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THE SURFACE INSTABILITY OF A GRADED THIN FILM INDUCED BY INTERACTION WITH A RIGID CONTACTOR

Gan-Yun Huang and Shou-Wen Yu

ABSTRACT

The present work deals with the van der Waals interaction between a graded film and a rigid contactor and the resultant surface instability the film may undergo. Using integral approach, the interaction between the graded film and the contactor (treated as a homogeneous half space) has been obtained explicitly by assuming the exponential dependence of the compositions on the depth. It has been shown that when the distance between the contactor and the film is small as compared to the thickness of the film, the van der Waals interaction is weakly dependent on the gradient of the compositions. The stability of the graded film has then been investigated by linear stability analysis. Numerical results indicate that gradient of the film's elastic properties only influences the critical loads for the onset of bifurcation modes. The effect of the film's elastic anisotropy has also been illustrated. The results may have implications for the control of the patterns of surface morphology.

Keywords: film, stability, van der Walls, interaction

1. INTRODUCTION

Graded materials, with the volume fractions of the constituents varying spatially have been a useful concept in many areas. As shown by Suresh *et al.* [28], if the elastic properties at the contact surfaces were tailored appropriately, they can be used as protective coatings to enhance contact-damage resistance. Another paradigm for the concept is the fabrication of hetero-structures in semiconductor industry where graded thin film layers have been introduced to suppress the development of detrimental surface roughness (Samavedam and Fitzgerald, [25]; Yoon *et al.* [36]. Consequently research on the effect of gradation in composites has been intensive but mainly focused oncharacterization of their mechanical properties such as description of elastic moduli (for example, Zuiker, [37] and Pindera and Dunn, [22], fracture analysis (see Erdogan and Delale, [5]; Erdogan, [9]; Gu and Asaro, [12]; Choi [3],; Rousseau and Tippur [24]; Dolbow and Gosz, [6]; Walters *et al.* [32] and contact mechanics analysis as done by Linss *et al.* [21]; Guler and Erdogan [13]; El-Borgi *et al.* [8] and Ke and Wang [20]. However, from the perspective of applications whether in coating systems or fabrication of hetero-structures, it may be significant to study surface morphology development and evolution in graded materials involved systems, since it has long been recognized that the surface morphology of coatings influences their tribological performance and failure behavior whilst graded buffer layers has been found to produce certain cross hatches that may be controlled through optimized design (Radu *et al.*, [23] and Yoon *et al.*, [36]. The surface morphology of homogeneous films has been well studied due to their fundamental significance in many technological applications, for example, in adhesion and friction, and in the growth of self-assembled nanostructures. According to the extensive experiments and theoretical simulations, the study of surface morphology can be reduced to stability problems and the various surface morphologies can be triggered or modified by many factors such as electric field (Du and Srolovitz, [7]; Huang, [14] and Chiu *et al.*, [2], applied mechanical force (Huang and Suo, [15]; Decuzzi and Demelio [4] and Trofimov [30], van der Waals force (Sarkar *et al.* [26]; Yoon *et al.* [36] and Gonuguntla *et al.* [11] and lattice mismatch (Jonsdottir and Freund, [17]). Since the graded materials are frequently used as coatings to their advantages in which context, the van der Waals

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interaction between the graded materials and contactors may be important, the present work, as a preliminary step, is to consider stability of a graded film due to such surface force. The effect of van der Waals force on the surface instability of liquid films has been well documented (see Kao *et al.* [18]; Tomar *et al.* [29] and Buxton and Clarke [1]). And for homogenous elastic solid films as analyzed by Shenoy and Sharma [27] and many others mentioned above, such interaction can also induce surface instability, especially for soft films. Thus the present work will focus on the effect of gradient in the films. Attention will also be paid to the effect of anisotropy.

Briefly, to consider the surface instability of graded films induced by van der Waals force, the present paper is organized as follows. The van der Waals interaction between a graded medium and a rigid half space will be derived in Section 2. In section 3, the surface instability will be analyzed and numerical results will be given in section 4.

2. VAN DER WAALS INTERACTION BETWEEN A HOMOGENEOUS HALF PLANE AND A GRADED MEDIUM

To study the problem as shown in Figure 1 where a graded film is perfectly bonded to a rigid substrate and interacts with a rigid contactor, we would like first to find out the van der Waal interaction between the contactor and the graded film. The contactor will be considered as a homogeneous half space and the contribution of the rigid substrate will be neglected. The presence of the gradation of constituents in the film may render the van der Waal interaction with other medium different from the homogeneous materials. In fact, the van der Waal interaction between nonhomogeneous media has been the focus of several recent studies, see for example, Kaya [19], van Benthem *et al.* [31]) and Genchev [10]. Here we would like to consider the volume of constituents vary along the thickness direction in a specific functional form, say, exponential function as frequently used to describe the modulus in functionally graded materials by Erdogan and Delale [5] and many other authors (e.g., Choi, [3]). More specifically, we assume the graded material is composed of two phases with each phase represented by one type of atom. The number density of the total atoms, i.e., the number of particles in a unit volume, is regarded as constant. Thus we can write the number density for the two types of atoms as

$$\rho_1 = \rho_1' + \rho_{10} \exp(\beta y) \; \rho_2 = \rho_0 - \rho_1, \tag{2.1}$$

with ρ_0 being the total particle number density, ρ'_1 and ρ_{10} being constants that are related to the number density of phase 1 and β a parameter characterizing the gradation of phase 1.

