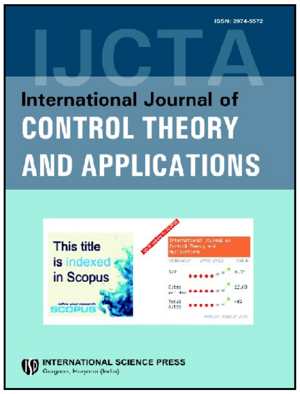

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Quasi-Cyclic Low Density Parity Check (QC-LDPC) Codes at Various Code Rates

[1]Nalini Prasad T. and [2]P. Subbarao

[1] P G Student, Department of E C E Sagi Rama KrishnamRaju Engineering College Bhimavaram, Andhra Pradesh, India
E-mail: naliniprasadt@gmail.com
[2] Professor, Department, of E C E Sagi Rama KrishnamRaju Engineering College Bhimavaram, Andhra Pradesh, India
E-mail: patsrao@gmail.com

Abstract: Quasi-Cyclic Low Density Parity Check codes (QC-LDPC) are the special class of forward error correcting code (FEC) and linear block code. QC-LDPC code has approaching the Shannon capacity limit for data transmission over the channel. QC-LDPC has better performance compared to other existing codes due to it's efficient and easy encoding process and effective iterative Min-Sum decoding Algorithm (MSA). In this paper performance evaluation results reported in between bit error rate (BER) and bit energy to the noise power spectral density (Eb/No) for distinct code rates when number of iterations were set, and also compared the QC-LDPC codes with turbo codes.

Index Terms: Parity check codes, LDPC codes, Quasi-Cyclic codes, QC-LDPC codes, Min-Sum Algorithm.

1. INTRODUCTION

During Transmission of data from sender to receiver some of the bits are occasionally flipped. Flipped bits are detected and possibly corrected by adding redundancy bits to the transmitted data [1]. These added redundancy bits are called parity bits. Parity bits are of two categories [2], even parity and odd parity. A transmitted data string has odd parity, if the count of 1's within the string is odd and count of 1's within the string is even then it is even parity. Group of large number of bits are organized in row and columns, for example consider, m x n parity check matrix. For this odd &even check matrix calculated the parity bits of each row & column, and then added to the row & column and also calculated the whole parity check matrix parity bits, and then these bits are also added to original data before the information (bits) transmitted to the receiver.

Low Density parity check codes contain parity check matrix with its entire row and columns have lowest number of one's compared to zero's i.e., number of one's reduced when compared with the total number of elements in the matrix. Low density of parity check matrix is defined by total number of one's in parity check matrix to the total number of elements (one's & zero's) in parity check matrix.

LDPC codes are originally introduced by Gallager in mid-sixties [3]. Up to three decades these code are neglected due to low technology development in early years. Mackay and Neal rediscovered these LDPC codes in mid-nineties. Ever since these codes are the subject of active analysis within the coding community impelled by better performance of those codes that is realized by exploitation repetitive coding schemes supported the Min-sum decoding algorithm (MSA).

In fact it has been shown that these codes are challenger to turbo codes in terms of performance and, if well designed, have higher performance than turbo codes. LDPC codes are one of the most effective code available codes present. Its Shannon capacity approaching limit performance [4, 5], made them use in many communication applications compared to other existing codes. LDPC Codes also have low complexity decoding parallel processing algorithms to get the best BER graphs.

LDPC codes are of two kinds, they are regular (Which have constant and equal row& column weight (w)) and irregular (Irregular type LDPC has the row and column weights are different).

The leftover part of the paper explains as, Section II explains the proposed QC-LDPC codes. The next section III proposed QC-LDPC decoding technique. In section IV different code rates performance results and appropriate comparisons, while conclusion in section V.

2. PROPOSED LDPC AND QC-LDPC CODES

(A) Representation of LDPC Code

Low density parity check codes (LDPC) are indicated with a sparse (most of the elements are zero in matrix) parity check matrix 6. LDPC codes are constructed by using Tanner (bipartite graph).It is easy to construct the tanner graph by using LDPC matrix. In tanner graph representation lower side bits are called bit nodes (variable nodes) and top side bits are called check bits (check node bits). The below Figure 1 shows, the low density parity check matrix denoted by letter H.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{3X7} \quad (1)$$

Figure 1: Parity bits represented in matrix form.

