Design and Analysis of PWM-Based Quasi-Sliding-Mode Controllers for Buck Converters

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ABSTRACT

In this paper, the design and analysis of a fixed-frequency pulse-width modulation (PWM)-based quasisliding mode voltage controller for dc-dc Buck converters is presented. The dc-dc Buck converter is operating in the continuous conduction mode (CCM). A practical design approach for selecting the sliding coefficients for Sliding Mode controller is also presented. The designed quasi-sliding- mode voltage controller based on pulse-width modulation technique is having the same structure as that of Proportional Derivative (PD) linear controller, with an additional component consisting of the instantaneous input voltage and instantaneous output voltage. The performance of the controller is verified by the simulation results of the converter with the fixed frequency bandwidth and without bandwidth. The simulation results show that the performance of the converter is satisfies with the designed approach.

Keywords: Buck converter, Hysteresis-Modulation (HM), Pulse-Width Modulation (PWM), Sliding Mode (SM), Quasi-Sliding-Mode (QSM), Sliding-Mode Voltage Controller (SMVC).

1. INTRODUCTION

Sliding-Mode Controllers are non-linear controllers which are used for controlling the variable structure systems (VSS's) [1]-[3]. The power converters are variable-structured systems due to their high switching action. Hence SM controllers are used for control of power converters [4]. The SM controllers are operated at infinite, varying and self-oscillating switching frequencies. Ideally the SM controllers are operated at an infinite switching Frequency such that the controlled variables exactly follow the reference path to achieve the desired dynamic response and steady-state operation [1]. The power converters operated at extremely high switching and varying frequencies which causes excessive switching losses, inductor and transformer core losses, and electromagnetic interference (EMF) noises [2]. Hence the SM controllers that are applicable to power converters will have the fixed switching frequency within a desirable range [3]-[11]. Since all the power converters are operated at fixed-frequency instead of variable switching frequency, this controller transform into a new type of controller which is called the Quasi-Sliding-Mode controller.

To get the desired fixed switching frequency operation, the previously proposed SM controllers for the power converters are Hysteresis-Modulation (or Delta-Modulation), constant sampling frequency, constant ON time, constant switching frequency and limited maximum switching frequency [5]. However, the above proposed SM controllers fail to practical design methods and implementation criteria. For keeping the switching frequency constant for HM-based SM controller, basically two types of approaches are there. First approach is adding a constant ramp or timing function directly into the controller [5]. The second approach uses an adaptive hysteresis band that varies with the parameter changes to control and fixes the switching frequency [6]. However, the above proposed approaches requires additional components and are not suitable for low voltage conversion applications. On the other hand, constant switching-frequency SM controllers can also be obtained by changing the modulation method of the SM controllers from HM modulation to pulse-width modulation (PWM). This method is also called as duty cycle control. This is similar to classical PWM control schemes in which the control signal V_c is compared with the ramp signal \hat{V}_{ramp} to generate a discrete gate pulse signal which is having the same frequency as that of ramp signal [7].

In the earlier paper Siew-Chong Tan *et al.* [8], presented a unified approach to the design of PWM based SMVC for basic dc-dc converters operating in the continuous conduction mode. In this paper based on [8], the control equations for the equivalent control and the duty cycle control and the relationship between the equivalent control and duty cycle for the SMVC Buck converter are presented. Here the Buck converter is having the under-damped response. This controller can be easily designed and implemented from the derived mathematical expressions. The simulation results are shown to verify the operation of this controller and the mathematical modelling for the selection of sliding coefficients. The performance of the controller is also verified with the fixed bandwidth and without bandwidth. The advantages of SM controllers are guaranteed stability and the robustness against the sudden changes in the line, load and parameter variations [1]. The SM controllers have high degree of flexibility in its design and are easy to implement as compared with the other types of non-linear controllers. Hence SM controllers are using in various industrial applications e.g., automotive control, furnace control, etc [9].

2. DESIGN APPROACH OF SMVC BUCK CONVERTER

The complete discussion about the theory of SM control, equivalent control, and the relationship of SM control and duty ratio control is presented in [1], [10] and [11]. Here we will discuss the dc-dc converter modelling and the detailed procedure for designing the SM controller for dc-dc Buck converter in continuous conduction mode (CCM) operation.

