{cases} \tag{46}$$

Substituting (46) into (45), the error dynamics is simplified as

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) + e_p(y_2 y_3 - x_2 x_3) - k_1 e_1 \\ \dot{e}_2 = e_b e_2 - k_2 e_2 \\ \dot{e}_3 = -e_c e_3 - k_3 e_3 \end{cases} \tag{47}$$

Differentiating (43) with respect to t, we obtain

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \\ \dot{e}_c = -\dot{\hat{c}}(t) \\ \dot{e}_p = -\dot{\hat{p}}(t) \end{cases} \tag{48}$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b, e_c, e_p) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right) \tag{49}$$

Differentiating V along the trajectories of (47) and (48), we obtain

$$\begin{aligned}\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_b\left[e_2^2 - \dot{\hat{b}}\right] \\ + e_c\left[-e_3^2 - \dot{\hat{c}}\right] + e_p\left[e_1(y_2y_3 - x_2x_3) - \dot{\hat{p}}\right]\end{aligned} \tag{50}$$

In view of (50), we take the parameter update law as follows.

$$\begin{cases}\dot{\hat{a}} = e_1(e_2 - e_1) \\ \dot{\hat{b}} = e_2^2 \\ \dot{\hat{c}} = -e_3^2 \\ \dot{\hat{p}} = e_1(y_2y_3 - x_2x_3)\end{cases} \tag{51}$$

Next, we state and prove the main result of this section.

Theorem 2. The novel 3-D chaotic systems (40) and (41) with unknown system parameters are globally and exponentially synchronized for all initial conditions $x(0), y(0) \in R^3$ by the adaptive control law (44) and the parameter update law (51), where k_1, k_2, k_3 are positive constants.

Proof. We prove this result by applying Lyapunov stability theory [174].

We consider the quadratic Lyapunov function defined by (49), which is positive definite on R^7.

By substituting the parameter update law (51) into (50), we obtain the time-derivative of V as

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \tag{52}$$

From (52), it is clear that $\dot{V}$ is a negative semi-definite function on R^7.

Thus, we can conclude that the synchronization error vector $e(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\left[\boldsymbol{e}(t) \quad e_a(t) \quad e_b(t) \quad e_c(t) \quad e_p(t)\right]^T \in L_\infty \tag{53}$$

We define $k = \min\{k_1, k_2, k_3\}$. Then it follows from (52) that

$$\dot{V} \le -k\|\boldsymbol{e}(t)\|^2 \tag{54}$$

Thus, we have

$$k\|\boldsymbol{e}(t)\|^2 \le -\dot{V} \tag{55}$$

Integrating the inequality (55) from 0 to t, we get

$$\int_0^t k\|\boldsymbol{e}(\tau)\|^2\, d\tau \le V(0) - V(t) \tag{56}$$

From (56), it follows that $e \in L_2$. Using (47), we can conclude that $\dot{e} \in L_\infty$.

Using Barbalat's lemma [174], we conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

For numerical simulations, we take the parameter values of the chaotic systems (40) and (41) as in the chaotic case (2), *i.e.*

$$a = 30, \quad b = 14, \quad c = 4.5, \quad p = 14 \tag{57}$$

We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

As initial conditions of the master system (40), we take

$$x_1(0) = 11.3, \quad x_2(0) = 8.2, \quad x_3(0) = -7.9 \tag{58}$$

As initial conditions of the slave system (41), we take

$$y_1(0) = 14.7, \quad y_2(0) = -7.2, \; y_3(0) = 10.4 \tag{59}$$

As initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 5.2, \quad \hat{b}(0) = 1.8, \quad \hat{c}(0) = 4.7, \quad \hat{p}(0) = 1.3 \tag{60}$$

Figures 7-9 depict the synchronization of the novel chaotic systems (40) and (41).

Figure 10 depicts the time-history of the complete synchronization errors e_1, e_2, e_3.