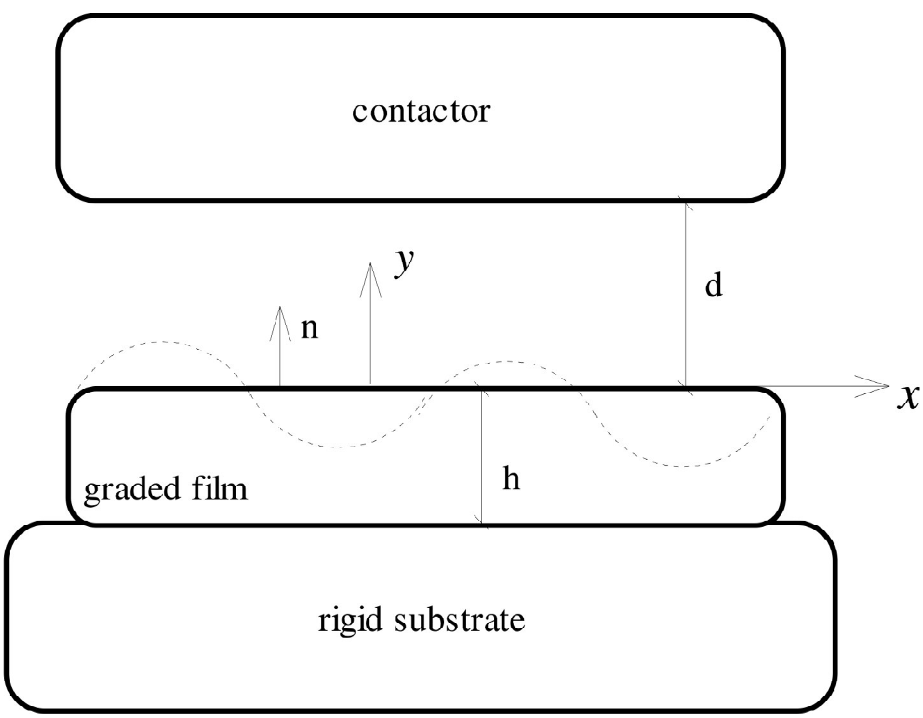

Figure 1: Schematic Illustration of a Graded film Bonded to a Rigid Substrate and in Proximity to a Contactor

If the van der Waals interaction between the atoms in the graded material and those in the homogeneous space can be described by the attractive Lennard-Jones potential

$$V_j = C_j / r^6 \quad j = 1, 2, \tag{2.2}$$

then the interaction between the graded material of thickness h and the homogeneous half space that are separated by d can be obtained through integral approach

$$\begin{aligned}
\Pi_{vdw} &= \int_{-h}^{0} dy \int_{d}^{\infty} dy^* \int_{0}^{\infty} \frac{2\pi r \rho_h \rho_1(y) C_1}{[r^2 + (y^* - y)^2]^3} dr + \int_{-h}^{0} dy \int_{d}^{\infty} dy^* \int_{0}^{\infty} \frac{2\pi r \rho_h \rho_2(y)] C_2}{[r^2 + (y^* - y)^2]^3} dr \\
&= \int_{-h}^{0} dy \int_{d}^{\infty} dy^* \int_{0}^{\infty} \frac{2\pi r \rho_h \rho_1' C_1}{[r^2 + (y^* - y)^2]^3} dr + \int_{-h}^{0} dy \int_{d}^{\infty} dy^* \int_{0}^{\infty} \frac{2\pi r \rho_h (\rho_0 - \rho_1') C_2}{[r^2 + (y^* - y)^2]^3} dr \\
&\quad + \int_{-h}^{0} dy \int_{d}^{\infty} dy^* \int_{0}^{\infty} \frac{2\pi r \rho_h \rho_{10} \exp(\beta y)(C_1 - C_2)}{[r^2 + (y^* - y)^2]^3} dr \\
&= \frac{\pi H_1}{12 d^2} + \frac{\pi H_2}{12 d^2} + \frac{\pi(\hat{H}_1 - \hat{H}_2)}{12 d^2} f(d, h, \beta)
\end{aligned} \tag{2.3}$$

where ρ_h is the number density of particles in the homogeneous half space and $H_1 = \rho'_1 \rho_h C_1$, $H_2 = (\rho_0 - \rho'_1) \rho_h C_2$, $\hat{H}_1 = \rho_{10}\rho_h C_1$, $\hat{H}_2 = \rho_{10}\rho_h C_2$ can be regarded as Hamaker constants, and $f(d,h,\beta) = 2(\beta d)^2 \exp(\beta d)[\Gamma(-2,\beta d) - \Gamma(-2,\beta(d+h))]$ with $\Gamma(a, z)$ being the incomplete Gamma function that reads $\Gamma(a,z) = \int_z^{\infty} t^{a-1} \exp(-t) dt$.

It can be seen from Eq. (3) that the van der Waals interaction between a homogenous half space and a graded medium can be considered as that due to two homogenous materials superimposed by a perturbed one that depends on the gradation. Specially, when $h \to \infty$, one may further find that $f(d,h,\beta) \to 1 - \beta d + (\beta d)^2 \exp(\beta d) \Gamma(0, \beta d)$. In Figure 2, the variation of $f(d, h, \beta)$ with βh has been graphically shown for different values of d / h. From the figure, one can find when d / h is small, $f(d, h, \beta)$ is very close to 1 as far as the range of βh in the graph is concerned. That is, the van der Waal interaction is almost independent of the gradient of compositions in the film. Since the van der Waals interaction generally plays a role when distance d is in the range of tens of nanometers, we may use $f(d, h, \beta) \approx 1$ for the graded film's thickness in the range of microns. Thus, equation (3) can be reduced to

$$\Pi_{vdw} \approx \frac{\pi H_e}{12 d^2}, \tag{2.4}$$

with $H_e = \rho_h \rho_0 [\nu_{10} C_1 + (1 - \nu_{10}) C_2]$ and ν_{10} is the fraction of particle 1 at $y = 0$. It should be noted that if d is comparable to h, the above approximation is no longer valid.

3. STABILITY ANALYSIS OF A GRADED FILM INTERACTING WITH A RIGID CONTACTOR

In this section, we will consider the stability of a graded film due to the interaction with a rigid contactor that is dominated by van der Waals potential. As mentioned previously, similar analysis has been conducted on the homogeneous film by Shenoy and Sharma (2002). Here we would like to focus on the effect of gradation in a graded film.