A. Mathematical Model of an Ideal SM PID Voltage Controlled Buck Converter

The first step to the design of an SM controller is to develop a state-space approach to the Buck converter model in terms of the desired control variables (i.e., voltage and/or current etc.) by applying the Kirchhoff's voltage and current laws [12]. Here the output voltage takes as the control variable for the controller. The SM controller presented here is a second-order proportional integral derivative (PID) SM voltage controller.

Fig 1.shows the schematic diagram of the PID SMVC Buck converter in the conventional HM configuration. Here C, L and R_L denote the capacitance, inductance, and instantaneous load resistance of the Buck converter respectively; i_C , i_L and i_R are the capacitor, inductor, and load currents respectively; V_{in} , δV_0 , and V_{ref} are the input voltage, the sensed output voltage and reference voltages, respectively; δ is the scaling factor which is defined as $\delta = V_{ref} / V_{od}$ and u = 0 or 1 is the switching state of the power switch *s*.

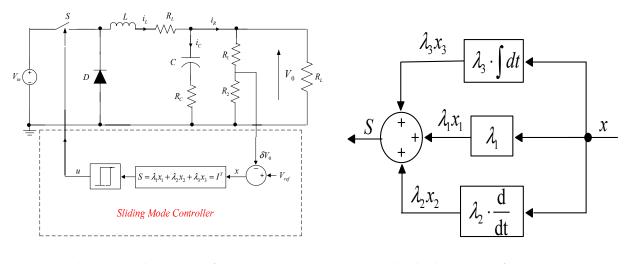


Fig 1.Schematic diagram of DC-DC Buck converter based on HM

Fig 2.Block diagram of instantaneous sliding surface S.

The basic expression for dc-dc buck, boost, and buck-boost PID SMVC converters, the control variable x is expressed in the general form as

$$x = \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} = \begin{bmatrix} V_{ref} - \delta V_{0} \\ \frac{d}{dt} (V_{ref} - \delta V_{0}) \\ \int (V_{ref} - \delta V_{0}) dt \end{bmatrix}$$
(1)

where the control variables x_1 , x_2 and x_3 represent the *voltage error*, the rate of change of *voltage error*, and the *integral of voltage error*, respectively. By substituting the buck converter's operational model under continuous conduction mode (CCM) into (1) gives the following control variable description:

$$x_{buck} = \begin{bmatrix} x_{1} = V_{ref} - \delta V_{0} \\ x_{2} = \frac{\delta V_{0}}{R_{L}C} + \int \frac{\delta (V_{0} - V_{in}u)}{LC} dt \\ x_{3} = \int x_{1}dt \end{bmatrix}$$
(2)

The next step is, time differentiation of (2) gives the state-space descriptions required for the controller of the Buck converter.

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \\ \dot{x}_{3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{R_{L}C} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\delta V_{in}}{LC} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{\delta V_{0}}{LC} \\ 0 \end{bmatrix}.$$
 (3)

The state-space representation shown in eq. (3) can be written in the standard form as:

$$\dot{x}_{buck} = Ax + By + D$$

where y=u or \overline{u} (depending on the mode of operation).

B. Controller Design:

The basic idea of SM control is to design a certain sliding surface in its control law that will follow the reference path state variables towards a desired equilibrium. The designed SM controller must satisfy the following three conditions [13].

1. To Meet Hitting Condition:

The control law which is based on satisfying the *hitting condition* [14] that follow switching functions such as:

$$u = \frac{1}{2}(1 + \operatorname{sgn}(S)) \tag{5}$$

(4)

and

$$u = \begin{cases} 1, & \text{when } S > 0 \\ 0, & \text{when } S < 0 \end{cases}$$
(6)

Where *S* is the instantaneous state variable's trajectory reference path, and is defined as

$$S = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = I^T x_{buck}$$
⁽⁷⁾

Here $I^T = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} \cdot \lambda_1, \lambda_2$, and λ_3 representing the control parameters which are also called as sliding coefficients. The equation in (7) can also be shown in diagrammatically in fig 2. By equating S = 0 which is shown in eq. (7) we will obtain the sliding surface.

