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# Speed Analysis of DC Motor under Load and no Load Condition using CHR Based PID and LQR Optimal Controller

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**ABSTRACT**

DC motors are having wide range of application from household appliances to large industrial machinery at various speed conditions. In this paper, speed control of a separately excited direct current (DC) motor under load conditions has been implemented using CHR tuning and LQR controllers. The Chien - Hrones - Reswick (CHR) is based on a Proportional Integral Derivative controller (PID) tuning and Linear Quadratic Regulator controller (LQR) is based on an optimal controller. Comparison of both the methods have been done based on transient response specifications. Simulations in MATLAB have shown that the performance of the DC motor has significantly improved quality response with the LQR method over the CHR method. It has also minimized deviation from the desired speed, with the LQR method. This can lead to significant improvement when implemented in applications viz. robotics, process control and cruise control.

***Keywords:*** Proportional Integral Derivative controller, Optimal controller, Linear Quadratic Regulator, Chien-Hrones-Reswick controller, transient response.

## 1. INTRODUCTION

DC motor is one of the most important actuator in any production industry or mobile devices. Due to its small size, low cost, and high power it is being used very commonly. Various industries may require different speed applications. Sometimes it is required to keep some constant speed, or track the speed with respect to the load variation, or variation due to environmental disturbances. Hence there exist different methods for controlling the speed like Fuzzy logic, Optimal controller, Genetic algorithm, PID controllers, etc. For the past few decades various industries has been implementing different control methods for speed controlling. PID controller is the most widely used feedback control that has been implemented successfully in various engineering industrial processes due to simplicity. An efficient desired performance is achieved by selecting a right mix of P, I, and D actions. The objective of the method is to minimize the transient specifications such as settling time, rise time, maximum overshoot. In the past years many advanced improvements in controlling the systems have been developed to meet the requirements of users. There exists several approaches for determining the PID parameters which is been documented in the literature. Among those, Ziegler- Nichols tuning [7], [8] is been used most widely due to its simplicity, stability, and robustness, [9]. Another controller is the fuzzy logic, which is able to model inaccurate models, and offers a simpler, faster and well grounded solution [10], other controllers found were the Genetic algorithm [11], [12] PSO etc.

LQR design is a modern, widely used optimal control theory technique. Large system with multiple inputs and output (MIMO) system can be controlled efficiently, economically and reliably with a very nice robustness property with this design. The LQR is an optimal controller that minimizes a given cost function,

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which is also called as the performance index. This is defined by two matrices Q and R. These matrices regulates the penalties on the excursion of state variables and control signal. Algebraic Riccati Equation (ARE) is first solved to find the control law, this will leads to optimal results calculating from the defined cost function is obtained [9-10].

## 2. SYSTEM MODEL

### 2.1. DC motor mathematical model

In this work a separately excited DC motor is used. Adaptability of DC motors to adjust the speed drives makes it more appealing than AC motors, which have a constant speed rotating fields. DC motors can be used to adjust operating speed over wide ranges by a diverse of methods. The method here used is armature voltage. In this method, the armature voltage V*a* is used to control the armature current and the field current (I*f* ) is held constant. The control equivalent circuit of a separately excited DC motor under load condition is shown in the Fig. 1.