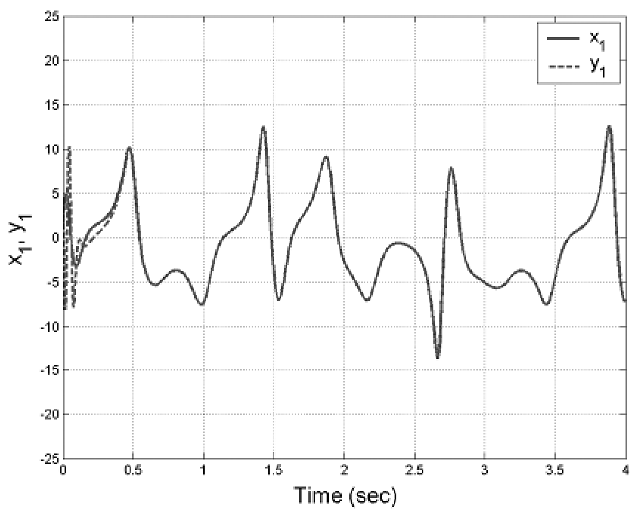


Figure 7: Synchronization of the states x_1 and y_1

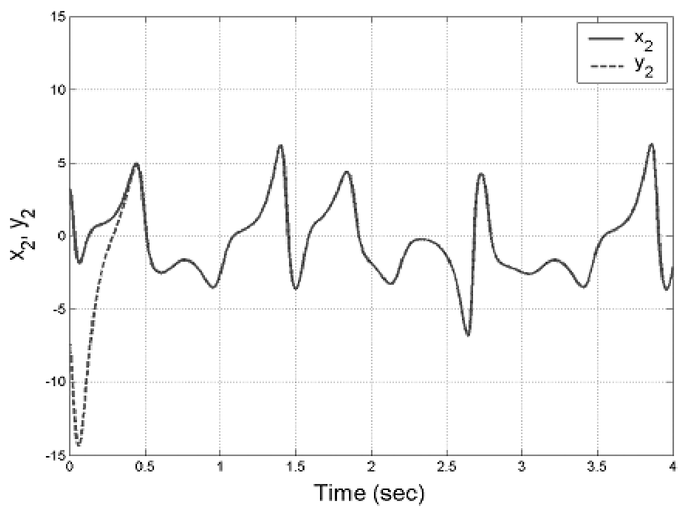


Figure 8: Synchronization of the states x_2 and y_2

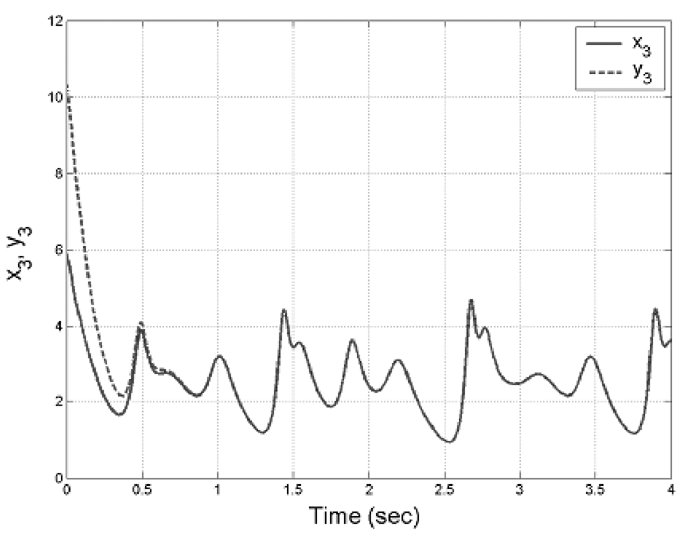


Figure 9: Synchronization of the states x_3 and y_3

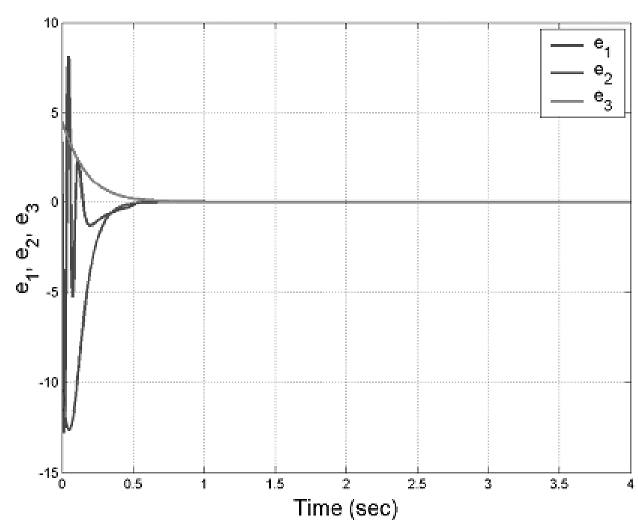


Figure 10: Time-history of the synchronization errors e_1, e_2, e_3

6. CIRCUIT SIMULATION AND LABVIEW IMPLEMENTATION

In this paper, the proposed Chaotic System is implemented in LabVIEW using the Control Design and Simulation Loop(CDS). Figure 11 shows the block diagram of the Chaotic System. The Simulation parameters are chosen to run the simulation loop with breakpoints. Figures 12 shows the time history of the states $X1$, $X2$, $X3$. Figure 13 shows the 2D phase portraits of states $X1X2$, $X2X3$, $X3X1$. The Adaptive controller is implemented in the CDS loop using the feedback-Summing methodology. Figure 14 shows the Block diagram of the Parameter update law. Figure 15 shows the designed adaptive controller.. The Master and the slave systems are identical chaotic systems with different initial conditions. Figure 16 shows the Time history of the synchronisation errors $e1$, $e2$, $e3$.

