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Design, Analysis of the Genesio-Tesi Chaotic System and its Electronic Experimental Implementation

Aceng Sambas*[1,5], Sundarapandian Vaidyanathan[2,5], Mustafa Mamat[1], Mada Sanjaya WS[3,5] and Rizki Putra Prastio[5]

Abstract: In this paper, a Genesio-Tesi chaotic system with one quadratic term has been proposed, and its qualitative properties have been detailed. The dynamic behavior of the Genesio-Tesi chaotic attractor is analyzed. Specially, the Lyapunov spectrum and eigenvalue structure are calculated and the bifurcation diagram is sketched. Chaotic electronic implementation of Genesio-Tesi attractor were designed and simulated in MultiSIM. The system was implemented as an electronic circuit whose behavior confirms the numerical predictions.

Keywords: Genesio-Tesi system, Lyapunov spectrum, bifurcation diagram.

1. INTRODUCTION

Chaos is one of the more interesting recent developments in the area of system dynamics. In the past it was generally assumed that a system must follow either purely deterministic or purely probabilistic laws. A chaotic system is predictable in the short-term but unpredictable in the long-term due to their extremely sensitive dependence on initial conditions, topologically mixing. and also with dense periodic orbits [1-2].

Chaos behaviour have been discovered in physical [3], economics [4], biology [5], ecology [6], psychology [7], and chemical reaction [8]. In many application engineering and computer science such as robotic [9], random bits generator [10], image encryption [11], video encryption [12], speech encryption [13] and secure communication system [14-17].

In 1963, Edward Lorenz, a meteorologist, studied a simplified model for thermal convection numerically [18], In 1976, O.E. Rössler constructed several three dimensional quadratic autonomous chaotic systems, which also have seven terms on the right-hand side but with only one quadratic nonlinearity [19]. In 1986, Moore-Spiegel found a model the irregular variability in the luminosity of stars [20]. In 1992, Genesio proposed A harmonic balance methods for the analysis of chaotic dynamics in nonlinear systems [21], In 1994, J.C. Sprott suggests 19 cases of chaotic systems: case A-S with five linear terms and two nonlinear terms [22], In 2000, Malasoma proposed the simplest dissipative jerk equation that is parity invariant [23], In 2011, Sprott presented a new chaotic Jerk circuit which has frequency spectrum wide enough up to 10 KHz but the strength is decaying fast with increasing frequencies[24], In 2009, Sun et al. construced a simple Jerk system with piecewise exponential nonlinearity [25], In 2010, Sprott gives a 3-D jerk chaotic system having six terms on the R.H.S. with one hyperbolic sinusoidal nonlinearity [26]. In 2013, Pandey modifes the system of Jerk equations into a system of simple quadratic equations [27-28], In 2015,

[1] Department of Information Technology, Universiti Sultan Zainal Abidin, Kuala Terengganu, Malaysia, *Email: must@unisza.edu.my*

[2] Research and Development Centre, Vel Tech University, Avadi, Chennai, India, *Email: sundarvtu@gmail.com*

[3] Department of Physics, Universitas Islam Negeri Sunan Gunung Djati Bandung, Indonesia, *Email: madasws@gmail.com*

[4] School of Electrical Engineering and Informatics, Bandung Institute of Technology, Indonesia, *Email: rppprastio@gmail.com*

[5] Bolabot Techno Robotic Institute, Sanjaya Star Group Corp, Bandung, Indonesia, *Email: acenx.bts@gmail.com*

Vaidyanathan created a 3-D novel Jerk chaotic system with two hyperbolic sinusoidal nonlinearities [29]. Moreover, Vaidyanathan constructed a novel 4-D hyperchaotic hyperjerk system [30] and a six-term novel Jerk chaotic system with two exponential nonlinearities [31].

This paper proposes a Genesio-Tesi chaotic system with one quadratic-term. The complex dynamical behaviors of the system are furthere investigated by Lyapunov eksponen, eigenvalue structure and bifurcation diagram. The chaotic system examined in MATLAB 2010. The Oscillator circuit of the chaotic system is afterwards designed by using MultiSIM software and a typical chaotic attractor is experimentally demonstrated.