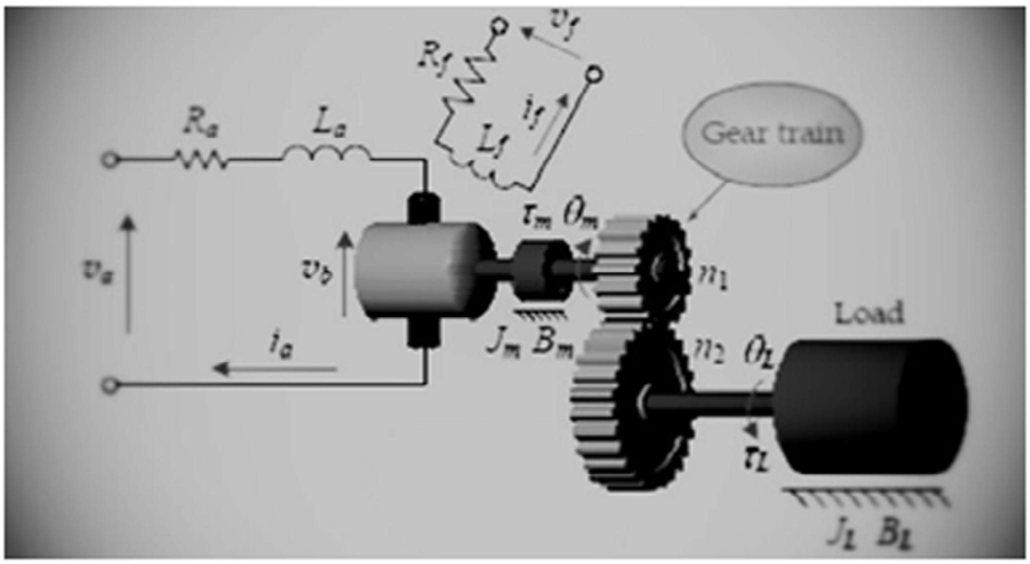


**Figure 1: DC motor equivalent model under load condition.**

The DC motor transfer function is,

$$G(s)=\frac{\omega(s)}{V_c(s)}=\frac{K_T}{(L_c s+R_c)(Js+B)+K_b K_T} \tag{1}$$

The controller output is given by,

$$K(s)=\frac{U(s)}{V(s)}=K_p+\frac{K_i}{s}+K_d s \tag{2}$$

### 2.2. DC motor model under load condition

The equivalent model of the motor is shown in Fig. 2. The motor network is given by,

$$v_a=L_a\frac{di_a}{dt}+R_a i_a+v_b \tag{3}$$

$$v_b=k_b\frac{d\theta_m}{dt}=k_b w_m \tag{4}$$

The new transfer function relating the unloaded motor torque T0 with the load position is computed as;

$$\frac{\Theta_L}{T_0} = \frac{n}{(J_0 s + B_0)s} \tag{5}$$

The associated transfer function of the equivalent system is,

$$G(s) = \frac{\Theta_L}{V_a} = \frac{k_a n}{\left[(L_a s + R_a)(J_0 s + B_0) + k_a k_b\right]s} \tag{6}$$

## 3. STUDY OF DC MOTOR SPEED CONTROL

In this paper, the speed control of DC motor is achieved by two methods namely CHR (based on PID controllers) method and LQR(an optimal controller) method. In the PID controllers the proportional term helps in stability set up and transient response enhancement. While the derivative term makes it enhance the closed loop response speed. The integrator term eliminates the offset.

Tuning of PID : Tuning refers to the adjusting of various parameters of PID to achieve optimum value for the control system response. There exists different types of speed control techniques such as Cohen Coon reaction curve method, Ziegler-Nichol's oscillation method, Ziegler-Nichol's reaction curve method, CHR tuning formula, Lambda tuning and LQR controllers etc. With the effective tuning of controller parameter Kp, Ki, Kd the dynamic system characteristics such as rise time, overshoot, settling time, steady state error are controlled.

1) *CHR tuning Method*: This method is for determining the values of the proportional gain Kp, integral time Ki and derivative time Kd based on the transient response characteristics of a given plant. From the response of a typical first order system (the S-curve), which is characterized by two parameters, namely the delay time L and the time constant T. The plant model is given by G(s) and the control parameter for this method is derived from the model below;

$$G(s) = \frac{Ke^{-sL}}{Ts+1} \tag{7}$$

Many varieties of plants for real time control systems can be approximately modeled by this equation. The values for Kp, Ki and Kd for open-loop system can be determined with the parameters K, L and T