2. To Meet Existence Condition:

After determining the switching states u = 1 or 0, the next stage is to see whether the sliding coefficients λ_1, λ_2 , and λ_3 follow the existence condition. Here Lyapunov's direct method [15] is used to determine the ranges of the employable sliding coefficients. This is possible by checking the approachability condition of the state trajectory reference path in graph theory, i.e.

$$\lim_{s \to o} S \cdot \dot{S} < 0 \tag{8}$$

By differentiating (7) with respect to time, \dot{S} is obtained. $\dot{S} = I^T \dot{x}$

By substituting the (4) in (9), it gives

$$\dot{S} = I^T \dot{x} = I^T (Ax + By + D) \tag{10}$$

By substituting the (5) in (10)

$$\dot{S} = I^{T} A x + \frac{1}{2} I^{T} B + \frac{1}{2} I^{T} B sgn(S) + I^{T} D$$
(11)

Multiplying (11) by (7) gives

$$S\dot{S} = S \left[I^{T} A x + \frac{1}{2} I^{T} B + \frac{1}{2} I^{T} B \operatorname{sgn}(S) + I^{T} D \right]$$
(12)

$$= S \left[I^{T} A x + \frac{1}{2} I^{T} B + I^{T} D \right] + \frac{1}{2} |S| I^{T} B$$
(13)

The equation (10) can be expressed as

$$\int \dot{S}_{S \to 0^{+}} = I^{T} A x + I^{T} B v_{S \to 0^{+}} + I^{T} D < 0$$
(14(a))

$$\dot{S}_{S \to 0^{-}} = I^{T} A x + I^{T} B v_{s \to 0^{-}} + I^{T} D > 0$$
 (14(b))

Case 1: $S \rightarrow 0^+, S < 0$:

By substituting the $v_{s \to 0^+} = u = 1$ and the state-space Matrices in the equation (3) into (14(a)) gives

$$I^{T}\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{R_{L}C} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} + I^{T}\begin{bmatrix} 0 \\ -\frac{\delta V_{in}}{LC} \\ 0 \end{bmatrix} u + I^{T}\begin{bmatrix} 0 \\ \frac{\delta V_{0}}{LC} \\ 0 \end{bmatrix} < 0.$$
(15)

$$= -\lambda_1 \frac{\delta i_C}{C} + \lambda_2 \frac{\delta i_C}{R_L C^2} + \lambda_3 (V_{ref} - \delta V_0) - \lambda_2 \frac{\delta V_{in}}{LC} + \lambda_2 \frac{\delta V_0}{LC} < 0$$
(16)

Case 2: $S \rightarrow 0^-, S > 0$:

By substituting the $v_{s\to 0^-} = u = 0$ and the state-space matrices in the equation (3) into (14(b)) gives

$$I^{T}\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{1}{R_{L}C} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \\ x_{3} \end{bmatrix} + I^{T}\begin{bmatrix} 0 \\ -\frac{\delta V_{in}}{LC} \\ 0 \end{bmatrix} u + I^{T}\begin{bmatrix} 0 \\ \frac{\delta V_{0}}{LC} \\ 0 \end{bmatrix} < 0.$$
(17)

$$= -\lambda_1 \frac{\delta i_C}{C} + \lambda_2 \frac{\delta i_C}{R_L C^2} + \lambda_3 (V_{ref} - \delta V_0) + \lambda_2 \frac{\delta V_0}{LC} > 0$$
(18)

Finally by combining the equations (16) and (18), it will give the existence condition as:

$$0 < -\delta L \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_L C}\right) i_C + L C \frac{\lambda_3}{\lambda_2} \left(V_{ref} - \delta V_0\right) + \delta V_0 < \delta V_{in} \cdot$$
(19)

where i_c =capacitor peak current.

Theoretically, the actual output voltage V_0 is ideally a pure dc whose magnitude is equal to the desired output voltage $V_{\alpha d} \equiv V_{ref} / \delta$. But due to limitation of switching frequency

and improper feedback loop, there is always some steady-state dc error between V₀ and V_{od} · So it is important to take this parameter into consideration for the design of the controller, because the factor $LC(\lambda_3 / \lambda_2)((V_{ref} / \delta) - V_0)$ is greater in comparison to V_0 · The above inequalities gives the conditions for existence condition and give the information about the range of employable sliding coefficients. The sliding coefficients will force the converter to stay in SM operation when its state trajectory is nearer to the sliding surface.