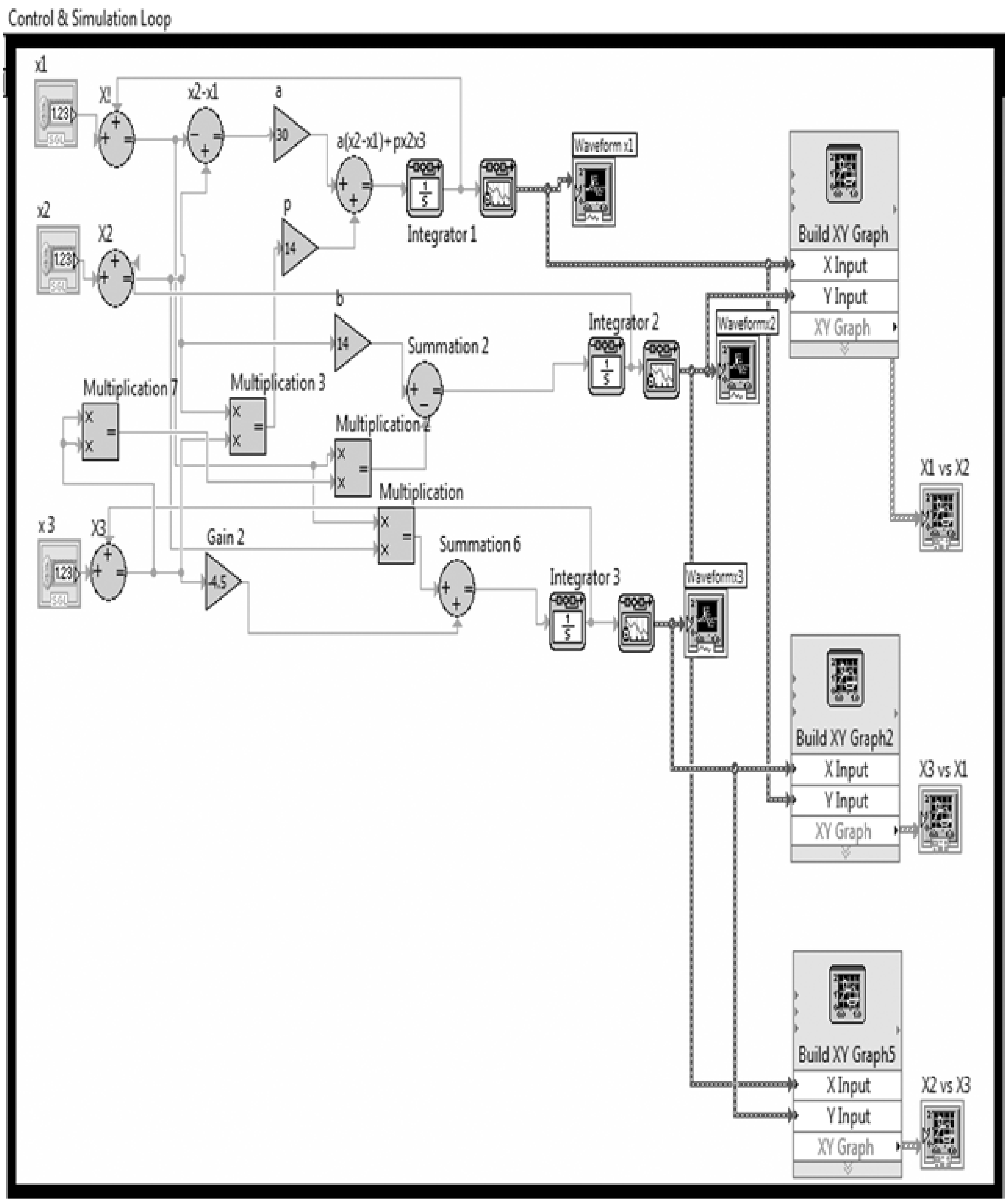


Figure 11: LabVIEW Blockdiagram of the Chaotic System