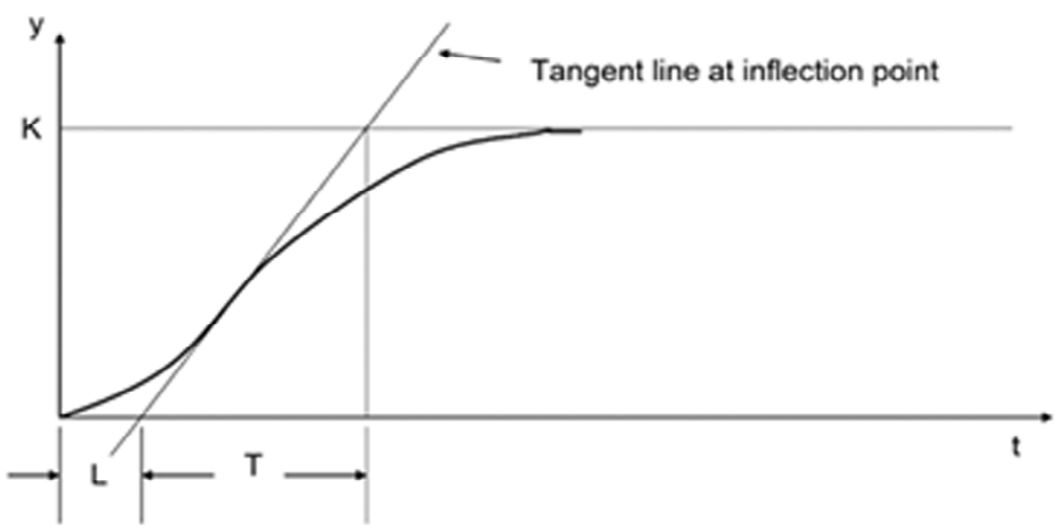


**Figure 2: Response curve for DC motor transfer function.**

**Table 1**
**CHR tuning method for closed loop system**

| *Controller type* | *Kp* | *Ti* | *Td* |
|---|---|---|---|
| P | 0.7/a | – | – |
| PI | 0.6/a | T | – |
| PID | 0.95/a | 1.4/T | 0.47T |

(a = KL/T) using the formula shown in the TABLE I. Then calculate the PID parameter according to table for closed loop system, and these parameter values can be used in the controller.

2) *Linear Quadratic Regulator*: It is basically a controller which uses mathematical algorithm to minimize the cost function with some weighting factors. Thus the algorithm minimizes the undesired deviations (deviations from desired temperature, speed or altitude). In feedback, each state variable is multiplied by a gain and results are summed to get a single actuation value. The result of LQR formulation is the set of gains based on relative weighting of errors and actuation in the performance index. In general, the state space equation of the system model can be written as,

$$*x = Ax + Bu \tag{8}$$

where *x is the state space representation. A is the state matrix of order nn and B is the control matrix of order nm. The performance index (J) for the design of a LQR controller is given by,

$$J = \int \left( x^T Q_x + u^T Ru \right) dt \tag{9}$$

where Q and R are the symmetric positive semi-definite and positive definite control weighting matrices respectively. The closed loop optimal control law and the ARE are given by:

$$*u = -K \tag{10}$$

$$A^T P + PA - PBR^{-1} B^T + Q = 0 \tag{11}$$

where P is a symmetric and positive definite matrix obtained by solution of the ARE . Then the feedback gain matrix K is given by:

$$K = Ax - BKx = (A - BK)x \tag{12}$$

Substituting the above equation (11) into equation (9) gives:

$$*x = Ax - BKx = (A - BK)x \tag{13}$$

## 4. IMPLEMENTATION

Both PID and LQR speed controlling techniques were implemented using a maxon DC motor, under load and no load conditions. The motor specifications are listed in Fig. 3. For PID controller, initially the L and T values were found using open loop response from the DC motor transfer function. From the value of L and T, PID parameters such as Kp and Ti were found. With Ti, Ki was calculated using the formula Ki = Kp/Ti and table. These values are used to plot the step response of the system in MATLAB. From the PID parameters like Kp and Ki, the PID controller transfer function is determined. Finally the product of PI controller and DC motor transfer function will gives the closed loop step response of the system. The algorithm for both CHR and LQR Optimal controllers are given in Fig. 4, 5.

$$TF = \frac{25.9}{3.9865s^2 + 20.516s + 0.918} \tag{14}$$

For LQR method, several trials were performed in simulation and the result of each trial was noticed, by changing Q and R values. Tuning of different point of the LQR parameters has been done. Among the various trials, the trial with Q=diag[1,1] and R=1 gave a better dynamic response. Both load and no load conditions of DC motor were performed in the same manner.

