

Entropy Generation in Cu-Water Nanofluid in a Cavity with Chamfer in the Presence of Magnetic Field

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ABSTRACT

This paper deals with the investigation of the irreversibility in nanofluid flow convection in a cavity with chamfer. The nanofluid is submitted to a thermal gradient and a magnetic field applied in different directions. The equations of continuity, momentum balance and energy are solved by using the Comsol software. It was studied the effect of the Rayleigh number, and the Hartmann number and the magnetic field inclination angle, on the entropy generation.

Keywords: Entropy generation, nanofluid, Rayleigh number, Hartmann number, magnetic field, etc.

1. INTRODUCTION

Over the last few decades many research projects have dealt with using nanofluids in the presence of magnetic field. H. K. Yang and C.P.Yu [1] combined forced and free convection (MHD) channel flow in the entrance region. They found that an applied transverse magnetic field may reduce the entrance length of the velocity considerably but has little effect on the temperature development. At high Hartmann number (Ha), the velocity entrance length is inversely proportional to Ha [2]. Mahmoudi et al [2] investigated the effect of magnetic field on nanofluid flow in a cavity with a linear boundary condition analyzed with Lattice Boltzmann Method, according to the Rayleigh number, the Hartmann number, the volume fraction and the direction of the magnetic field. For various Rayleigh numbers and for all magnetic field direction, heat transfer and fluid flow decline with the increase in Hartmann number. The magnetic field direction controls the effect of nanoparticles in the fluid. H. Heidary et al [3] conducted a numerical study of the magnetic field's effect on nanofluid forced convection in a channel.

A parametric study of the effect of nanofluid volume fraction and magnetic strength on the enhancement of heat exchange between an isothermal duct and the core flow was carried out. They observed that the presence of a magnetic field and the addition of nanoparticles to a pure fluid can significantly enhance the heat exchange between the wall and the fluid. Abolbashari et al [4] used the Homotopy analysis method (HAM), to study the entropy analysis in an unsteady magneto hydrodynamic nanofluid regime adjacent to an accelerating stretching permeable surface with water as the base fluid and four different types of nanoparticles. HAM is successfully applied to solve the system of ordinary differential equations.

Many other works related to MHD effect on heat transfer and entropy generation of nanofluid in mixed, forced and natural convection flow are conducted by Mehrez et al. [5], Das and Jana [6], Hatami et al. [7], Rahman et al. [8] and Teamah and El-Maghlany [9]. Some recent control methods are discussed in [18-25].

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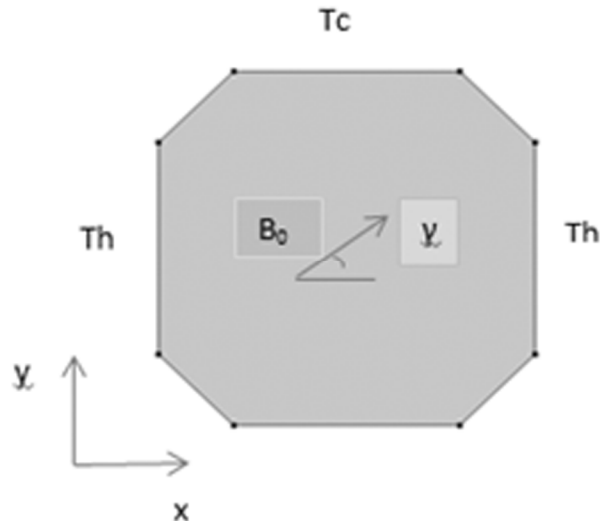


Figure 1: Geometric configuration of the problem

2. PROBLEM STATEMENT

The schematic of the system under consideration is presented in Fig. 1. The temperature T_h is uniformly imposed along the vertical walls. T_c along the top wall and the bottom surface as well as the four chamfers are assumed to be adiabatic. A magnetic field with uniform strength B_0 is applied in different angle inclination γ . Also the enclosure is filled with a water based nanofluid. It is assumed that the nanoparticles are in thermal equilibrium, the nanofluid is Newtonian and incompressible and the flow is laminar.