3. To Meet Stability Condition:

In addition to the existence condition, the selected sliding coefficients must ensure the stability condition. The selection of sliding coefficients is based on the desired dynamic response of the converter. In our example, the sliding surface equation (7) is relating the sliding coefficients to the dynamic response of the converter during SM operation is

$$\lambda_1 x_1 + \lambda_2 \frac{dx_1}{dt} + \lambda_3 \int x_1 dt = 0$$
⁽²⁰⁾

The equation (19) can be rearranged into a standard second-order form as:

$$\frac{d^2 x_1}{dt^2} + \frac{\lambda_1}{\lambda_2} \cdot \frac{dx_1}{dt} + \frac{\lambda_3}{\lambda_2} x_1 = 0$$
(21)

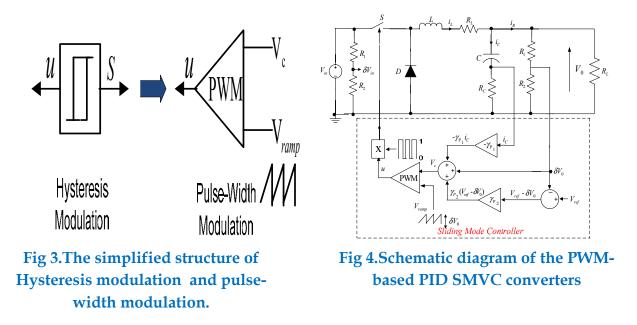
$$=\frac{d^{2}x_{1}}{dt^{2}}+2\xi\omega_{n}\frac{dx_{1}}{dt}+\omega_{n}^{2}x_{1}=0$$
(22)

where $\omega_n = \sqrt{(\lambda_3 / \lambda_2)}$, is the undamped natural frequency and $\xi = (\lambda_1 / 2\sqrt{\lambda_2 \lambda_3})$ is the damping ratio. There are three possible types of responses are there in a linear-second order system: under-damped ($0 \le \xi \le 1$), critically- damped ($\xi = 1$), and over-damped ($\xi > 1$). Here, by taking the dynamic response of the converter for critically-damped system ($\xi = 1$), the following expressions can be derived.

From
$$\omega_n = \sqrt{(\lambda_3 / \lambda_2)}$$
, and $\xi = (\lambda_1 / 2\sqrt{\lambda_2 \lambda_3})$, we can derive $\omega_n = \sqrt{(\lambda_3 / \lambda_2)} = 2\pi f$
$$\frac{\lambda_3}{\lambda_2} = 4\pi^2 f^2, \frac{\lambda_1}{\lambda_2} = 4\pi f$$
(23)

By observing the equations (23) and (24), it is clear that the sliding coefficients are dependent on the bandwidth with the existence condition (19) for the PWM-based

controllers. The design equations mentioned in (23) and (24) are applicable to all other types of second-order SMVC converters.



3. IMPLEMENTATION OF PWM BASED SMVCBUCK CONVERTER

A. Derivation of PWM-Based SM Control Law:

The simplified structure of the modulation technique from HM to linear PWM is shown in fig 4. The HM technique in SM control requires only the control equations (5) and (6).The linear PWM based SM controller requires the relationship of the two control techniques which are to be developed as shown in the fig. 3 of the pulse-width modulation technique. The PWM-based PID SMVC buck converter controller structure is shown in fig 4. The design of the linear PWM based SM controller can be performed in two steps.

• The equivalent control signal u_{eq} [1] is used instead of u, which is a function of discrete input function is derived from the invariance condition by setting the time differentiation of (7) as $\dot{s} = 0$.

• The equivalent control function u_{eq} is mapped on the duty cycle function of the pulsewidth Modulator. For the PWM based SMVC Buck converter, the derivations for the equivalent control and duty cycle control techniques are as follows [8]:

1) Equivalent Control:

as

By equating $\dot{S} = I^T A x + I^T B u_{eq} + I^T D = 0$, it gives the equivalent control signal [1]

$$u_{eq} = -\left[I^T B\right]^{-1} I^T \left[Ax + D\right]$$
(24)

$$= -\frac{\delta L}{\delta V_{in}} \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_L C} \right) x_2 + \frac{\lambda_3 L C}{\lambda_2 \delta V_{in}} x_1 + \frac{V_0}{V_{in}}$$
(25)

Here u_{eq} is continuous and $0 < u_{eq} < 1$.