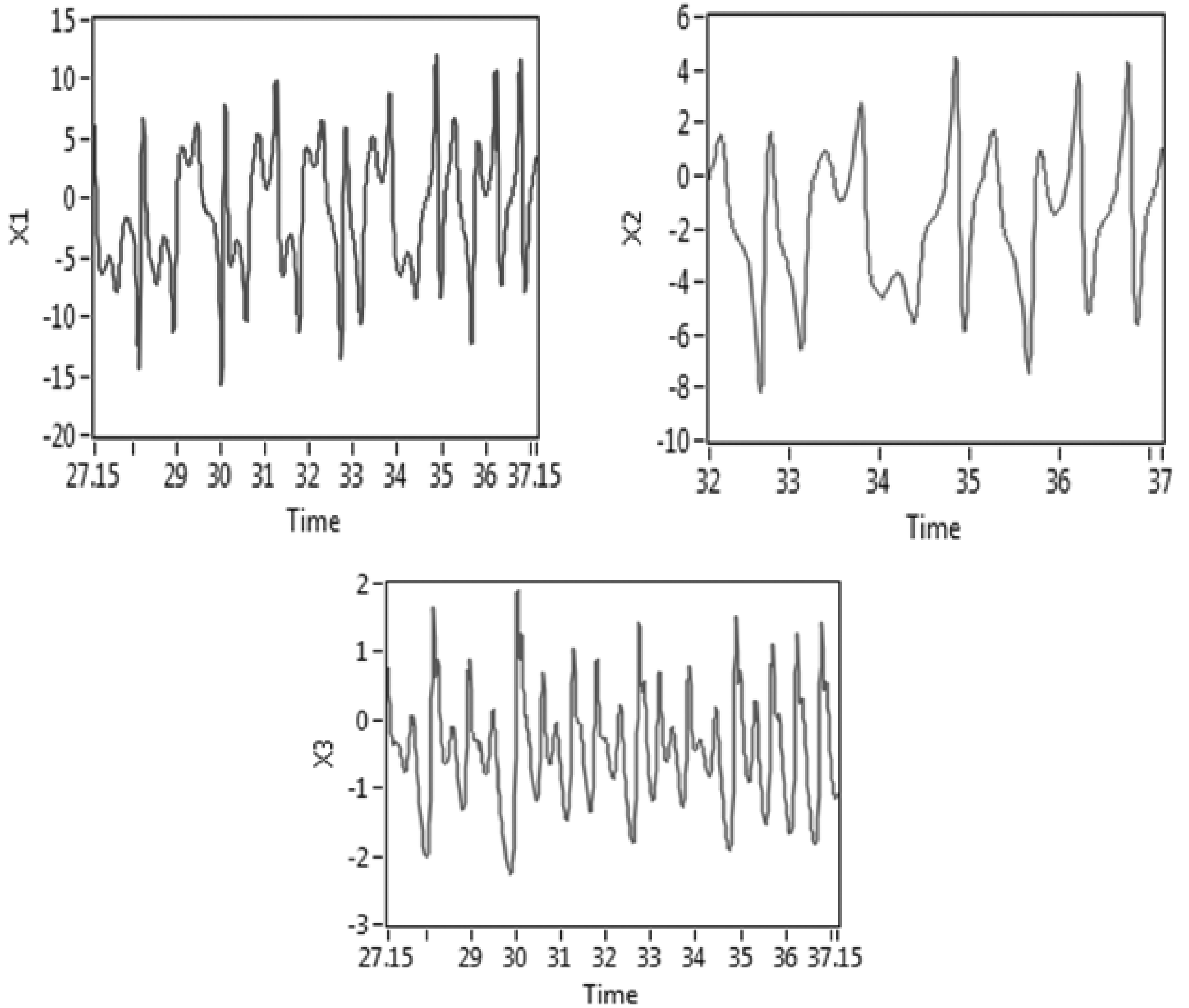
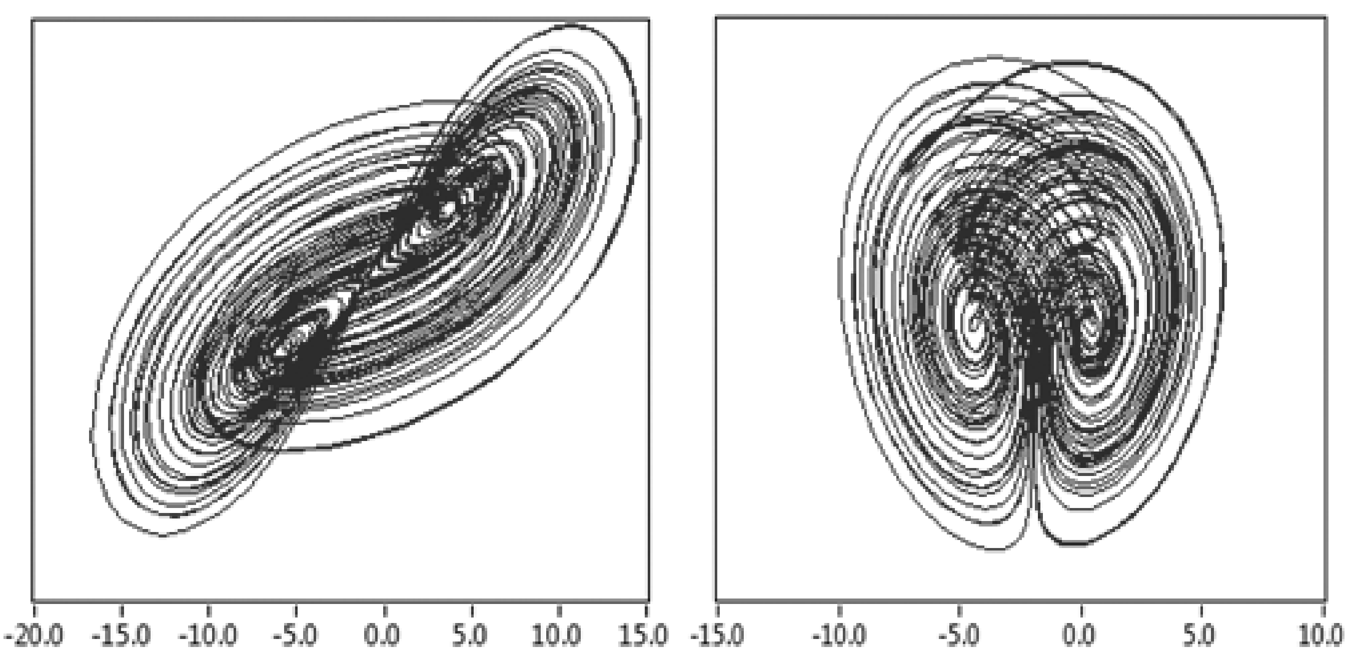