3.1. Basic Assumptions and Equations

Due to the gradation of the constituents in such a film, the physical properties in general also vary spatially. Although one may approximate the physical properties by multilayered medium in a piecewise homogeneous manner as done

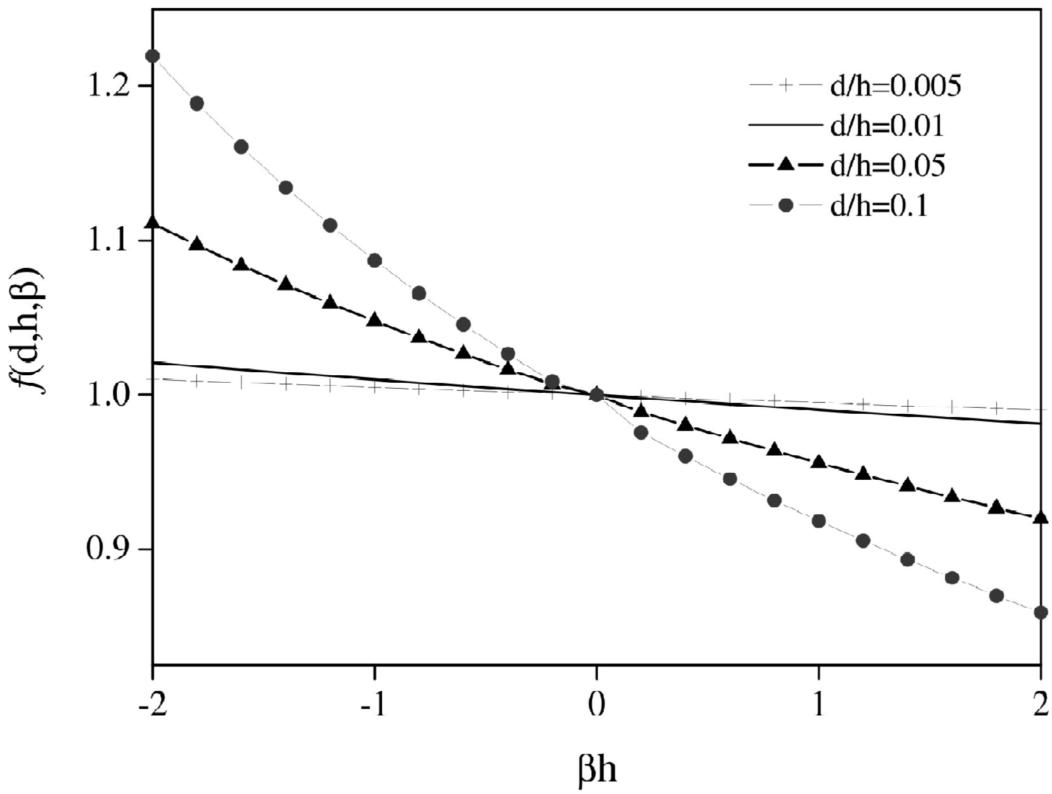

Figure 2: Graphical Illustration of $f(d, h, \beta)$ as a Function of βh for Different Values of d/h

by Wang et al (2000) or in a linear manner by Wang and Gross (2000), as a preliminary investigation on the effect of the gradients of material properties, the exponential functions will be yielded to describe the properties if equation (1) and the mixture rule like $Q_{eff} = Q_1 \nu_1 + Q_2 (1 - \nu_1)$ are taken into account. To make the subsequent analysis tractable, the relevant material properties are assumed to have the following forms

$$c_{11}(y) = c_{110} \exp(\beta y), \ \ c_{12}(y) = c_{120} \exp(\beta y), \tag{3.1a}$$

$$c_{22}(y) = c_{220} \exp(\beta y), \ c_{44}(y) = c_{440} \exp(\beta y), \tag{3.1b}$$

if the graded film is orthotropic. The prefactors in the above expressions are the moduli at $y = 0$. The stress and strain can be correlated through the moduli as

$$\sigma_{11} = c_{11}\varepsilon_{11} + c_{12}\varepsilon_{22}, \ \sigma_{22} = c_{12}\,\varepsilon_{11} + c_{22}\varepsilon_{22}, \ \sigma_{12} = \sigma_{21} = 2c_{44}\varepsilon_{12}, \tag{3.2}$$

with $\varepsilon_{jl} = 1/2(u_{l,j} + u_{j,l}), \ (l, j = 1,2)$.

For the system shown in Figure 1, one may put the total potential as

$$\Pi = \int_V W(\boldsymbol{\varepsilon})dV + \int_S \gamma\sqrt{1+u_{3,y}^2}\,dS - \int_S \Pi_{vdw}(\mathbf{u}\cdot\mathbf{n})dS \tag{3.3}$$

in which $W(\varepsilon)$ is the bulk strain energy density that relates stress components to strain as $\sigma_{lj} = \dfrac{\partial W(\varepsilon)}{\partial \varepsilon_{lj}}$, γ is the surface energy and the van der Waals potential is dependent on the displacement of the graded film such that

$$\Pi_{vdw}(\mathbf{u}\cdot\mathbf{n}) = \frac{\pi H_e}{12(d - \mathbf{u}\cdot\mathbf{n})^2}, \tag{3.4}$$

with $\mathbf{n}$ being the direction normal to the surface of the graded film. If infinitesimal deformation and $d >> \|u\|$ are assumed, then as approximations, one may make use of the following expressions

$$\sqrt{1+u_{3,y}^2} \approx 1+u_{3,y}^2/2, \tag{3.5}$$

$$\Pi_{vdw}(\mathbf{u}\cdot\mathbf{n}) \approx \Pi_0 + F\mathbf{u}\cdot\mathbf{n} + \frac{1}{2}Y(\mathbf{u}\cdot\mathbf{n})^2, \tag{3.6}$$

$$\Pi_0 = \Pi_{vdw}(0),\ \ F = \Pi'_{vdw}(0),\ \ Y = \Pi''_{vdw}(0), \tag{3.7}$$

The prime in the above equations denotes partial differential with respect to $\mathbf{u}\cdot\mathbf{n}$. With the approximations, we may obtain the related equations for the present problem by setting the variational $\delta\Pi = 0$ and they read

$$\nabla\cdot\boldsymbol{\sigma} = 0, \tag{3.8}$$

$$\boldsymbol{\sigma}\cdot\mathbf{n} = \gamma u_{2,xx}\mathbf{n} + F\mathbf{n} + Y(\mathbf{u}\cdot\mathbf{n})\mathbf{n} \ \text{ at } y = 0, \tag{3.9}$$