Substitution of (24) into the inequality (25), it gives

$$0 < u_{eq} = -\frac{\delta L}{\delta V_{in}} \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_L C} \right) i_C + \frac{\lambda_3 L C}{\lambda_2 \delta V_{in}} \left(V_{ref} - \delta V_0 \right) + \frac{V_0}{V_{in}}$$
(26)

Now multiplying the inequality in equation (26) by δV_{in} ,

$$0 < u_{eq}^{*} = -\delta L \left(\frac{\lambda_{1}}{\lambda_{2}} - \frac{1}{R_{L}C} \right) i_{C} + LC \frac{\lambda_{3}}{\lambda_{2}} \left(V_{ref} - \delta V_{0} \right) + \delta V_{0} < \delta V_{in}$$
 (27)

2) *Duty Cycle Control:* In the linear PWM-based SM controlled system, the instantaneous duty cycle d is expressed as

$$d = \left(\frac{V_c}{\hat{V}_{ramp}}\right) \tag{28}$$

Where V_c is the control signal to the pulse-width modulator or comparator and \hat{V}_{ramp} is the peak magnitudes of the ramp signal with constant switching frequency. Since *d* is also continuous and it should be bounded by 0 < d < 1, this is also expressed as:

$$0 < V_c < \hat{V}_{ramp}$$
 (29)

Where $0 < d = (V_c / \hat{V}_{ramp}) < 1$, gives the following relationships for the control signal V_c and the ramp signal \hat{V}_{ramp} for the practical implementation of the PWM-based SM controller.

3) Comparing Equivalent control and Duty Cycle Control:

By comparing the equivalent control and duty ratio control [11], the following relationships can be derived for the practical application of PWM-based SMVC buck converters.

$$V_{c} = u_{eq}^{*} = -\delta L \left(\frac{\lambda_{1}}{\lambda_{2}} - \frac{1}{R_{L}C} \right) i_{C} + LC \frac{\lambda_{3}}{\lambda_{2}} \left(V_{ref} - \delta V_{0} \right) + \delta V_{0}$$
(30)

$$V_{c} = u_{eq}^{*} = -\gamma_{p_{1}}i_{C} + \gamma_{p_{2}}\left(V_{ref} - \delta V_{0}\right) + \delta V_{0}$$
(31)

Where
$$\gamma_{p_1} = -\delta L \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_L C} \right)$$
 and $\gamma_{p_2} = LC \frac{\lambda_3}{\lambda_2}$ (32)

and

to

 $\hat{V}_{ramp} = \delta V_{in}$

4. RESULTS AND DISCUSSION

The design of SMVC Buck converter is verified with the simulation results. The specifications of the SMVC Buck converter are shown in table I. Here the Buck converter is designed to operate in the continuous conduction mode with the input voltage of $V_{in} = 24V$ and a load current of $I_0 = 2A \cdot$ The maximum allowable peak-topeak ripple voltage is 50mV. The LC filter frequency should be less than the switching frequency to reduce the harmonics in the output voltage and output current.

TABLE I SPECIFICATION OF BUCK CONVERTER			TABLE-II	
Description	Parame	Nomina		
	ter	l Value	S.NO.	Derived
Input Voltage	V_{in}	24 V		Expressions
Inductance	L	150 μH	1.	$\gamma_{p_1} = -\delta L \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_1 C} \right)$
Inductor resistance	R_{L}	0.12 Ω		$(\lambda_2 - \kappa_L C)$
Capacitance	С	200 µF		1
Capacitor ESR	R_{C}	21 mΩ	2.	$\gamma_{p_2} = L C \frac{\lambda_3}{\lambda_2}$
Minimum load	$R_{L(\min)}$	3 Ω		
resistance	D		3.	$\hat{V}_{ramp} = \delta V_{in}$
Maximum load resistance	$R_{L(\max)}$	24 Ω		ramp
Switching frequency	F_{s}	200 KHz	4.	V
ownering nequency	1' _S	200 1112	4.	V_c
Desired output voltage	V_{od}	12 V		

Here the PWM-based SM controller is designed to give a critically-damped response at an angular frequency of 3.8Krad/sec. Now from the equations (23) and (24) the sliding parameters are determined as $(\lambda_1 / \lambda_2) = 14439735 \cdot 52$ and $(\lambda_1 / \lambda_2) = 7610.104$ For

(33)

the design of the sliding parameters at full condition, $(\lambda_1/\lambda_2) \gg (1/R_{L(\min)}C)$, i.e., 7610.104 >> 1666 · 67 The reference voltage taken as $V_{ref} = 2.5 V$, and the calculated scaling factor is $\delta = 0.208$ · Finally the control parameters are determined from (32) are

$$\gamma_{p_1} = -\delta L \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_L C} \right) = 0.2053$$

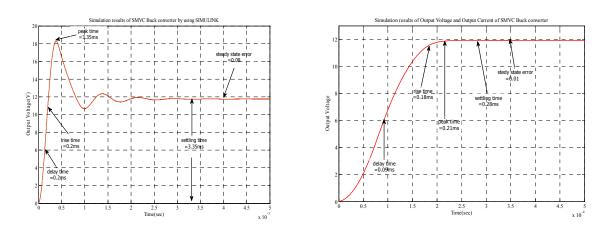
$$\gamma_{p_2} = L C \frac{\lambda_3}{\lambda_2} = 0.53319.$$
(34)