Figure 12: Time-history of the states



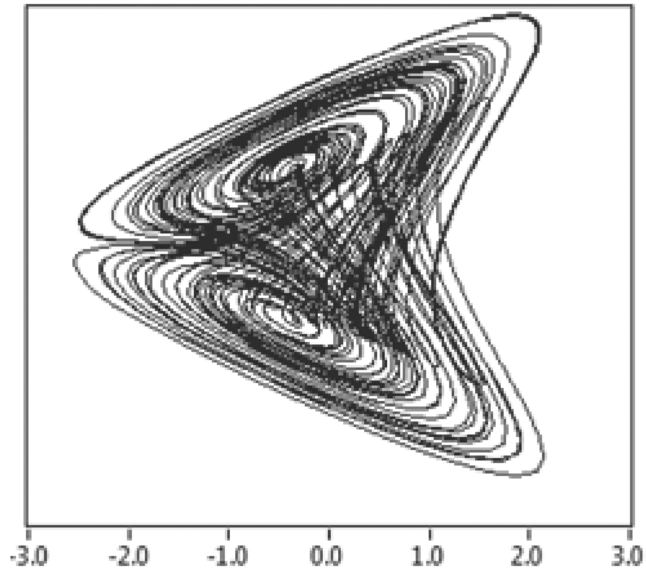


Figure 13: 2D Phase Portraits of the states X1X2, X2X3, X3X1.