3. MATHEMATICAL FORMULATION

Hence and in two-dimensional Cartesian coordinate system, the dimensionless equations of continuity, momentum and energy are written in steady state as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + \frac{\sigma_{nf}}{\sigma_f} Pr_f Ha^2 (V \sin \gamma \cos \gamma - U \sin^2 \gamma) \tag{2}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} Ra_f Pr_f T + \frac{\sigma_{nf}}{\sigma_f} Pr_f Ha^2 (U \sin \gamma \cos \gamma - V \cos^2 \gamma) \tag{3}$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) \tag{4}$$

Here X, Y dimensionless coordinates; U, V dimensionless velocity; P dimensionless pressure. Pr, Ha and Ra denoted the Prandtl number, the Hartmann number and the Rayleigh number respectively.

$$Pr_f = \frac{\nu_f}{\alpha_f}; Ha = B_0 H \sqrt{\frac{\sigma_f}{\mu_f}}; Ra_f = \frac{g \beta_f (T_h' - T_c') H^3}{\nu_f \alpha_f}$$

The expressions of density, thermal expansion, specific heat coefficient, dynamic viscosity and electrical conductivity of the Nanofluid (Maxwell Model [10]) are given as follows [11]. In the equations below, ϕ is the nanoparticles volume fraction.

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s \quad (5)$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \quad (6)$$

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s \quad (7)$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1 - \phi)^{2.5}} \quad (8)$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} = 1 + \frac{3(\alpha - 1)\phi}{(2 + \alpha) - (\alpha - 1)\phi}; \alpha = \frac{\sigma_s}{\sigma_f} \quad (9)$$

There have been many reviews of nanofluid thermal conductivity (Lee et al. [12] and Das et al. [13]). The thermal conductivity model of Patel et al. [14] was used in this work. It can be given by:

$$k_{nf} = k_f \left(1 + \frac{k_s A_s}{k_f A_f} + C k_p Pe \frac{A_s}{k_f A_f} \right) \quad (10)$$

The parameter C is set equal to 25000. The parameters Pe and A_s/A_f are defined as:

$$Pe = \frac{u_s d_s}{\alpha_f}; \frac{A_s}{A_f} = \frac{d_f}{d_s} \frac{\phi}{1 - \phi} \quad (11)$$

In Eq. (11), d_f is the molecular size of water, which is taken 2\AA , d_s is the diameter of solid particles and k_b is the Boltzmann constant. The variable u_s , which depends on the temperature, is the Brownian motion velocity of particles and is given by H.E. Patel et al [14].

$$u_s = \frac{2k_b T}{\Pi \mu_f d_s^2} \quad (12)$$

The appropriate initial and boundary conditions of the problem are:

- At dimensionless time equal to zero, $T = 0$ and $U = V = 0$ in the whole cavity.
- Along the left and the right walls the dimensionless temperature is $T = 1$.
- Along the top wall, the dimensionless temperature is $T = 0$.

- Along the horizontal insulator walls: $\frac{\partial T}{\partial y} = 0$

- Along the vertical insulator walls and the chamfers: $\frac{\partial T}{\partial x} = 0$

- Along the isothermal, the chamfers and insulator walls $U = V = 0$.