Low density parity check matrix, number of rows represent the check node bits and number of column bits represent the bit node (variable node bits) bits. Tanner graph representation of the parity check matrix (H), shown below Figure 2.

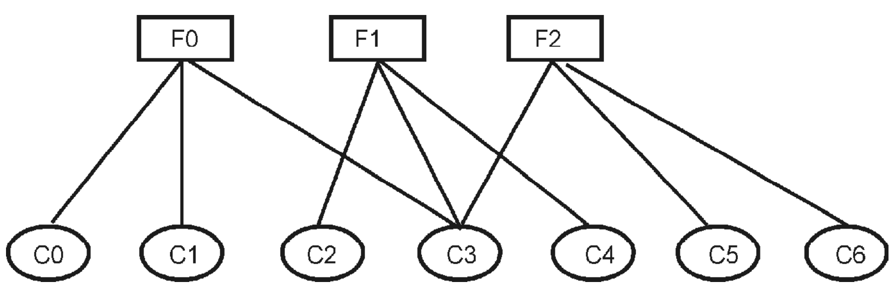

Figure 2: Tanner graph representation

LDPC codes are very easy to construct by bipartite graph it is also called as tanner graph and tanner graph itself used as a decoding of the transmitted data which is one of the advantage of the LDPC code, no need of other decoding structure for decoding of QC-LDPC code.

A regular low density parity check sparse m × n matrix H satisfying the low density properties[6].

1. There are wr(row weight) 1s in each roe of H and
2. There are wc(column weight) 1s in each column of H.

(B) Quasi-Cyclic Codes

Quasi-cyclic codes are the main class of linear block codes, if a code word is cyclically rotated then it yields a another codeword [7]. Quasi-cyclic (QC) codes are the generalized form of cyclic codes in which cyclically rotate of a codeword by ‘t’ places result other codeword. The below figure 3 shows the Quasi-Cyclic code (QC) represented in circulant matrices (circulant is a square matrix of m × m order).

$$QC=\begin{bmatrix} QC_0 & QC_1 & QC_2 & .... & QC_{t-1} \\ QC_{t-1} & QC_0 & QC_1 & .... & QC_{t-2} \\ . & . & . & . & . \\ . & . & . & . & . \\ QC_1 & QC_2 & QC_3 & .... & QC_0 \end{bmatrix} \quad (3)$$

Figure 3: Quasi Cyclic Code matrix

The total block length ‘n’ of a QC Code is multiple of‘t’ (i.e., n = m× t). QC-LDPC Code structures allows for low storage requirement. We need not require representing total QC-LDPC matrix. Only part of the code is enough for storing the entire code. The favor of cyclic codes compared to remaining codes is that they’re simple to encode. Moreover, cyclic codes possess a well outlined mathematical process that drives the decoding schemes very efficiently. A binary code is said to be a cyclic code if it is satisfies the below two basic properties 7.

Linearity Property and

Cyclic Property

According to Linearity property, modulo-2 addition of two code words within the code is again a code word. Similarly, according to cyclic property, any cyclically rotate of a code word within the code word is again called other code word.

(C) Quasi-Cyclic LDPC Codes

QC- LDPC code [8] the code matrix represented in matrix H shown below figure 4.

$$H_{M X N}=\begin{bmatrix} h_{11} & h_{12} & h_{13} & .... & h_{1N} \\ h_{21} & h_{22} & h_{23} & .... & h_{2N} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ h_{M1} & h_{M2} & h_{M3} & .... & h_{MN} \end{bmatrix} \quad (4)$$

Figure 4: QC-LDPC code matrix

(D) Encoding procedure of LDPC Codes

Encoding operation is done by add message with generator matrix (G), then we get a codeword (c) (i.e., c = message × Generator matrix).Here generator matrix (G) is obtained from LDPC H matrix ($H = \lfloor I \mid P \rfloor$) i.e.,

$G = [P^T \mid I]$.Where P & I are the parity and Identity matrices. The codeword (c) is valid if it is satisfies the syndrome calculation $S = c.H^T = 0$.