In this study, mathematical model Genesio-Tesi chaotic system is introduce in Section 2. The Oscillator circuit using MultiSIM software in Section 3. In Section 4, the oscillator circuit of the Genesio-Tesi chaotic system is designed and a chaotic attractor is implemented via an electronic circuit. Finally, some conclusion remarks are drawn in the last Section

2. GENESIO-TESI CHAOTIC SYSTEM

Genesio–Tesi chaotic system can be represented by following set of nonlinear differential equations [21, 32]:

$$\left.\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -cx - by - az + x^2 \end{aligned}\right\} \quad (1)$$

where x, y, z are state variables, and a, b and c are the positive real constants satisfying $ab < c$. The parameters and initial conditions of the Genesio-Tesi system (1) are chosen as: $a = 1$, $b = 3.03$, $c = 5.55$ and $(x_0, y_0, z_0) = (0.1, 0.1, 0.1)$, so that the system shows the expected chaotic behavior.

2.1. Equilibrium Analysis

The equilibrium points of (1) denote by $E\,(\bar{x}, \bar{y}, \bar{z})$, are the zeros of its non-linear algebraic system which can be written as:

$$\left.\begin{aligned} 0 &= y \\ 0 &= z \\ 0 &= -cx - by - az + x^2 \end{aligned}\right\} \quad (2)$$

The Jerk system has two equilibrium points E_0 (0, 0, 0) and E_1 (5.550, 0, 0). The dynamical behavior of equilibrium points can be studied by computing the eigenvalues of the Jacobian matrix J of system (1) where:

$$J(\bar{x}, \bar{y}, \bar{z}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5.55 + 2x & -3.03 & -1 \end{bmatrix} \quad (3)$$

For equilibrium points E_0 (0, 0, 0), the Jacobian becomes:

$$J(0,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5.55 & -3.03 & -1 \end{bmatrix} \quad (4)$$

The eigenvalues are obtained by solving the characteristic equation, det[$\lambda I - J_1$] = 0 which is: $\lambda^3 + \lambda^2 + 3.03 * \lambda + 5.55$

$$\lambda^3 + \lambda^2 + 3.03 * \lambda + 5.55 \tag{5}$$

Yielding eigenvalues of $\lambda_1 = -1.4821$ $\lambda_2 = 0.2410 + 1.9200i$, $\lambda_3 = 0.2410 - 1.9200i$ for $a = 1$, $b = 3.03$, $c = 5.55$. For equilibrium points (5.550, 0, 0), the Jacobian becomes:

$$J(5.550, 0, 0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5.550 & -3.03 & -1 \end{bmatrix} \tag{6}$$

The eigenvalues are obtained by solving the characteristic equation, det[$\lambda I - J_1$] = 0 which is:

$$\lambda^3 + \lambda^2 + 3.03 * \lambda - 5.550 \tag{7}$$

Yielding eigenvalues of $\lambda_1 = 1.0627$, $\lambda_2 = -1.0313 + 2.0392i$, $\lambda_3 = -1.0313 - 2.0392i$, for $a = 1$, $b = 3.03$, $c = 5.55$

The above eigenvalues show that the system has an unstable spiral behavior. In this case, the phenomenon of chaos is presented.