4. ENTROPY GENERATION

According to Woods [15], the dimensionless local entropy generation can be expressed by:

$$S_{gen} = \frac{k_{nf}}{k_f} \left[\left(\frac{\partial T}{\partial X} \right)^2 + \left(\frac{\partial T}{\partial Y} \right)^2 \right] + \phi \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial Y} \right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right] + \frac{\sigma_{nf}}{\sigma_f} Ha^2 \phi (U \sin \gamma - V \cos \gamma)^2 \quad (13)$$

In the right side of Eq. (13), the first term represents the heat transfer irreversibility, the second is the viscous effect irreversibility and the third is the magnetic irreversibility. The distribution irreversibility ratio is given by:

$$\phi = \frac{\mu_{nf} T_0}{k_f} \left(\frac{\alpha_f}{H \Delta T} \right)^2 \quad (14)$$

5. NUMERICAL PROCEDURE AND VALIDATION

In the dimensionless form and taking into account the initial and the boundary conditions, the flow governing equations were solved using the finite volume method and COMSOL Multiphysics software. Results given by numerical calculation using licensed version of COMSOL software were validated with the works of Magherbi et al. [16] in terms of isentropic lines related to a pure fluid (air) and of Ozotop et al. 17 in terms of streamline and isotherms related to a nanofluid (Cu-water).

6. RESULTS AND DISCUSSION

In this study, the Prandtl number (Pr) is kept constant at Pr = 6.2 with solid volume fraction $\tilde{\Omega} = 4\%$ (water-Cu nanofluid). The numerical results for the streamlines and isothermal contours for various values of Rayleigh number Ra and Hartmann number Ha. In addition, results for thermal entropy, viscous entropy, magnetic and total entropy, at various conditions, are presented and discussed.

6.1. Effect of Hartmann and Rayleigh numbers on streamlines and isotherms

Fig. 2 represents the effect of Rayleigh number and Hartmann number on the streamlines and isothermal contours for, $\tilde{\Omega} = 0.04$ and Pr = 6.2.