$$\mathbf{u}(x, -h) = \mathbf{0} \ \text{ at } y = -h. \tag{3.10}$$

3.2. Linear Stability Analysis

The present stability problem formulated by equations (3.8)-(3.10) can be solved by linear stability analysis in which the solution is the superposition of a homogenous one and a perturbed one. Since the homogenous solution in essence is one dimensional, it can be easily sought as

$$u_1^h = 0,\ \ u_2^h = \frac{F}{Y-Y_m}\frac{\exp(-\beta y)-\exp(\beta h)}{\exp(\beta h)-1}, \tag{3.11}$$

with $Y_m = c_{220}\,\beta/(\exp(\beta h) - 1)$. It should be noted that Y is smaller than Y_m, or else approximations in equations (9) and (10) will be invalid. For the perturbed solution, it can be found that it should satisfy the equilibrium equation (3.8) together with the following boundary conditions

$$\boldsymbol{\sigma}^b\cdot\mathbf{n} = \gamma u_{2,xx}^b\mathbf{n} + Y(\mathbf{u}^b\cdot\mathbf{n})\mathbf{n} \ \text{ at } y = 0, \tag{3.12}$$

at

$$y = -h. \tag{3.13}$$

Assuming the displacement components have the following forms

$$u_1^b(x,y) = f_1(y)\exp(ikx),\ \ u_2^b(x,y) = f_2(y)\exp(ikx), \tag{3.14}$$

with k being the perturbed wave number and $i=\sqrt{-1}$, we can obtain the general solutions for f_1 and f_2 from the equilibrium equations (3.7) as

$$\{f_1, f_2\}^{\mathrm{T}} = \{X_j, 1\}^{\mathrm{T}} A_j \exp(\lambda_j y),\ j = 1, 2, 3, 4, \tag{3.15}$$

in which

$$\lambda_j = -\frac{\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 2(\eta \pm \sqrt{\eta^2 - 4c_{220}c_{440}^2(k^4c_{110} + k^2\beta^2c_{120})})/c_{220}c_{440}},$$

$\eta = k^2c_{120}^2 - k^2c_{110}c_{220} + 2k^2c_{120}c_{440}$ $X_j = -(c_{220}\lambda_j^2 + c_{220}\lambda_j\beta - c_{440}k^2)/((c_{120}+c_{440})ik\lambda_j + c_{120}\beta ik)$ and A_j are constants to be determined.

Substituting equation (3.15) into (3.14) and then into the boundary conditions (3.12) and (3.13), one has a set of linear equations with respect to A_j as

$$X_j \exp(-\lambda_j h)\, A_j = 0 \tag{3.16}$$

$$\exp(-\lambda_j h)\, A_j = 0, \tag{3.17}$$

$$(X_j\lambda_j + ik)\, A_j = 0, \tag{3.18}$$

$$\alpha_j A_j = 0, \tag{3.19}$$

and $\alpha_j = (ikc_{120}X_j + c_{220}\lambda_j + \gamma k^2 - Y)$. Since A_j should be nontrivial, it is required that

$$\begin{vmatrix} X_1\exp(-\lambda_1 h) & X_2\exp(-\lambda_2 h) & X_3\exp(-\lambda_3 h) & X_4\exp(-\lambda_4 h) \\ \exp(-\lambda_1 h) & \exp(-\lambda_2 h) & \exp(-\lambda_3 h) & \exp(-\lambda_4 h) \\ X_1\lambda_1 + ik & X_2\lambda_2 + ik & X_3\lambda_3 + ik & X_4\lambda_4 + ik \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{vmatrix} = 0, \tag{3.20}$$

from which the relationship between the van der Waals force related parameter Y and wave number k can be obtained as

$$Y = \frac{D_1}{D_2 - D_3 + D_4 - D_5}, \tag{3.21}$$

with $D_1 = \tilde{\alpha}_1 D_2 - \tilde{\alpha}_2 D_3 + \tilde{\alpha}_3 D_4 - \tilde{\alpha}_4 D_5$, $\tilde{\alpha}_j = ikc_{120}X_j + c_{220}\lambda_j + \gamma k^2$, and

$$D_2 = \begin{vmatrix} X_2\exp(-\lambda_2 h) & X_3\exp(-\lambda_3 h) & X_4\exp(-\lambda_4 h) \\ \exp(-\lambda_2 h) & \exp(-\lambda_3 h & \exp(-\lambda_4 h \\ X_2\lambda_2 + ik & X_3\lambda_3 + ik & X_4\lambda_4 + ik \end{vmatrix},$$

$$D_3 = -\begin{vmatrix} X_1\exp(-\lambda_1 h) & X_3\exp(-\lambda_3 h) & X_4\exp(-\lambda_4 h) \\ \exp(-\lambda_1 h) & \exp(-\lambda_3 h) & \exp(-\lambda_4 h) \\ X_1\lambda_1 + ik & X_3\lambda_3 + ik & X_4\lambda_4 + ik \end{vmatrix},$$

$$D_4 = -\begin{vmatrix} X_1\exp(-\lambda_1 h) & X_2\exp(-\lambda_2 h) & X_4\exp(-\lambda_4 h) \\ \exp(-\lambda_1 h) & \exp(-\lambda_2 h) & \exp(-\lambda_4 h) \\ X_1\lambda_1 + ik & X_2\lambda_2 + ik & X_4\lambda_4 + ik \end{vmatrix},$$

$$D_5 = -\begin{vmatrix} X_1\exp(-\lambda_1 h) & X_2\exp(-\lambda_2 h) & X_3\exp(-\lambda_3 h) \\ \exp(-\lambda_1 h) & \exp(-\lambda_2 h) & \exp(-\lambda_3 h) \\ X_1\lambda_1 + ik & X_2\lambda_2 + ik & X_3\lambda_3 + ik \end{vmatrix}.$$

From equation (3.20), one may expect the relationship between k and Y will be influenced by the gradient parameter β. It is worth pointing out that Y in equation (3.21) only represents the loads required for the instability to occur and is different from that in equation (3.7). Generally, one can determine the occurrence of instability if the value calculated by equation (3.7) is larger than that by equation (3.21).