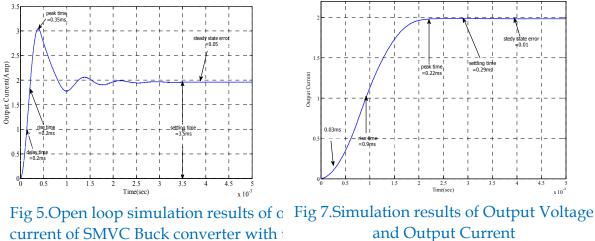
Substituting the equations (35) and (36) in equation (31), we get the resultant control signal V_c amplitude.

$$V_c = u_{eq}^* = 0.2053i_C + 0.53319 (V_{ref} - \delta V_0) + 0.208V_0$$
(35)

The theoretical descriptions of the signals which are derived for SMVC Buck converter are shown in TABLE-II.

The open-loop and closed loop simulation results of output voltage and output current of SMVC Buck converter with the time domain specifications are shown in fig 5. and fig 7. with the parameter values taken from TABLE-I. By comparing the open loop simulation results and SMVC simulation results with the time domain specifications, the SMVC Buck converter has very less setting time and steady state error when compared with the open loop. The fig 6. Shows the generation of PWM signal by comparing the V_c with the ramp signal. The dynamic performance of the controller can be explained by using a load resistance that changes between 3Ω and 24Ω at a constant switching frequency. The output current waveform when the step changes in the load current when an external load of 20Ω is added to 3Ω and the 24Ω is added to 10Ω are shown in fig. 8.and fig 9.





specifications.

and Output Current Of SMVC Buck converter with the time domain specifications.

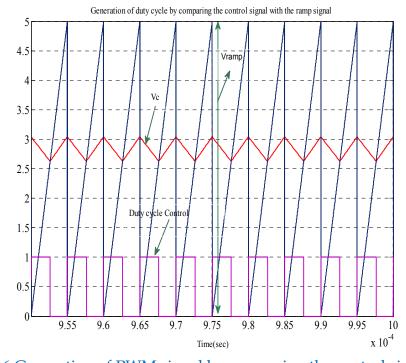
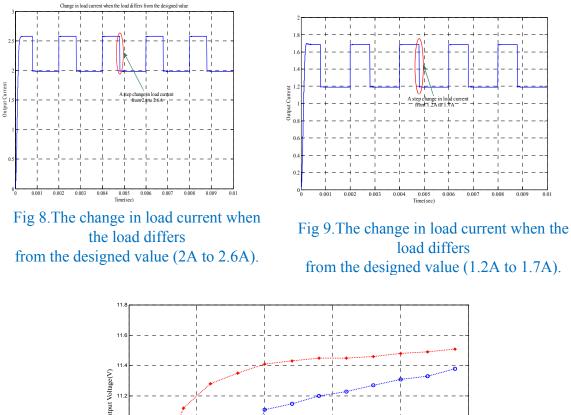


Fig 6.Generation of PWM signal by comparing the control signal and the ramp signal

The variation of measured output voltage (V) with respect to change in load is shown in fig 10. With the fixed 5 KHz frequency bandwidth the measured output voltage is approximately equal. So by using the fixed frequency bandwidth, we will get the constant duty cycle control, which will gives the constant output voltage.



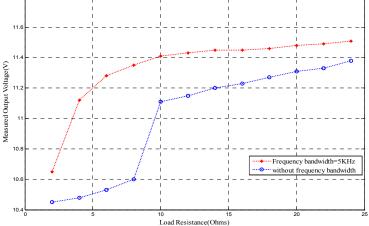


Fig 10.Plot of measured output voltage against load resistance without the frequency bandwidth and fixed frequency bandwidth.

4. CONCLUSION

In this paper, a unified approach to the fixed-frequency PWM-based sliding mode voltage controller for a Buck converter operating in continuous conduction mode is presented. A theoretical procedure for the design of sliding coefficients is also described. The control equations for the implementation of PWM-based SM voltage controller are also derived. The variation of output voltage with the change in load resistance is also verified with the fixed bandwidth and without bandwidth. The simulation results shows, the response of the converter agrees with the theoretical design.

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