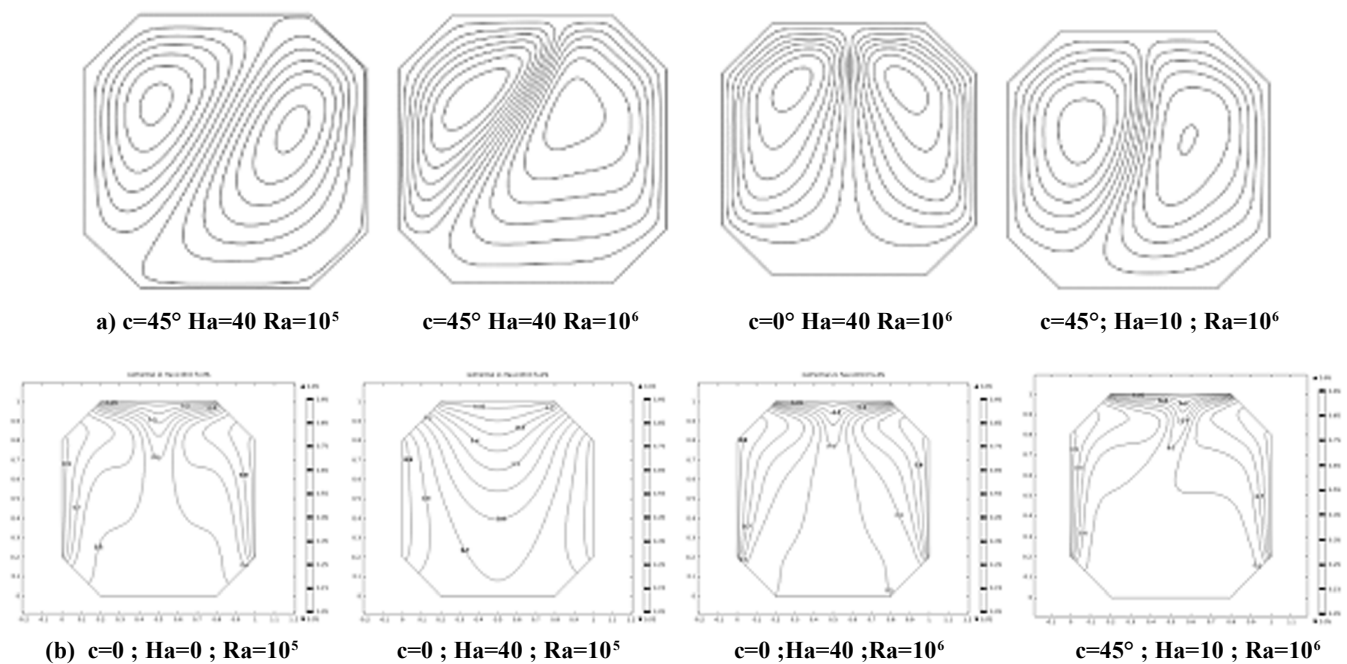


Figure 2: a) Streamlines and b) Isotherms

For all Rayleigh and Hartmann numbers, Fig.2a shows a pair of cells in rotation, one in clockwise direction the other in anticlockwise, are formed inside the cavity. Fig.2 shows that, the symmetry of streamlines and isotherms is broken since magnetic field inclination angle is not zero. The strength of these cells increases as the Rayleigh number increases and decreases as the Hartmann number increases and the streamlines are elongated in the direction of the magnetic field. Remark that the bottom cell is more elongated than the upper one. For all values of Rayleigh number, the application of the magnetic field tends to slow down the movement of the fluid in the cavity.

6.2. Effect of inclination angle on different cause of irreversibility

6.2.1. Magnetic irreversibility

The values of the magnetic entropy of different inclinations of the magnetic field are illustrated in figure 3

As seen in Fig.3, at fixed magnetic field inclination angle (γ), the irreversibility increases for relatively small Hartman number and reaches a maximum value at critical Hartman number (H_{ac}) then it decreases towards a minimum value at maximum value of Ha . The decrease of magnetic irreversibility, when Ha exceeds H_{ac} can be the results of the significant decrease of the velocity in the cavity due to the important