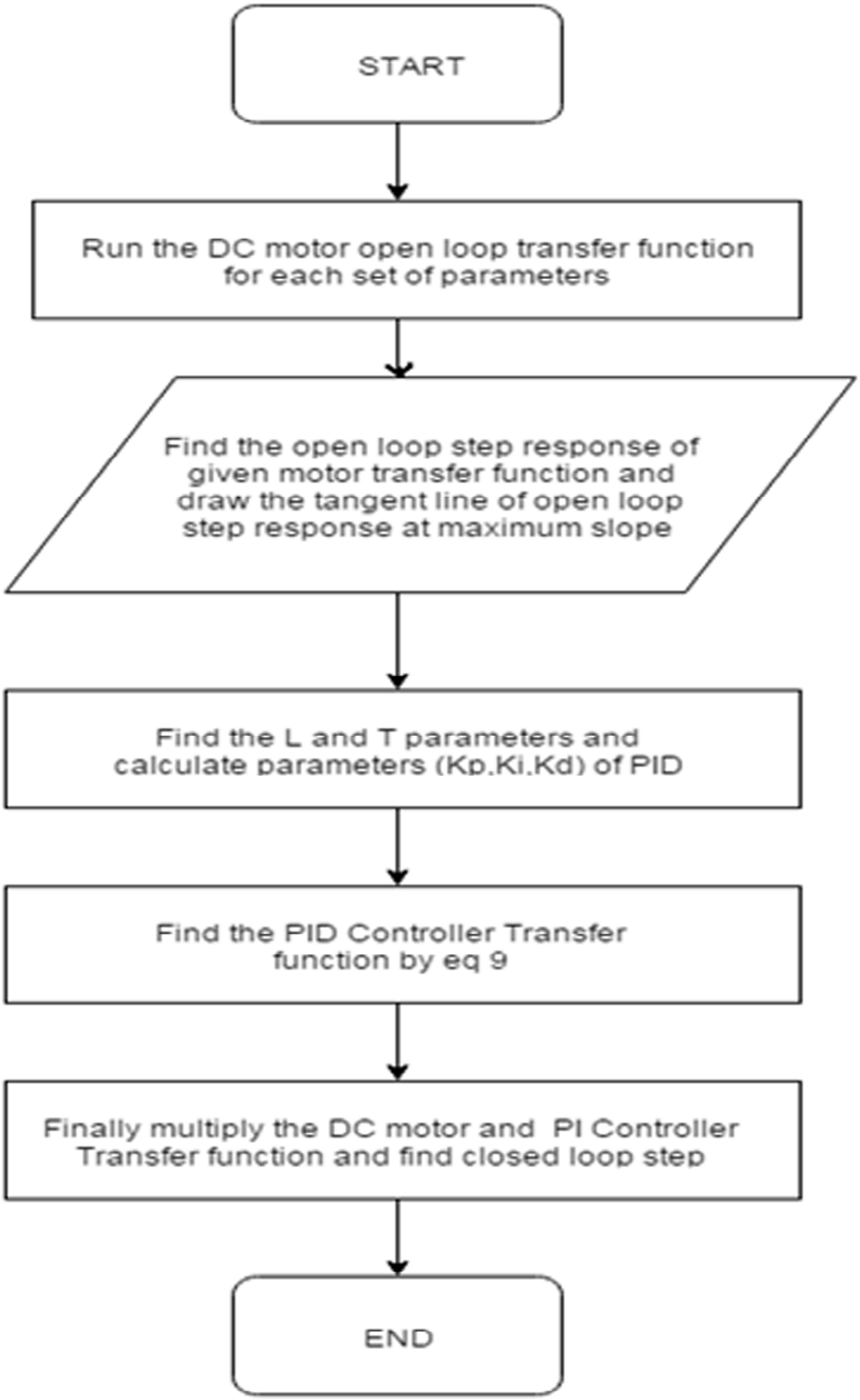


**Figure 3: Algorithm for CHR method**

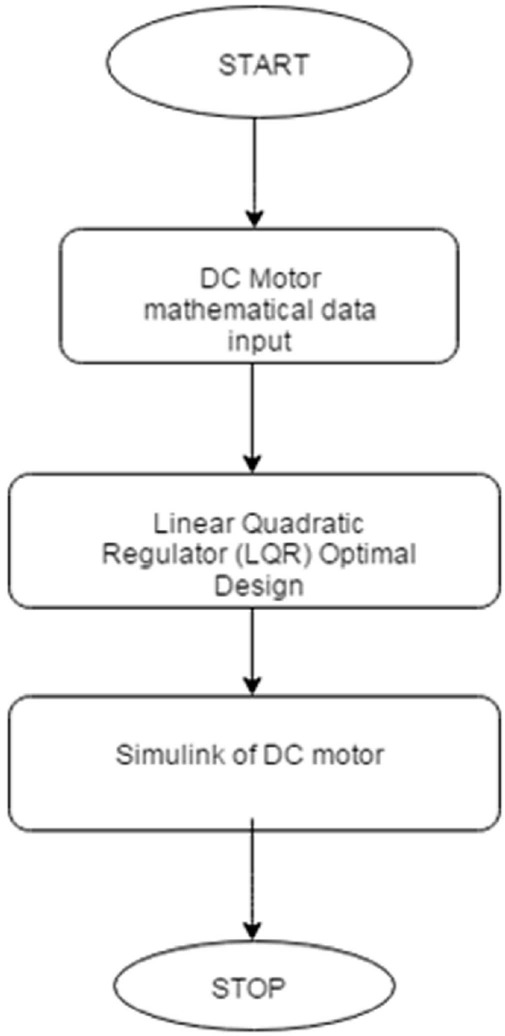


**Figure 4: Algorithm for LQR method**

| Parameter | Motor rating |
|---|---|
| Armature Resistance (Ra) | 0.611 ohm |
| Armature Inductance (La) | 0.119 mH |
| Moment of inertia (J) | 33.5 gcm2 |
| Friction constant (B) | 0.4 |
| Torque constant (Kt) | 25.9mNm/A |
| EMF constant (Kb) | 0.02601 |

**Figure 5: DC motor specifications**

## 5. RESULTS AND ANALYSIS

The step response of DC motor under no load and load conditions with CHR and LQR controllers are shown in Fig. [6-9] respectively. The comparison of the two methods is made based on modeling and MATLAB coding, taking rise time and settling time under load and no-load conditions are shown as in TABLE II. And from that it has been analyzed that LQR provides better performance. On comparing the tuning methods implemented in this paper, with a PID controller without the implementation of a tuning method is usually time consuming, prone of error, manually intensive which cannot be commissioned in a plant wide basis. From the simulation results, the LQR tuning method under load condition showed 8.65% increase in rise time and 47.63% in settling time compared to CHR method.