2.2. Numerical Simulations

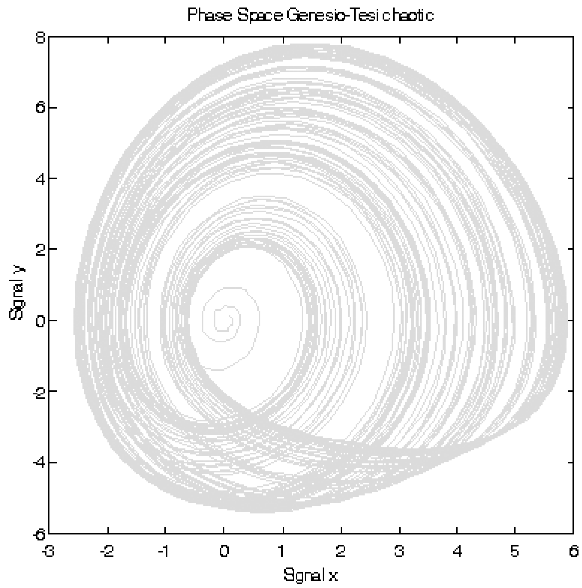
In this section, we present the numerical simulation to illustrate the dynamical behavior of Genesio-Tesi of system (1). For numerical simulation of chaotic system defined by a set of differential equation such as Genesio-Tesi system, different integration techniques can be used. In the MATLAB 2010 numerical simulation, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used. Figures 1(a)-(c) show the projections of the phase space orbit on to the *x*–*y* plane, the *y*–*z* plane and the *x*–*z* plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, Genesio-Tesi system presents chaotic attractors of Rössler type.

The Lyapunov exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories. The sensitive dependence on initial conditions of a dynamical system is characterized by the presence of a positive Lyapunov exponent. A positive Lyapunov exponent reflects a direction of stretching and folding and along with phase-space compactness indicates the presence of chaos in adynamical system.

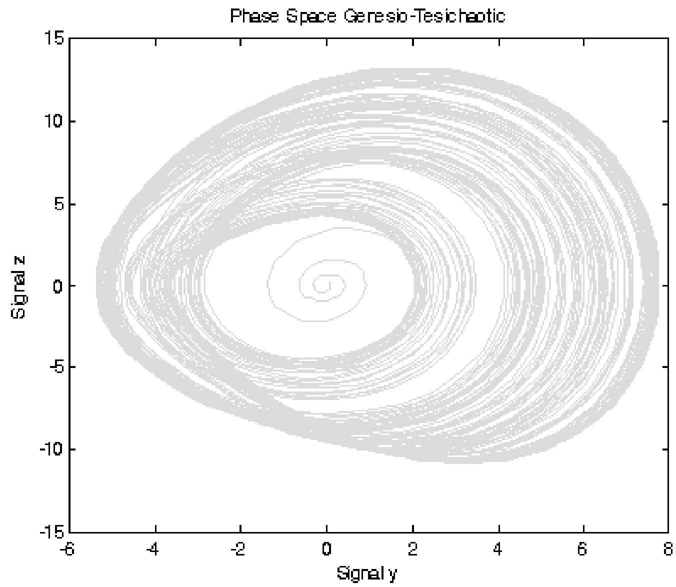
In a three dimensional system, like this, there has been three Lyapunov exponents (LE_1; LE_2; LE_3). In more details, for a3D continuous dissipative system the values of the Lyapunov exponents are useful for distinguishing among the various types of orbits. So, the possible spectra of attractors, of this class of dynamical systems, can be classied in four groups, based on Lyapunov exponents [33-36].

- (LE_1, LE_2, LE_3) → (–, –, –): a fixed point
- (LE_1, LE_2, LE_3) → (0, –, –): a limit point
- (LE_1, LE_2, LE_3) → (0, 0, –): a two-torus
- (LE_1, LE_2, LE_3) → (+, 0, –): a strange attractor (Figure.2).

So, in Figure 2 the dynamics of the proposed system's Lyapunov exponents for the variation of the parameter $c \in [4–6]$, is shown. For $5.30 \leq c \leq 6.00$ a strange attractor is displayed as the system has one positive Lyapunov exponent, while for values of $4.00 \leq c \leq 5.29$ is a transition to limit point behavior as the system has two negative Lyapunov exponents.