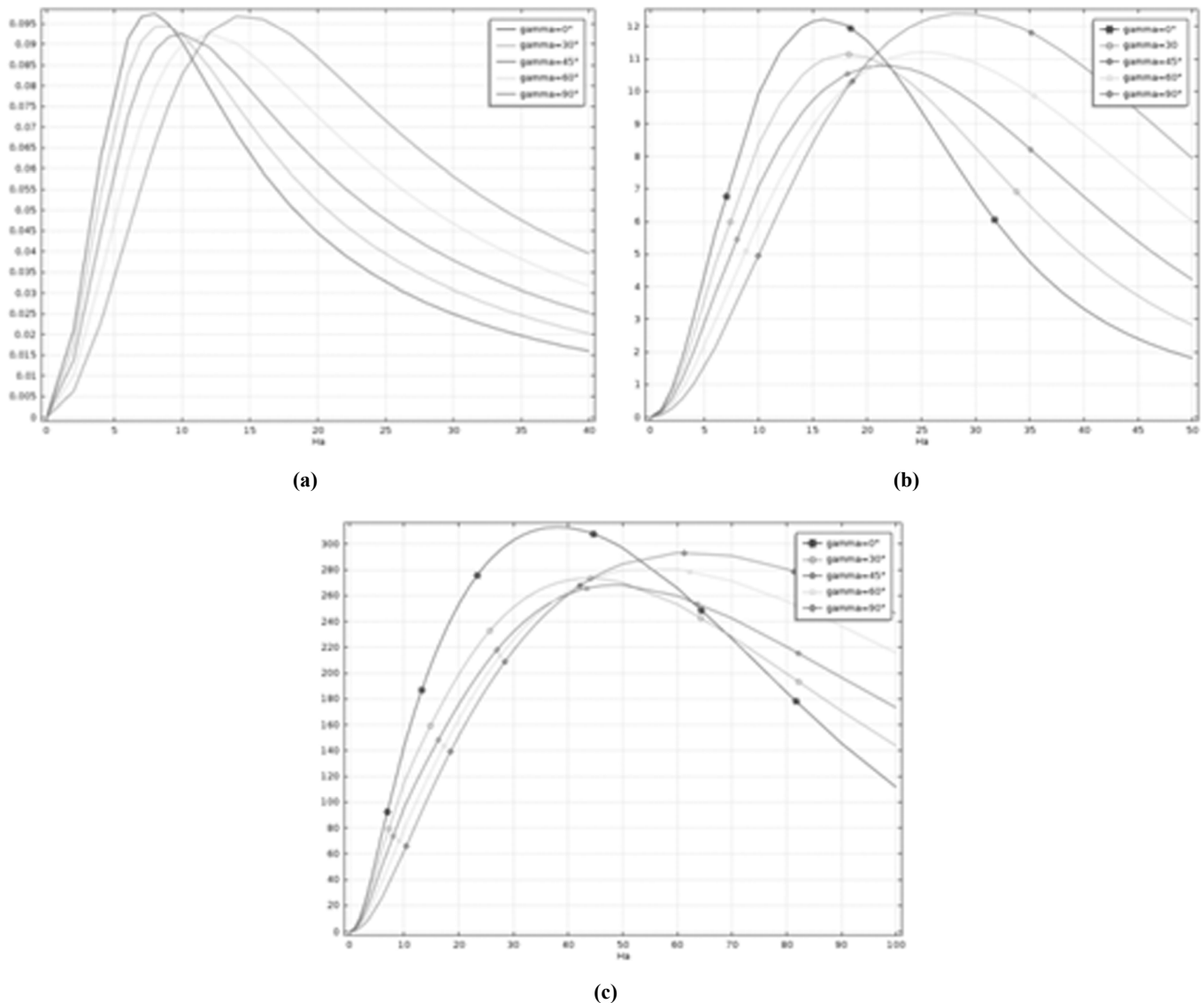


Figure 3: Variation of magnetic entropy as a function of Hartmann number for different angle inclination of magnetic field with $\phi = 0.04$ a) $Ra=10^4$; b) $Ra=10^5$; c) $Ra=10^6$

slowing effect induced by the Lorentz force at high Hartman number. Whereas, at small Ha value, although the slowdown effect of magnetic field exists, it remains insignificant. In this case the increase of the magnetic entropy generation is the consequence of the intrinsic effect of Hartman number on the magnetic entropy generation equation. It is important to note that the critical Hartman number for which entropy generation is maximum depends on the inclination angle (γ). One can see, at $Ra = 105$ for example, that the critical Hartman number increases from 16 to 29 when γ increases from zero to 90° . The critical Hartman number (Ha_c) represents the frontier between, first the case where Ha is preponderant via it's intrinsic effect on the magnetic entropy generation equation and secondly, the case where the fluid velocity becomes dominant through the extrinsic effect of Ha on the magnetic entropy generation through the momentum equation balance. Additionally, the minimum of entropy generation at maximum Ha is as important as γ is important.

6.2.2. Thermal and viscous irreversibilities

Figs. 4 and 5 show that, at fixed Ra and (γ), the thermal and viscous entropies generations decrease when Hartmann number increases. In fact, when Ha increases the convection in the cavity diminishes that leads