4. NUMERICAL RESULTS AND DISCUSSION

In this section, we will attempt to illustrate quantitatively the effect of gradation on the stability of a graded film. Attention will also be paid to the anisotropy of the film. During all the calculation, the surface energy γ will be set to zero. Since the expressions in section 3 can be readily used for an isotropic graded film, we would like to consider the effect of gradient first. In this case, $c_{11} - c_{12} = c_{22} - c_{12} = 2c_{44}$ and one can use shear modulus μ and Poisson's ratio ν as the basic elastic parameters. By taking $\nu = 0.4$ and 0.25 with $\beta h = -0.005$ and 0.005 respectively, the wave number k (normalized by $1/h$) as a function of Y(normalized by μ_0 / h with μ_0 being the shear modulus at $y = 0$) has been calculated and results are presented in Figure 3 by discrete symbols. The results for a homogeneous film with the same Poisson's ratio are also given in the figure by solid lines. It can be seen that the results for a graded film are identical to those for a homogeneous one, as one can expect since the selected values of the gradient parameter are very small. It is worth mentioning that the results for the homogeneous film are almost the same as

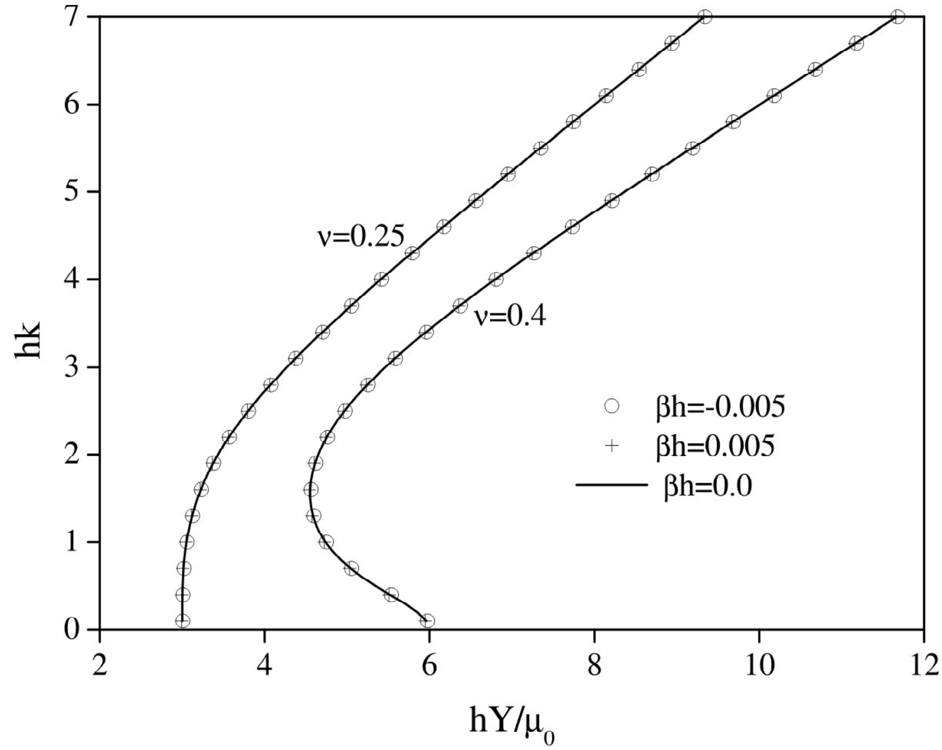

Figure 3: Bifurcation Modes k as a Function of Y in an Isotropic Graded film for Selected Values of Values of βh and Poisson's Ratio

those obtained by Shenoy and Sharma [27] and Huang *et al.* [14] for similar problems. So we take respectively, $\beta h = -1.0$, -0.5, 0.5 and 1.0 with Poisson's ratio being 0.4 and again calculated the wave number k as a function of Y. The results are plotted in Figure 4. From the figure it can be found that for different values of βh, the critical loads Y_c at which bifurcation modes begin to occur are different decrease with the increase of βh. That is, graded films with larger values of βh would bifurcate more easily. It is reasonable because larger value of βh means that the graded film is softer as a whole given the coordinate system used in the present work. It should be pointed out that in a homogeneous film the occurrence of bifurcation mode is mainly governed by Poisson's ratio, as is shown by Shenoy and Sharma [27]. The similar conclusion holds true for a graded film as can be seen from Figure 5 where wave number k as a function of Y is demonstrated for selected values of Poisson's ratio and $\beta h = 1.0$. So the gradient in the nonhomogeneous film only influences the magnitude of the critical Y_c. Due to the difficulty in obtaining a simple analytical expression for Y_c that can be calculated by solving $dY / dk = 0$, we have numerically computed the dependence of Y_c on βh for $\nu = 0.4$. The results are given in Figure 6. Numerical values of Y_m as a function of βh have also been given in the figure by dash line. Evidently the condition for the approximations (3.5) and (3.6) to be valid is satisfied since Y_c is less than Y_m. If H_e is known, one may use equations (3.4) and (3.7) to find the real Y of the graded film and thus can determine whether bifurcation mode occurs or not. According to equation (8), Y should be independent of βh. Consequently, the graded film can be designed to avoid or cater for the bifurcation instability which depends on the specific background of usage.