(a)



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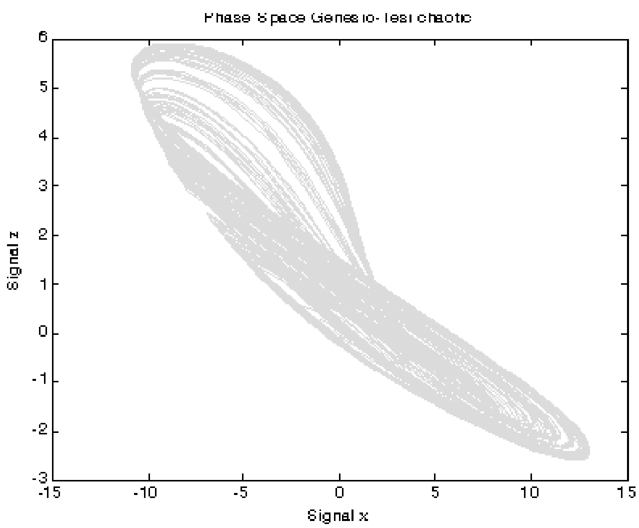


(c)

Figure 1: Numerical simulation results using MATLAB 2010, for a = 1, b = 3.03, c = 5.55, in (a) x-y plane, (b) y-z plane, (c) x-z plane.

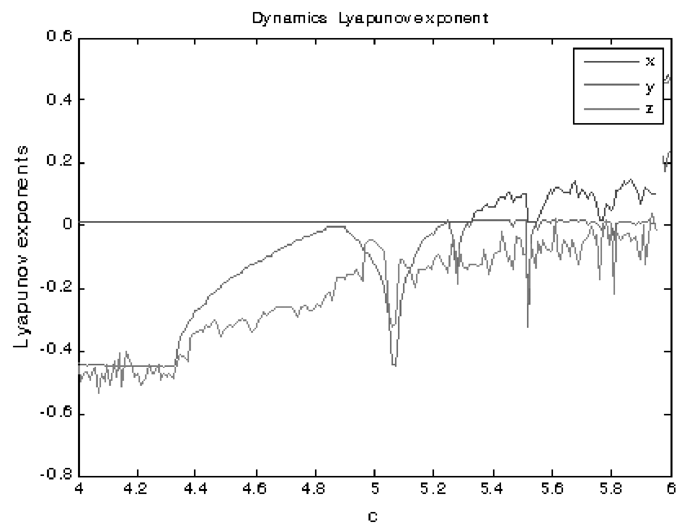


Figure 2: Nonlinear dynamics of system (1) for specific values set b=3.03, Lyapunov exponents versus the parameter control c[4–6], with MATLAB 2010

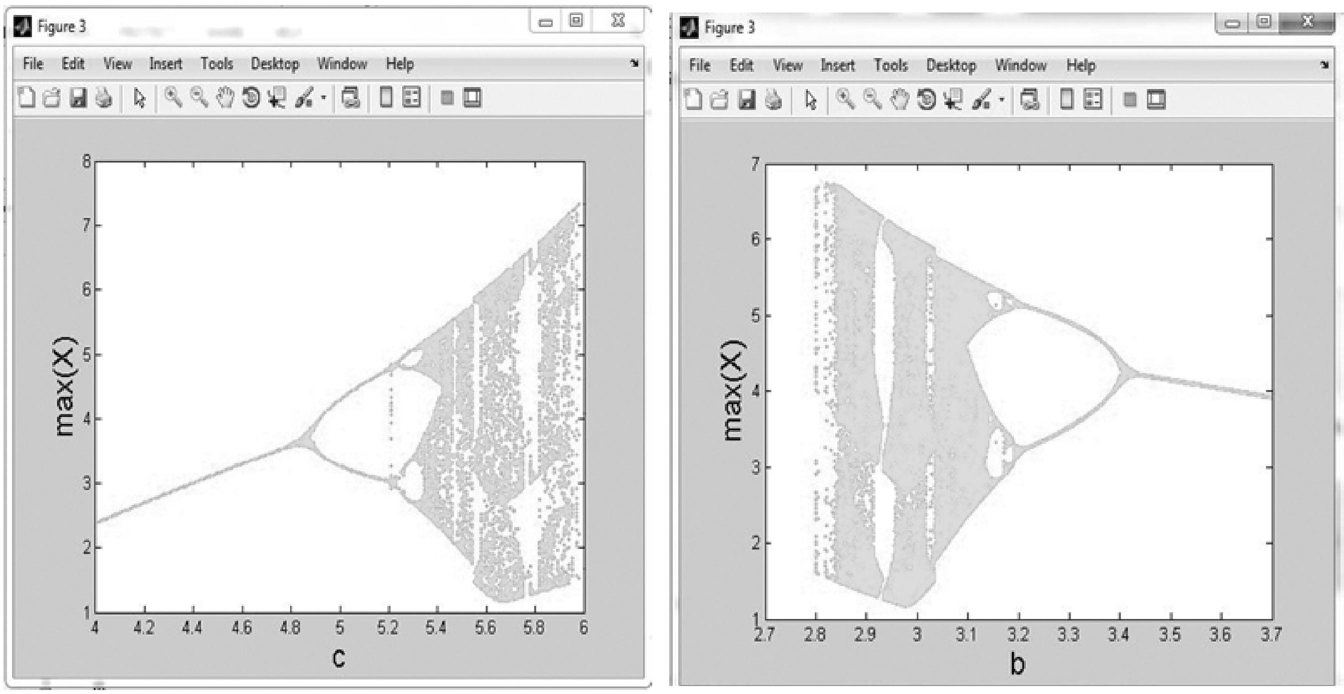
Also, the word bifurcation denotes a situation in which the solutions of a nonlinear system of differential equations alter their character with a change of a parameter on which the solutions depend. Bifurcation theory studies these changes (e.g. appearance and disappearance of the stationary points, dependence of their stability on the parameter etc.). A MATLAB program was written to obtain the bifurcation diagrams for Genesio-Tesi of Figure 3 (a) and (b).