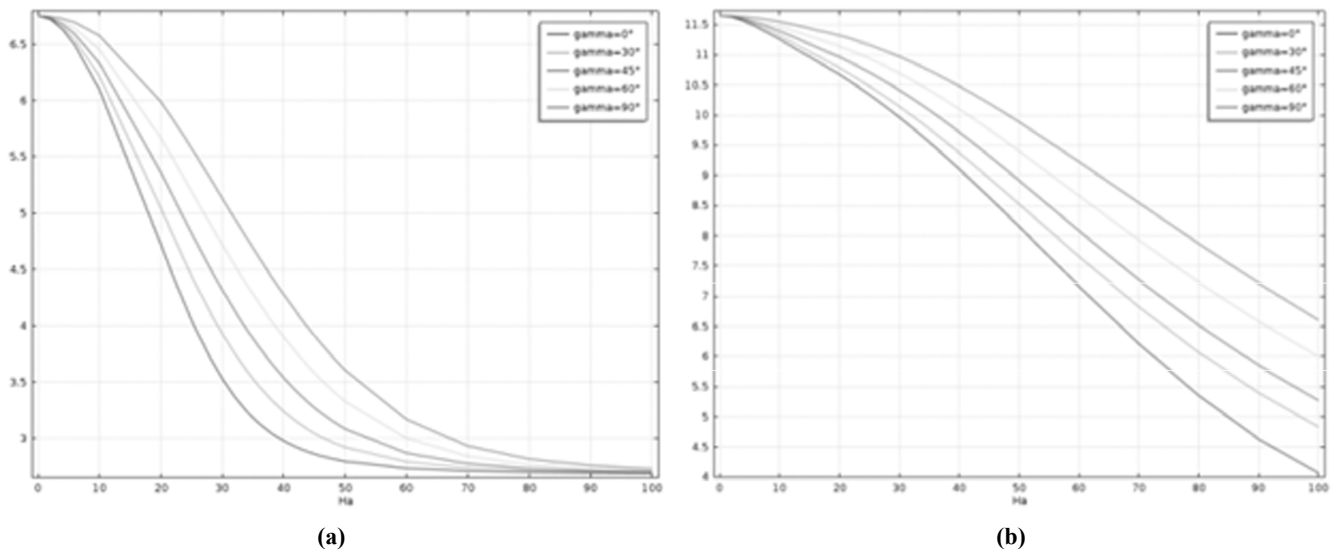


Figure 4: Variation of thermal entropy generation with Hartmann number for different angle inclination of magnetic field a) $Ra=10^5$; b) $Ra=10^6$

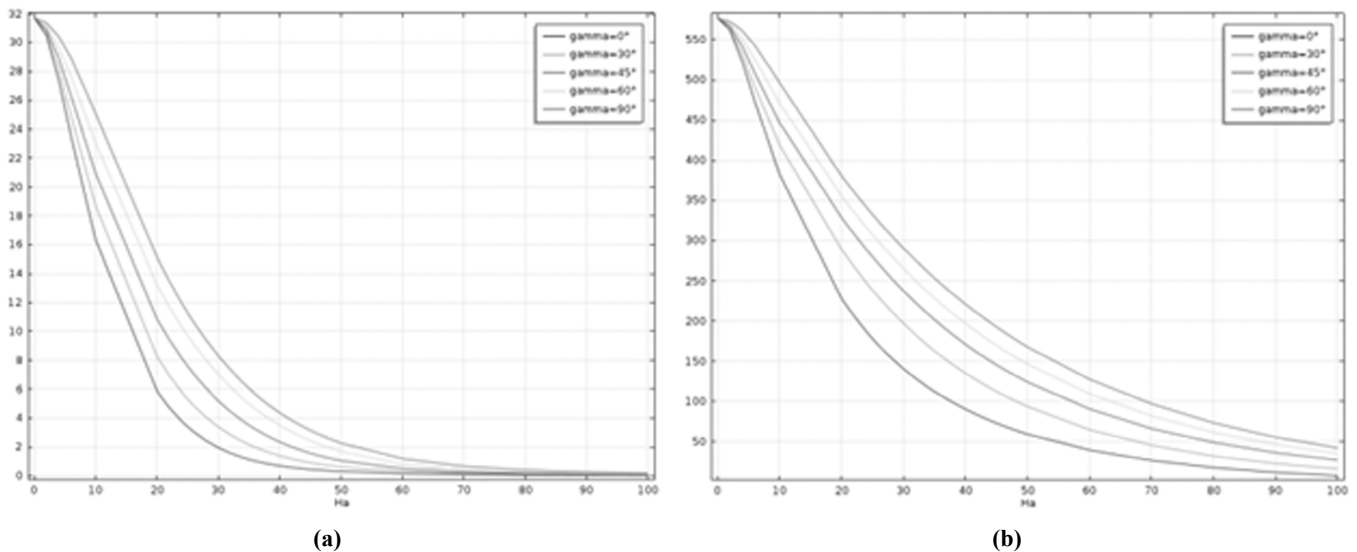


Figure 5: Evolution of viscous entropy with Hartmann number for different angle inclination of magnetic field a) $Ra=10^5$ b) $Ra=10^6$