Now we come to find out the effect of anisotropy of the film. Since in an isotropic film, the occurrence of bifurcation modes is determined by Poisson's ratio, one may expect c_{120} / c_{110} will govern the existence of bifurcation modes in an orthotropic film. To check this point, we take $\beta h = 1.0$ and $c_{110} / c_{440} = c_{220} / c_{440} = 5.0$, and calculate k as a function of Y (normalized by c_{440} / h in this case) for different values of c_{120}. Results are presented in Figure 7 which shows that only the value of c_{120} amounts to a certain critical value does the bifurcation mode occur. Figure 8 demonstrates the wave number k as a function of Y for $c_{120} / c_{440} = 3.0$, $\beta h = 1.0$, $c_{110} / c_{440} = 5.0$ and different values of c_{220} / c_{440}. It can be seen that for selected values of the parameters, the critical load Y_c for bifurcation mode increases with c_{220} / c_{440}, which again can be understood that larger value of c_{220} / c_{440} means stiffer film as whole. Just as has been done for the isotropic case, we have also calculated the critical Y_c as a function of βh and c_{220} for $c_{120} / c_{440} = 4.0$ and $c_{110} / c_{440} = 6.0$. Results are plotted in Figure 9. It can be seen that dependence of Y_c on βh behaves

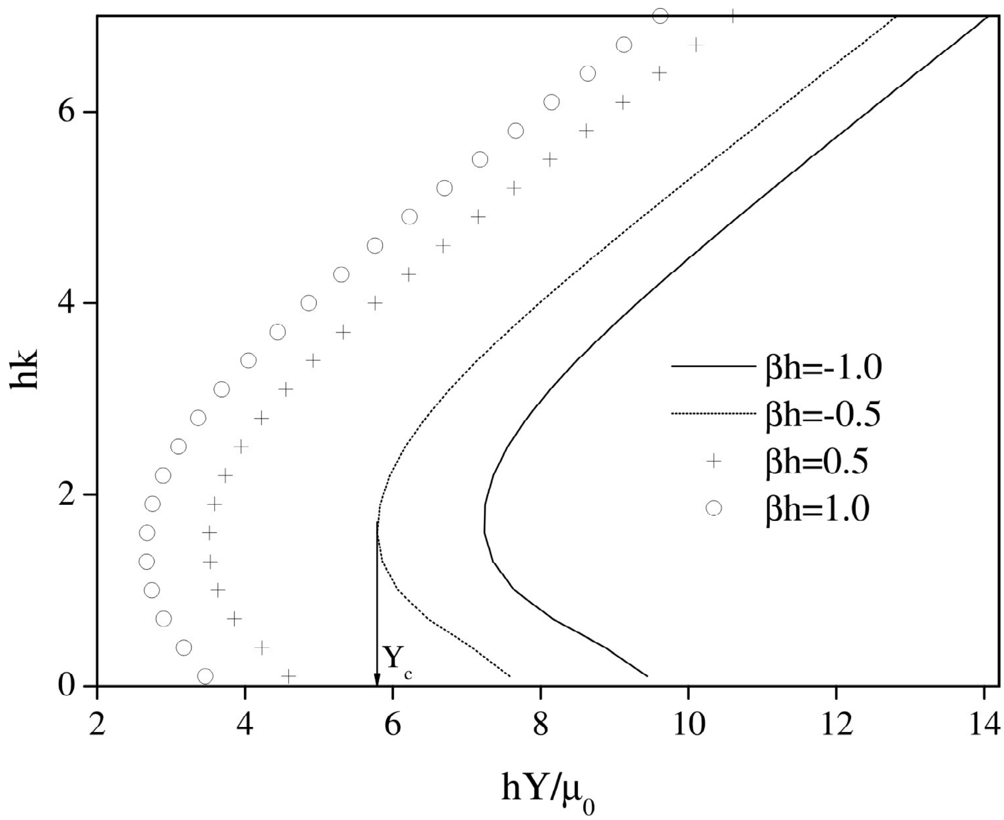

Figure 4: Bifurcation Modes *k* as a function of *Y* in an Isotropic Graded film for Selected Values of βh with Poisson's Ratio ν = 0.4

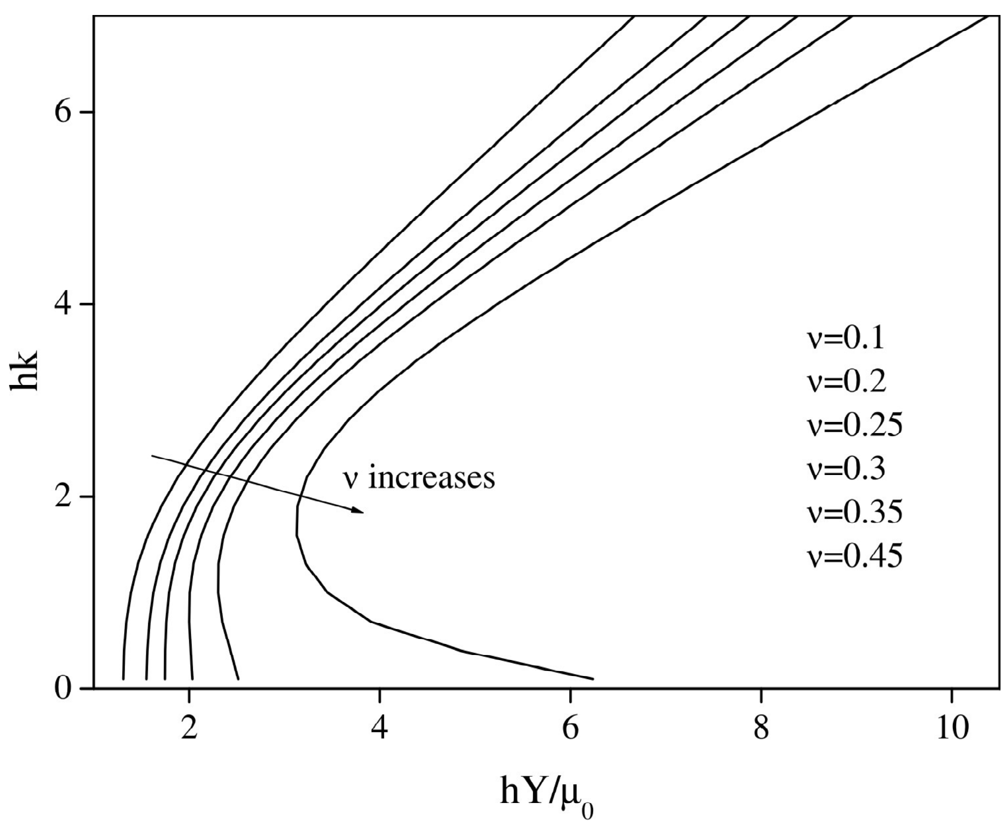

Figure 5: Bifurcation Modes *k* as a Function of *Y* in an Isotropic Graded film for Selected Values of Poisson's Ratio with β = 1.0