3. ANALOG CIRCUIT SIMULATION USING MULTISIM

In this section, an electronic circuit is designed to realize the Genesio-Tesi attractor system. Switched Genesio-Tesi system Eq. (1) can be realized by the circuit of Figure 4, which consists of three channels to realize the integration, addition, and subtraction of the three state variables x, y, and z, respectively. The circuit employs simple electronic elements, such as resistors, capacitors, multiplier and operational amplifiers. In Figure 4, the voltages of C_1, C_2, C_3 are used as x, y and z, respectively. The nonlinear term of system (1) are implemented with the analog multiplier. The corresponding circuit equation can be described as:

$$\left.\begin{aligned}\dot{x} &= \frac{1}{C_1R_1}y\\ \dot{y} &= \frac{1}{C_2R_2}z\\ \dot{z} &= -\frac{1}{C_3R_3}x-\frac{1}{C_3R_4}y-\frac{1}{C_3R_5}z+\frac{1}{10C_3R_6}x^2\end{aligned}\right\} \quad (8)$$

We choose $R_1 = R_2 = R_5 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12}$ = 100 kΩ, R_3 = 18 kΩ, R_4 = 33 kΩ, R_6 = 10 kΩ $C_1 = C_2 = C_3$=1 nF. The circuit has three integrators (by using Op-amp TL082CD) in a feedback loop and a



(a) **(b)**

Figure 3: Nonlinear dynamics of system (1), (a) Bifurcation diagram of *x* vs. the control parameter *c* ε [4–6], (b) Bifurcation diagram of *x* vs. the control parameter *b* ε [2.7–3.7], with MATLAB 2010.