to a decrease in the thermal and velocity gradients and consequently to a decrease in the thermal and viscous entropies generations. It's important to note that, at fixed magnetic field inclination angle, the decrease of thermal irreversibility is less important when Ra increases. This is due to the enhancement of the convection phenomenon when increasing Ra, which opposes the slowing effect of the magnetic field. Similar observations are conducted for the effect of Ra on the viscous irreversibility. It's important to notice that, at fixed Ha, both viscous and heat transfer irreversibilities increase when the magnetic field inclination angle increases.

7. CONCLUSION

This paper investigates the effect of an inclined magnetic field on the entropy generation on a natural convective heat transfer of CuO-Water nanofluid. The main findings are:

- The symmetry of streamlines and isotherms is broken since magnetic field inclination angle is not zero.
- The streamlines are elongated in the direction of the magnetic field.
- At fixed magnetic field inclination angle (γ), the irreversibility increases for relatively small Hartman number and reaches a maximum value at critical Hartman number (Ha_c) then it decreases towards a minimum value at maximum value of Ha.
- The critical Hartman number for which entropy generation is maximum depends on the magnetic field inclination angle.
- The critical Hartman number (Ha_c) represents the frontier between, first the case where Ha is preponderant via its intrinsic and extrinsic effects on the magnetic entropy generation equation.
- The thermal and viscous entropies generations decrease when Hartmann number increases.
- The decrease of thermal irreversibility is less important when Ra increases..

REFERENCES

- [1] H.K. Yang and C.P.Yu, "Combined forced and free convection MHD channel flow in entrance region," *International Journal of Heat and Mass Transfer*, **17** (6), 681-691, 1974.
- [2] A. Mahmoudi, I. Mejri, M.A. Abbassi and A. Omri, "Lattice Boltzmann simulation of MHD natural convection in a nanofluid-filled cavity with linear temperature distribution," *Powder Technology*, **256**, 257-271, 2014.
- [3] H.Heidary, R. Hosseini, M. Pirmohammadi and M. J. Kermani, "Numerical study of magnetic field effect on nanofluid forced convection in a channel," *Journal of Magnetism and Magnetic Materials*, **374**, 11-17, 2015.
- [4] M.H. Abolbashi, N. Freidoonimehr, F. Nazari and M.M. Rashidi, "Entropy analysis for unsteady MHD flow past a stretching permeable surface in nanofluid," *Powder Technology*, **267**, 256-267, 2014.
- [5] Z. Mehrez, A.E. Gafsi, A. Belghith and P.L. Quere, "MHD effects on heat transfer and entropy generation of nofluid flow in an open cavity," *Journal of magnetism and Magnetic Materials*, **374**, 214-224, 2015.
- [6] S. Das and R.N.Jana, "Entropy generation due to MHD flow in a porous channel with Navier slip," *Ain Shams Engineering Journal*, **5** (2), 575-584, 2014.
- [7] M. Hatami, R. Nouri and D.D. Ganji, "Forced convection analysis for Al₂O₃-water nanofluid flow over a horizontal plate," *Journal of Molecular Liquids*, **187**, 294-301, 2013.
- [8] M. M. Rahman, H.F. Ozotop, R. Saidur, S. Mekhilef and K. Al-Salem, "Finite element solution of MHD mixed convection in a channel with a fully or partially heated cavity," *Computers&Fluids*, **79**, 53-64, 2013.
- [9] M.A.Teamah and W.M. El-Maghlany, "Augmentation of natural convective heat transfer in square cavity by utilizing nanofluids in the presence of magnetic field and uniform heat generation/absorption," *International Journal of Thermal Sciences*, **58**, 130-142, 2012.
- [10] J.C. Maxwell, *A Treatise on Electricity and Magnetism*, Oxford University Press, Second Edition, Cambridge, U.K. 1904.