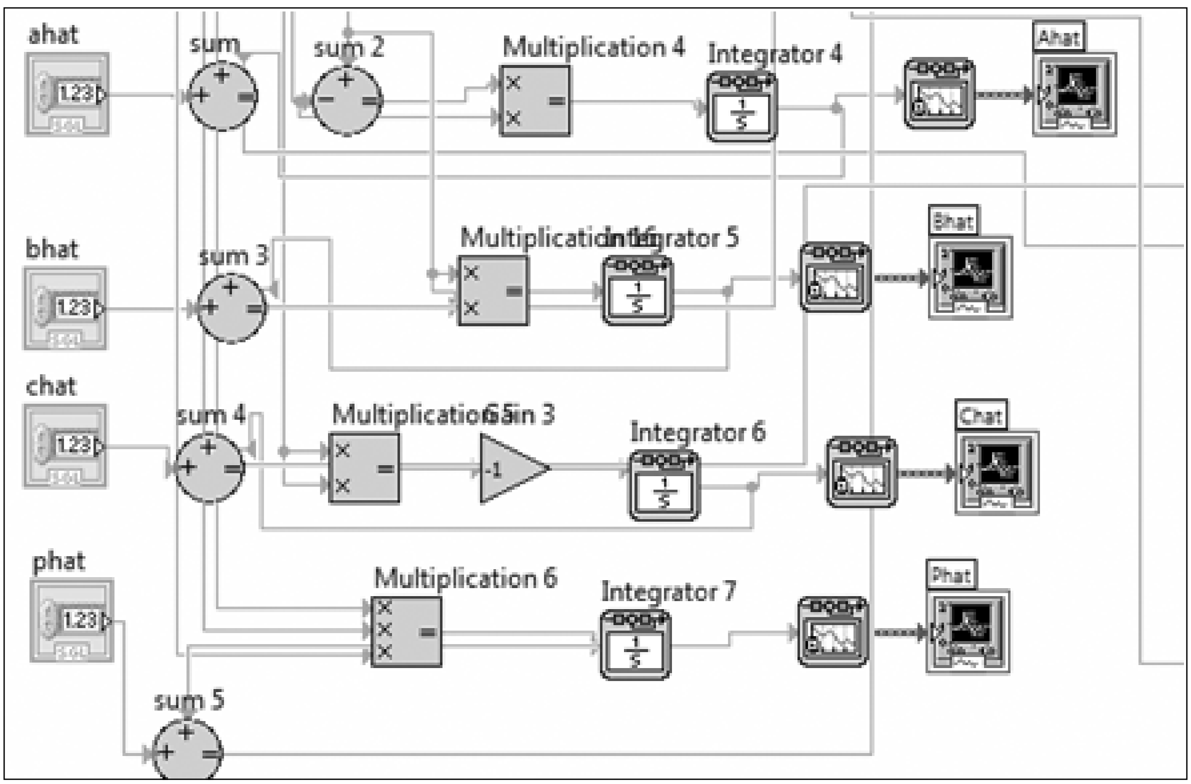


Figure 14: LabVIEW Block diagram of the Parameter Update Law

7. CONCLUSIONS

In this paper, we have proposed a seven-term novel 3-D chaotic system with a cubic nonlinearity and two quadratic nonlinearities. The qualitative properties of the novel chaotic system have been discussed. The proposed novel 3-D chaotic system has three equilibrium points, which are all unstable. We showed that the equilibrium point at the origin is a saddle point, while the other two equilibrium points are saddle-foci. The Lyapunov exponents of the novel 3-D chaotic system were obtained as and Also, the Kaplan-Yorke dimension of the novel 3-D chaotic system was derived as Next, an adaptive controller was designed to globally stabilize the novel 3-D chaotic system with unknown parameters. Moreover, an adaptive controller