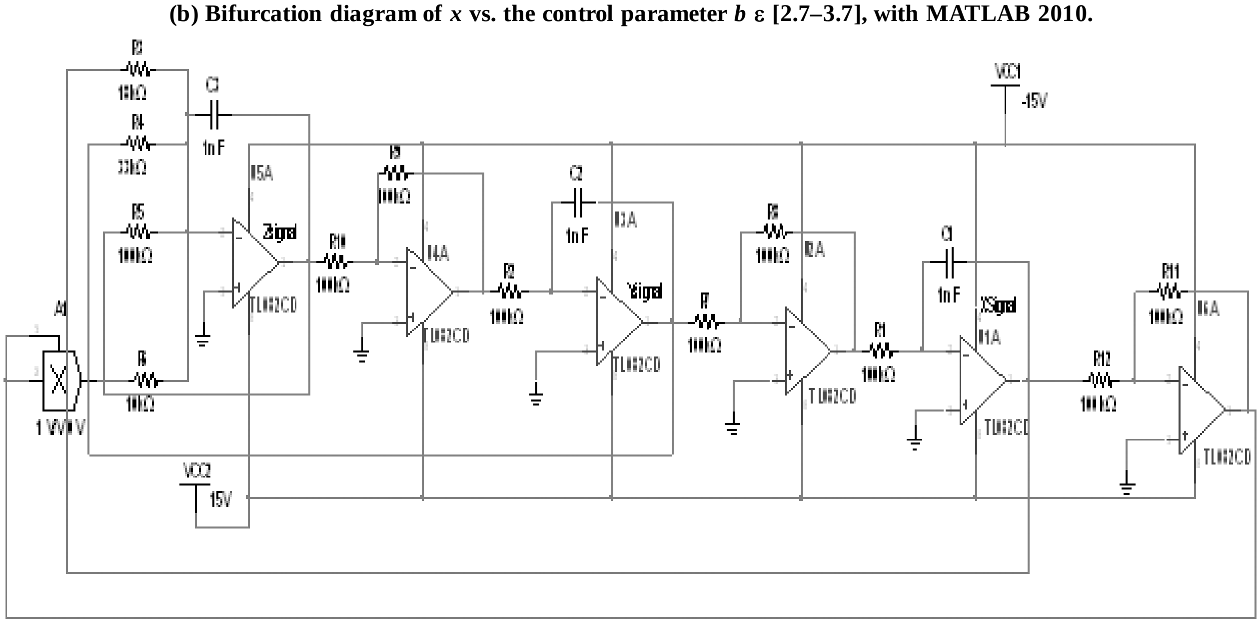
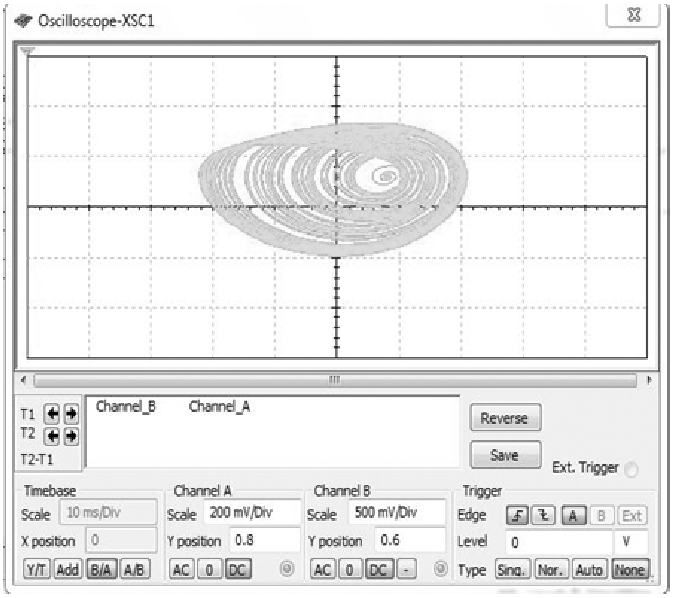
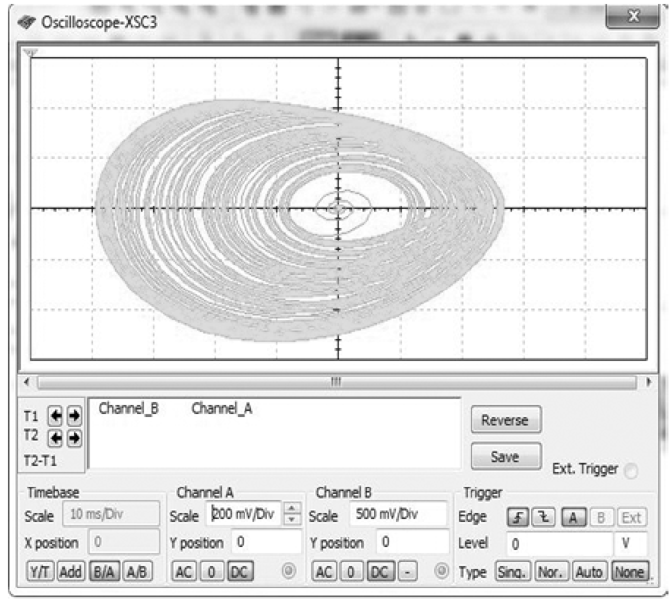


Figure 4: Schematic of the proposed Genesio-Tesi circuit by using MultiSIM 10.0.