- [11] O.Mahian, A. Kianifar, C. Kleinstreuer, M.A. Al-Nimr, I. Pop, S. Wongwises and A.Z. Sahin, "A review of entropy generation in nanofluid flow," *International Journal of Mass and Heat Transfer*, **65**, 514-532, 2013.
- [12] S. Lee, S.U.S. Choi, S. Li and J.A. Eastman, "Measuring thermal conductivity of fluids containing oxide nanoparticles," *Journal of Heat Transfer*, **121**, 1999, 280-289, 1999.
- [13] S.K.Das, N. Putta, P. Thiesen and W. Roetzel, "Temperature dependence of thermal conductivity enhancement for nanofluids," *Journal of Heat Transfer*, **125**, 567-574, 2003.
- [14] H.E. Patel, T. Sundarajan, T.Pradeep and S.K.Das, "A micro-convection model for thermal conductivity of nanofluids," *Pramana*, **65**, 863-869, 2005.
- [15] L.C. Woods, *The Thermodynamics of Fluid Systems*, Oxford University Press, Oxford, U.K., 1975.
- [16] M. Magherbi, H. Abbasi and A.B. Brahim, "Entropy generation on the onset of natural convection," *International Journal of Heat and Mass Transfer*, **64**, 3441-3450, 2003.
- [17] H.F. Oztop and E. Abu-Nada, "Numerical study in partially heated rectangular enclosure filled with nanofluids," *International Journal of Heat and Fluid Flow*, **29**, 1326-1336, 2008.
- [18] S. Vaidyanathan, "A novel 3-D conservative chaotic system with sinusoidal nonlinearity and its adaptive control", *International Journal of Control Theory and Applications*, **9** (1), 115-132, 2016.
- [19] S. Vaidyanathan and S. Pakiriswamy, "A five-term 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control method", *International Journal of Control Theory and Applications*, **9** (1), 61-78, 2016.
- [20] S. Vaidyanathan, K. Madhavan and B.A. Idowu, "Backstepping control design for the adaptive stabilization and synchronization of the Pandey jerk chaotic system with unknown parameters", *International Journal of Control Theory and Applications*, **9** (1), 299-319, 2016.
- [21] A. Sambas, S. Vaidyanathan, M. Mamat, W.S.M. Sanjaya and R.P. Prastio, "Design, analysis of the Genesio-Tesi chaotic system and its electronic experimental implementation", *International Journal of Control Theory and Applications*, **9** (1), 141-149, 2016.
- [22] S. Vaidyanathan and A. Boulkroune, "A novel hyperchaotic system with two quadratic nonlinearities, its analysis and synchronization via integral sliding mode control," *International Journal of Control Theory and Applications*, **9**(1), 321-337, 2016.
- [23] S. Sampath, S. Vaidyanathan and V.T. Pham, "A novel 4-D hyperchaotic system with three quadratic nonlinearities, its adaptive control and circuit simulation," *International Journal of Control Theory and Applications*, **9** (1), 339-356, 2016.
- [24] S. Vaidyanathan and S. Sampath, "Anti-synchronization of identical chaotic systems via novel sliding control method with application to Vaidyanathan-Madhavan chaotic system," *International Journal of Control Theory and Applications*, **9** (1), 85-100, 2016.
- [25] A.T. Azar and S. Vaidyanathan, *Chaos Modeling and Control Systems Design*, Springer, Berlin, 2015.