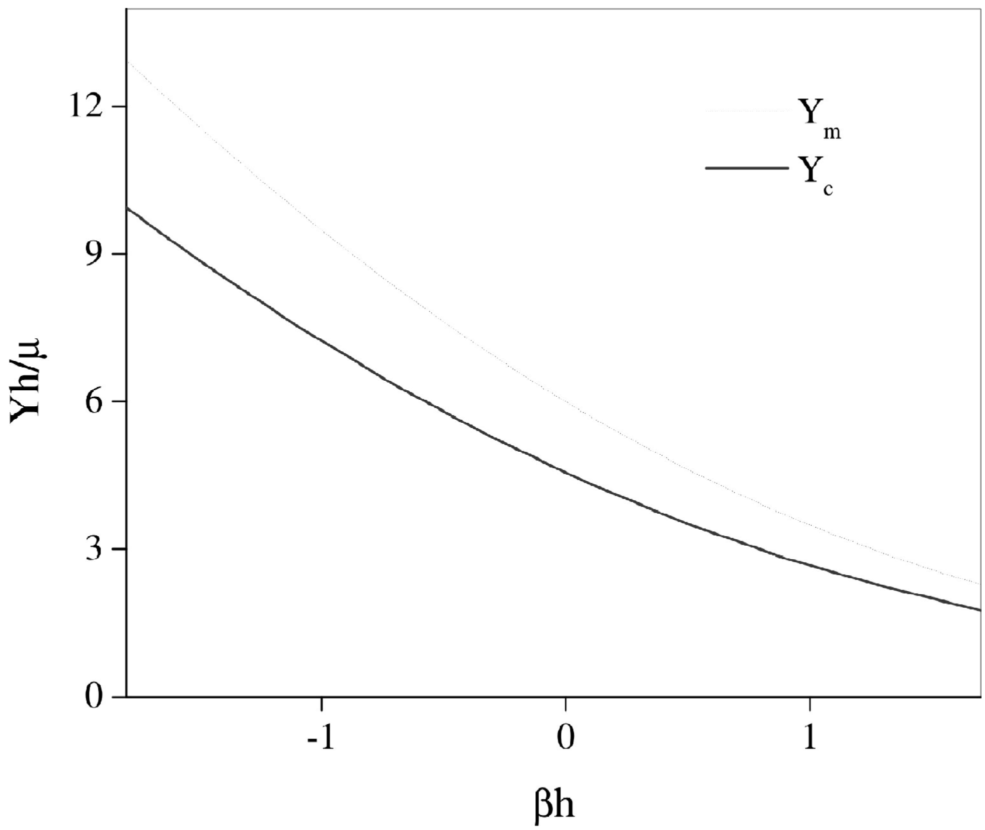

Figure 6: Y_c and Y_m as a Function of βh in an Isotropic Graded Film with Poisson's Ratio $\nu = 0.4$

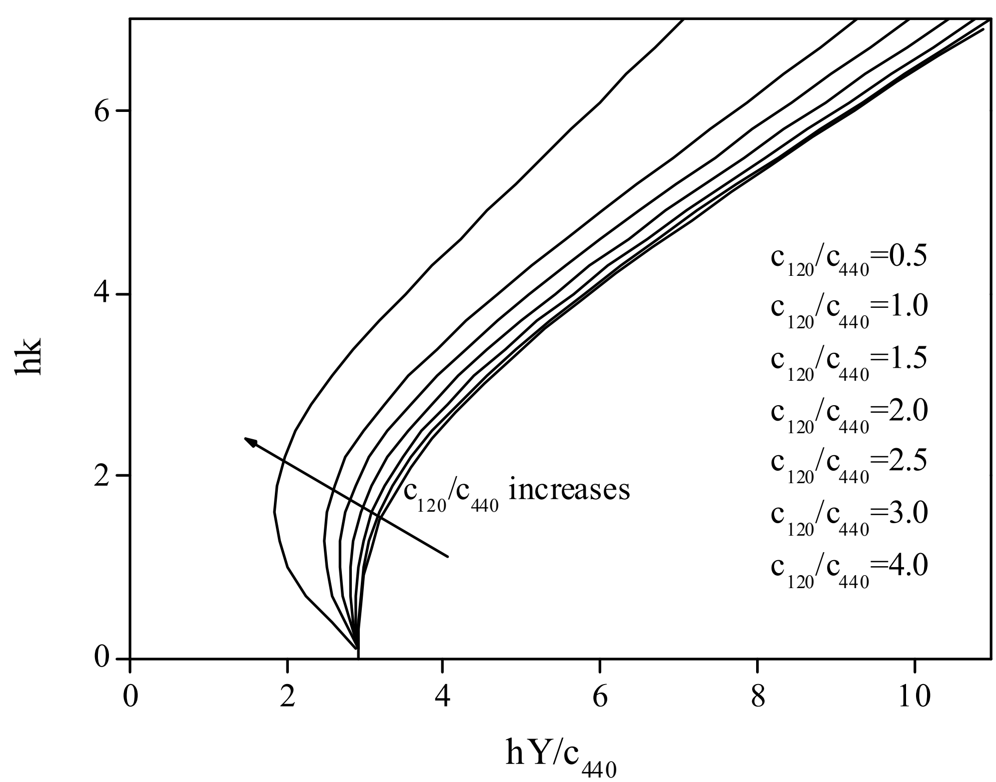

Figure 7: Bifurcation Modes k as a Function of Y in an Orthotropic Graded film for Selected Values of c_{120} / c_{440} with $c_{110} / c_{440} = c_{220} / c_{440} = 5.0$ and $\beta h = 1.0$