(a)



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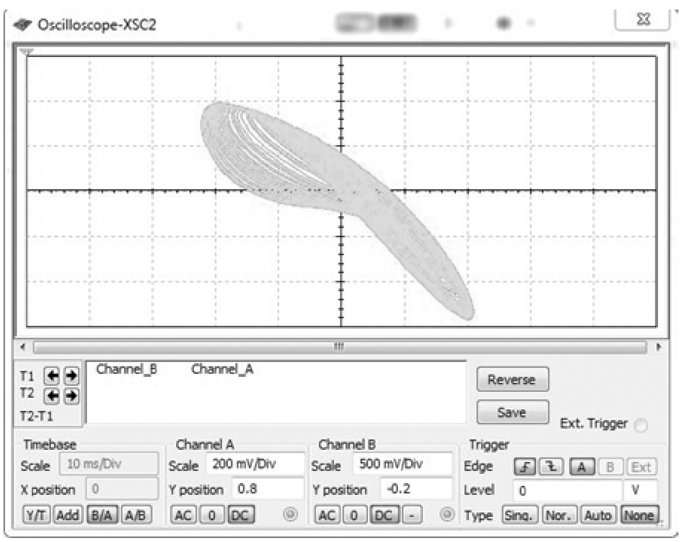


(c)

Figure 5: Various projections of the chaotic attractor using MultiSIM in (a) *x-y* plane, (b) *y-z* plane and (c) *x-z* plane.

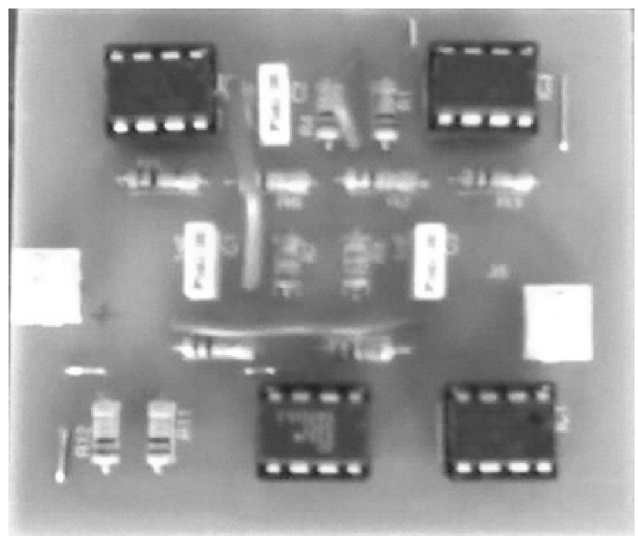
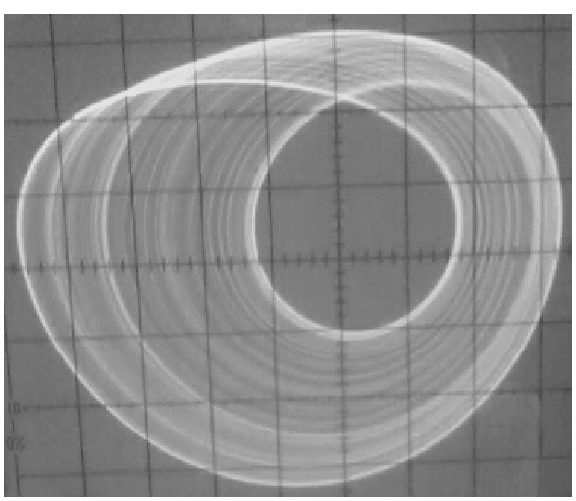
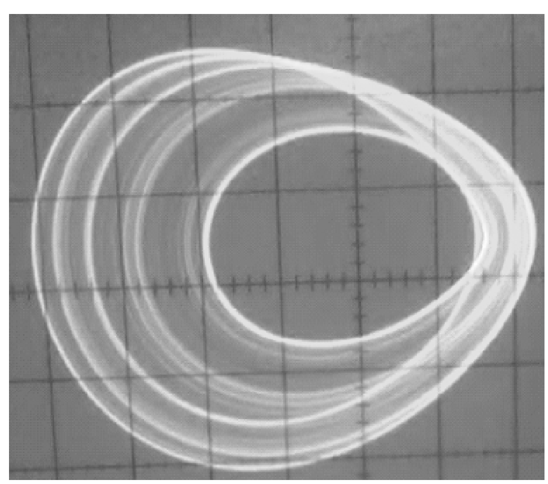