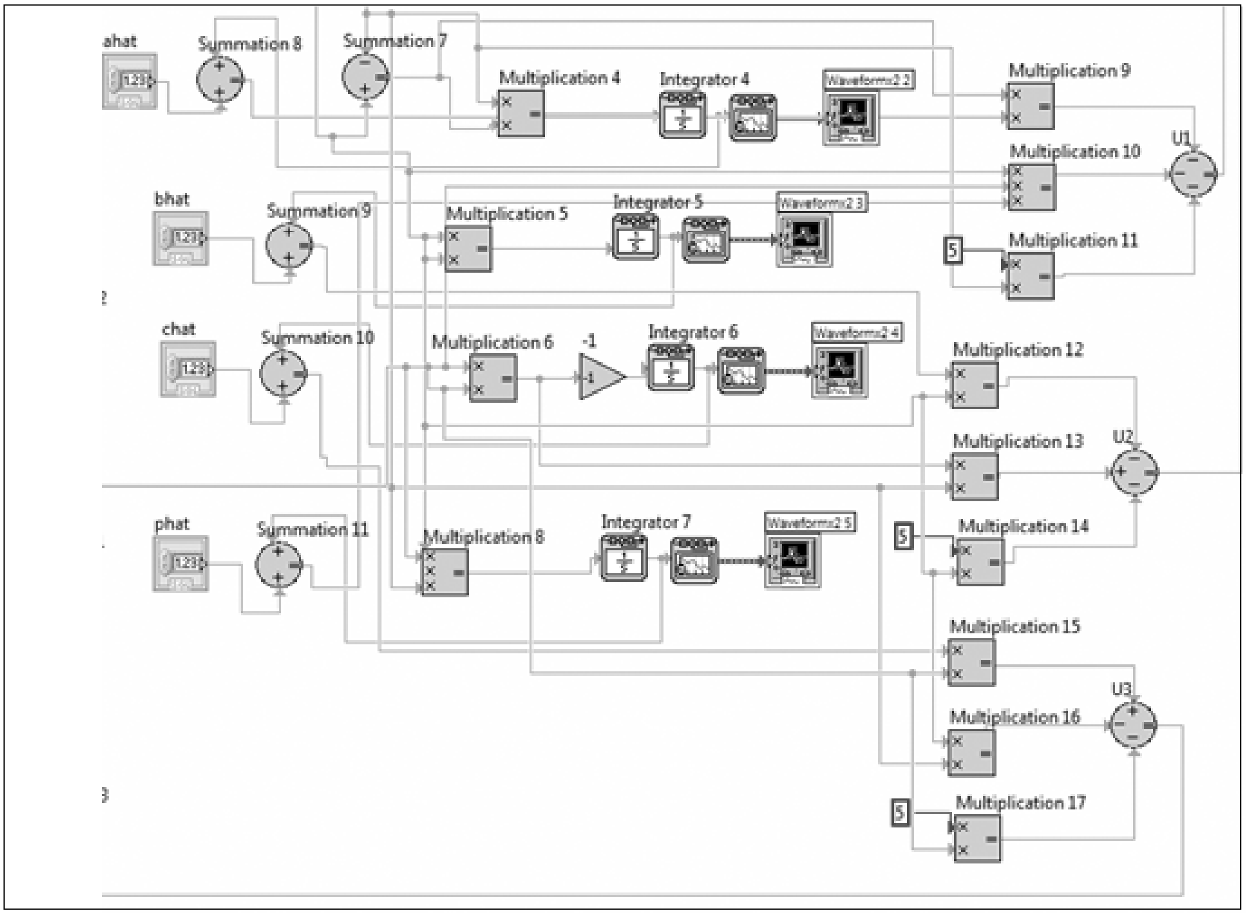


Figure 15: LabVIEW Block diagram of the Controller

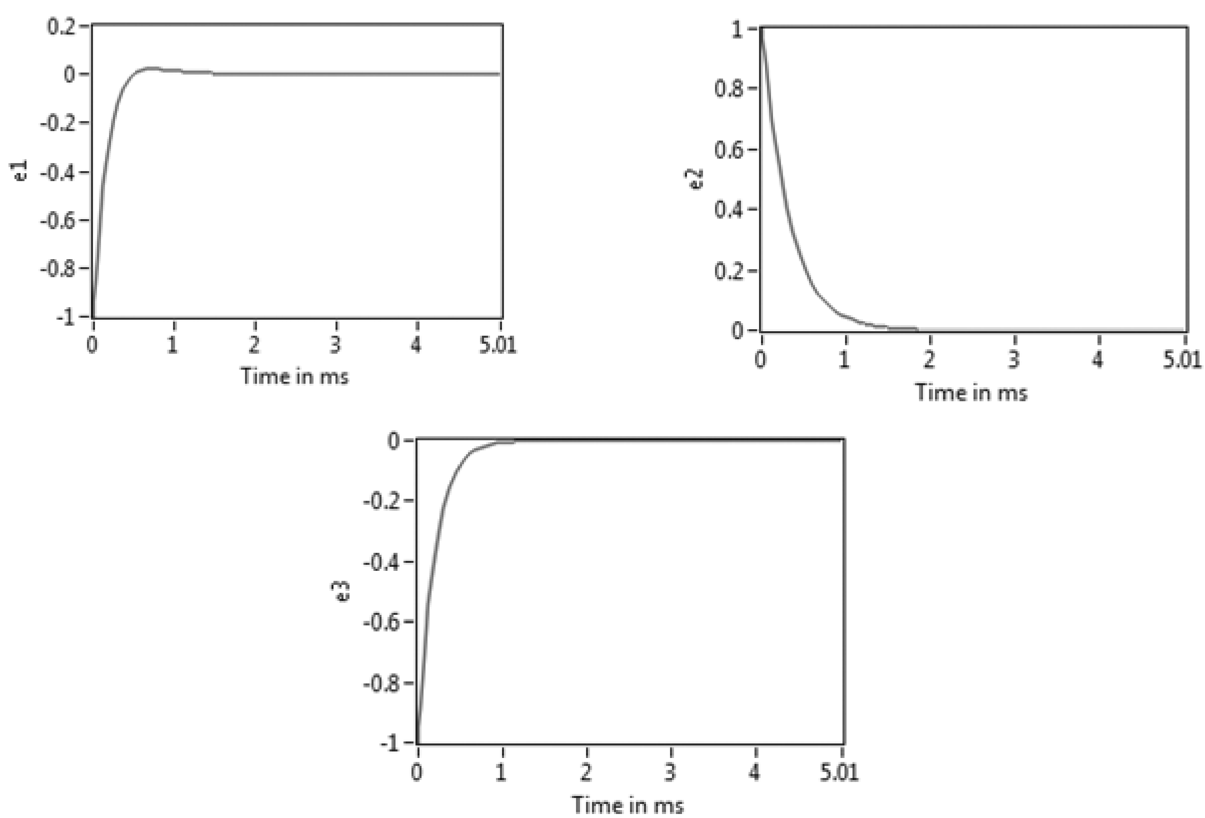


Figure 16: Time-History of the Synchronisation errors

was designed to achieve global and exponential synchronization of the identical novel 3-D chaotic systems with unknown parameters. The main adaptive results for stabilization and synchronization were established using Lyapunov stability theory. MATLAB simulations have been shown to illustrate all the main results derived in this work. Finally, a circuit design of the novel 3-D chaotic system has been implemented in LabVIEW to validate the theoretical chaotic model.

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