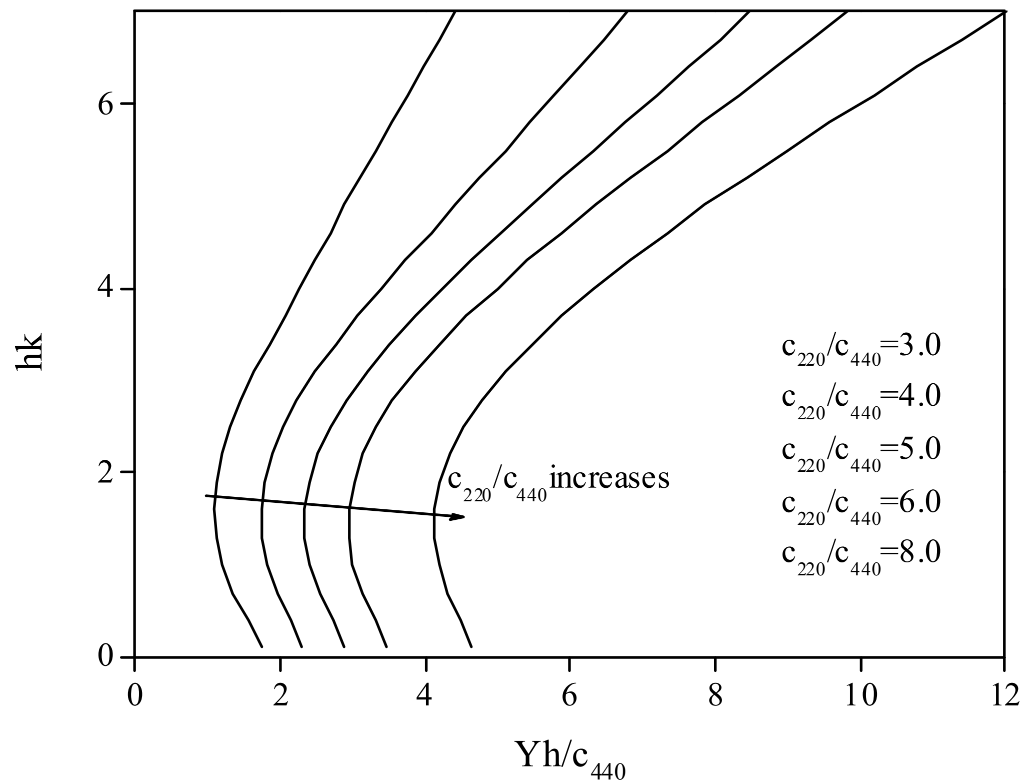

Figure 8: Bifurcation Modes k as a Function of Y in an Orthotropic Graded film for Selected Values of c_{220} / c_{440} with c_{120} / c_{440} = 3.0, and βh = 1.0

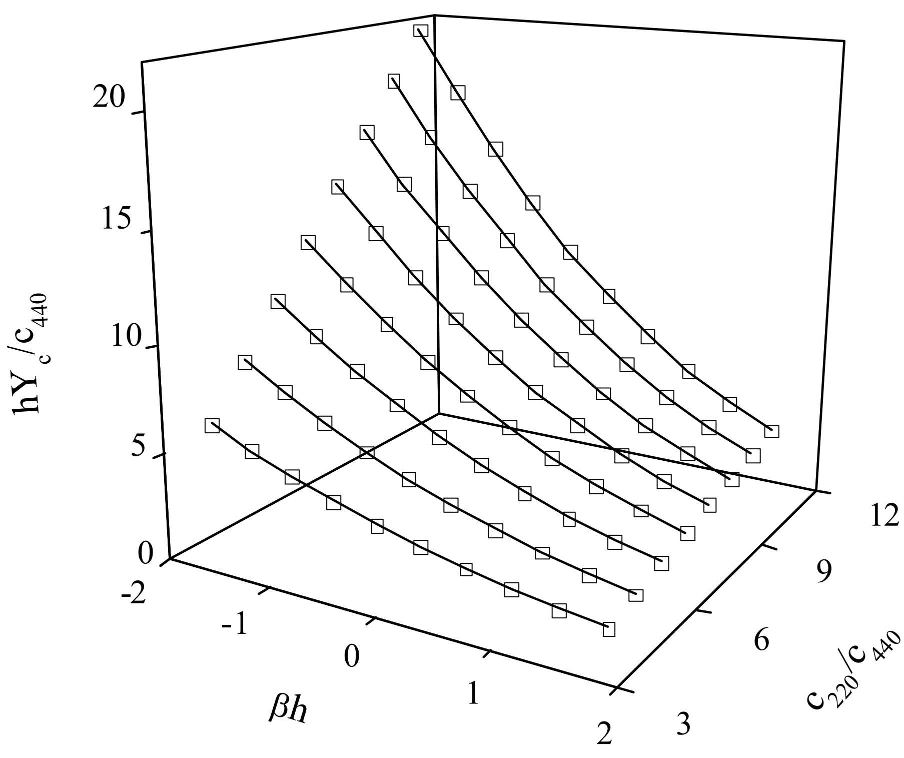

Figure 9: The Critical Y_c as a Function of βh and c_{220} / c_{440} for c_{120} / c_{440} = 4.0 and c_{110} / c_{440} = 6.0

similarly to that for isotropic case. And Y_c increases linearly with the increase of c_{220}. From this, one may also make use of anisotropy to control the patterns of surface instability.

5. CONCLUDING REMARKS

In this work, the van der Waals interaction between a graded film and a rigid contactor has been studied and the resultant surface instability of the graded film has been analyzed. Exponential form of gradation and elastic moduli has been assumed, based on which the solution can be obtained relatively easily. Through quantitative illustration of the effect of gradient, we have found that when the distance between the contactor and the graded film is small as compared with the thickness of the film, the van der Waals interaction is almost independent of the gradient in the film. Since according to the numerical results, the existence of bifurcation modes is independent of the gradient in elastic moduli but solely on Poisson's ratio for an isotropic film and c_{120} / c_{110} (also Poisson's ratio in essence) for an orthotropic film and the gradient only influences the critical load for bifurcation mode if it exists, we expect he patterns of the surface morphology may be controlled through the design of the gradation of the compositions in the film and its anisotropy. It is worth mentioning that all the elastic moduli have been assumed to possess the same exponential form and in the case of isotropic films, the Poisson's ratio has been taken to be constant. It is unclear how the different forms of elastic moduli or variable Poisson's ratio would influence the surface instability of graded films, which may remain for future study.

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