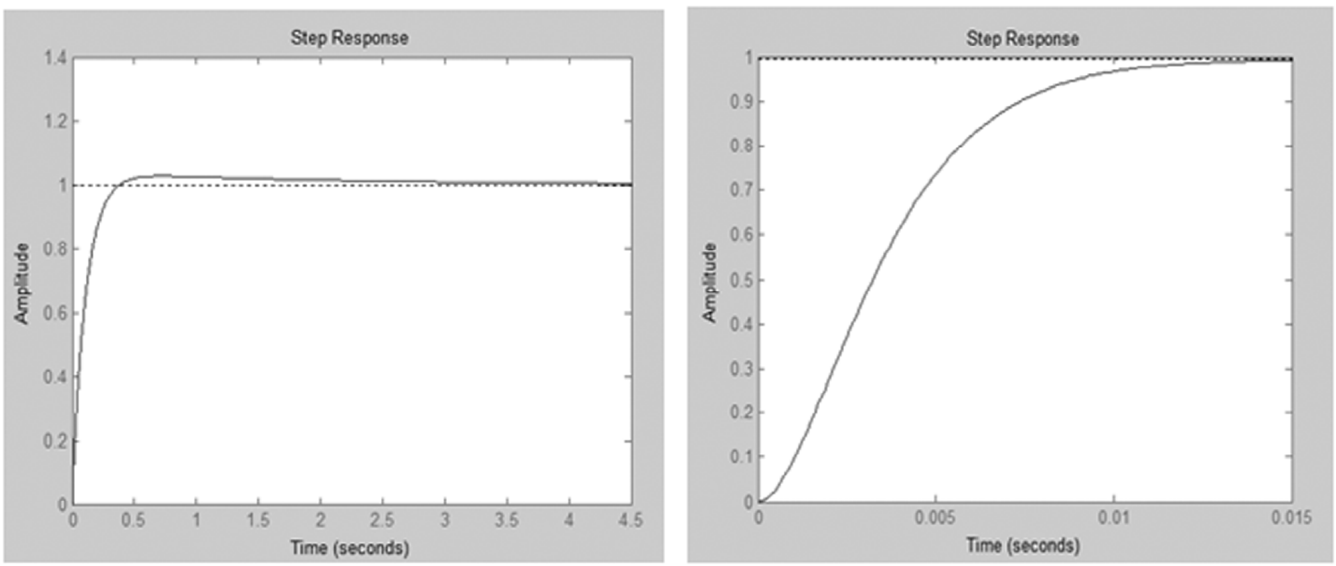


**Figure 6, 7: Transient response of DC motor using CHR and LQR methods Without load condition**

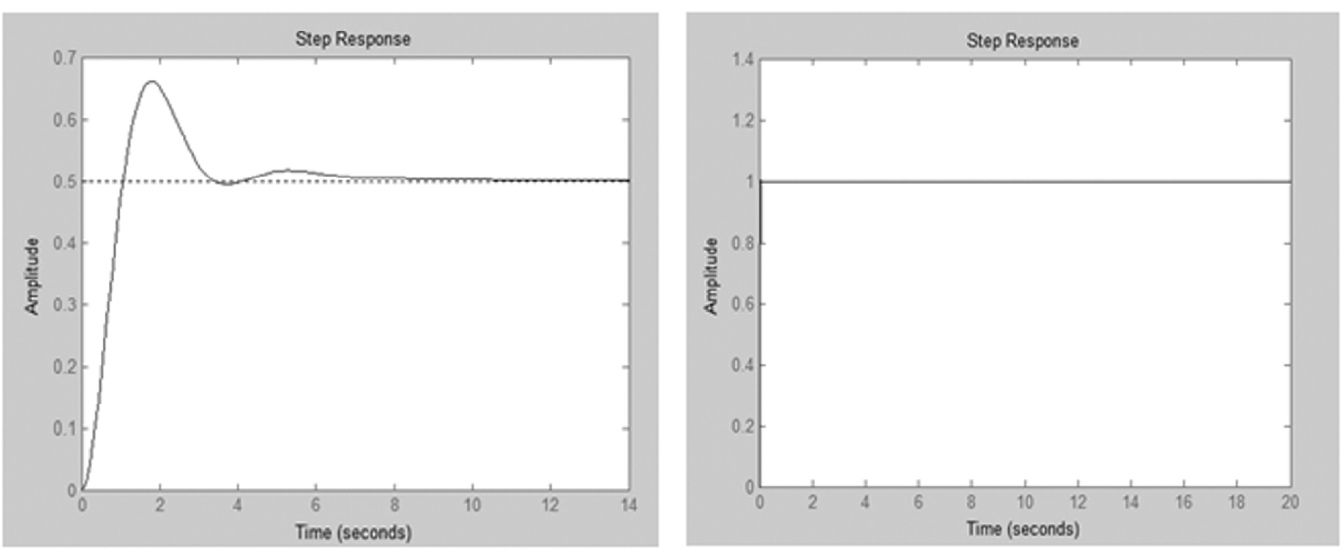


**Figure 8, 9: Transient response of DC motor using CHR and LQR methods With load condition**

## 6. CONCLUSION

In this paper, a comparative study of speed control of DC motor has been done using CHR tuning and LQR controller both under load and no load conditions. The methods have been implemented in MATLAB software. The simulation results prove that the proposed LQR method offers an improved dynamic performance in terms of rise time by 8.65% and settling time by 47.63, when compared with the CHR method.

## 7. FUTURE SCOPE

This can lead to significant improvement when implemented in applications viz. robotics, process control and cruise control. More research on the selection of the Q and R in the LQR method could prove to be significant in the understanding of a generic system transient response.

**Table 2**
**Comparison of transient responses**

| *Transient parameter* | *Rise time (sec)* | *Settling time (sec)* |
|---|---|---|
| CHR-with load | 1.04 | 8.1 |
| CHR-without load | 0.34 | 2.61 |
| LQR-with load | 0.0012 | 0.0017 |
| LQR-without load | 0.01 | 0.015 |

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