Figure 6 Electronic circuit realization for Genesio-Tesi circuit



(a)



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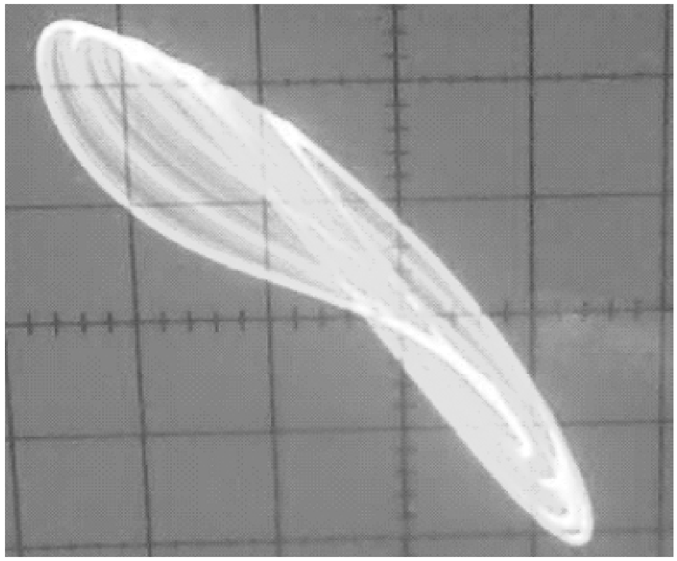

Figure 7: Experimental observations of the chaotic attractor in different planes (a) *x-y* plane (b) *y-z* plane (c) *x-z* plane

multiplier (IC AD633). The supplies of all active devices are ±15 V. With MultiSIM 10.0, we obtain the experimental observations of system (1) as shown in Figure 5. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Genesio-Tesi circuit is confirmed.

4. CIRCUIT REALIZATION OF THE GENESIO-TESI CIRCUIT

The chaotic dynamics of system (1) have also been realized by an electronic circuit. The designed circuitry realizing system (1) is shown in Figure 6. It consists of three channels to conduct the integration of the three state variable *x*, *y* and *z*, respectively. The operational amplifiers TL082CD and circuitry perform the basic operations of addition, subtraction and integration. The nonlinear term of system (1) are implemented with the analog multipliers AD633. We obtain the experimental observations of system (1) as shown in Figure 7. As compared with Figure 1 and Figure 5, a good qualitative agreement between the numerical simulation using MATLAB 2010, MultiSIM simulation and the experimental realization is confirmed.

5. CONCLUSIONS

In this work, we have mathematically constructed and electronically built the Genesio-Tesi system. The system has rich chaotic dynamics behaviors. The complex dynamics have also been explored in detail, including various periodic and chaotic motions, by means of Lyapunov exponent spectrum, eigen value structure and diagram bifurcation analysis. Moreover, it is implemented via a designed circuit with MultiSIM and tested experimentally in laboratory, showing very good agreement with the simulation result. Therefore, the proposed Genesio-Tesi system generating scheme may have a good application value in the field information technology such as secure communication and encryption

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