(E) Decoding of QC-LDPC codes

Min-Sum Decoding Algorithm (Msa).

Min Sum Aalgorithm [9, 10] is a one of the efficient algorithm to decode the QC-LDPC codes. Decoding is done by using tanner graph. It is a iterative soft decoding algorithm, in each iteration we get the estimated version of the transmitted data. There are three main steps in MSA decoding algorithm, Initialization, Check node update and Bit (Variable) node update. Three steps as follows.

i) Check node to variable node update: For a n^{th} variable node, check node finds the absolute minimum values, except n^{th} node, and send that value with sign such that modulo 2 sum is satisfied.

ii) Variable node to Check node update: Variable node sums up all the received message bits at the end of the iteration, except the message that came from the m^{th} check node.

iii) Above process is repeated until the best estimation of transmitted data occurs or maximum iterations were reached.

Typically, there are two mainmethods that to deliver messages in LDPC coding. One is to use probabilities, and also the alternative is to use log-likelihood ratios (LLR's). In general, exploitation LLR's is favored since that enables to interchange complex multiplication operations within easy addition operations. That's why in this decoding of QC-LDPC codes LLR's used.

Let contemplate a frame [1 0 1 1 0 0 1] being sent over the channel exploitation bi-polar mapping. Let the frame sent is [-1 1 -1 -1 1 1 -1]. Suppose the channel adds AWGN noise with a variance of 0.5 in order that LLR s of received frame is [-8 -6 -11 -5 8 9 -12].The second received sample has an error. The decoding stages on the Tanner graph are illustrated by Figure 5 shown below.

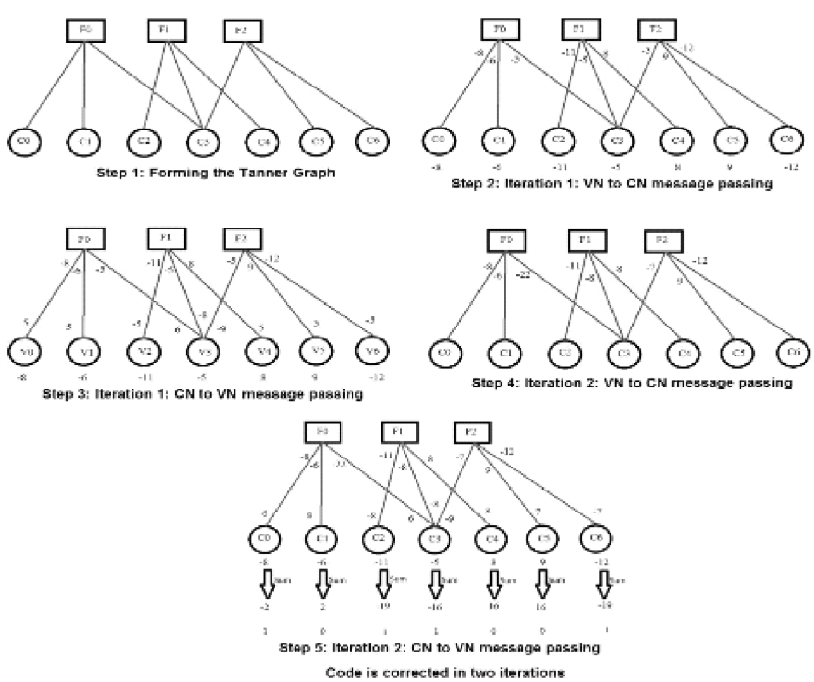

Figure 5: Example of min-sum decoding algorithm explained with Tanner graph

MSA algorithm have easy check node and bit (variable) node updating that reduce the implementation complexity. And complex exponential or logarithmic calculations are also avoided.

3. PERFORMANCE EVALUATION RESULTS OF QC-LDPC CODES AT VARIOUS CODE RATES

Performance evaluation results of QC-LDPC codes shown in between Bit Error Rate (BER) and bit energy (Eb) to the noise power spectral density (No). Here, code rate (k/n), gives the amount of information added to the transmitted data bits. Higher redundancy will detect the more errors and will correct the large number of errors. i.e., low code rate gives the better performance results compared to higher code rates. In this QC-LDPC scheme, Binary Phase Shift Keying (B.PSK) modulation is used and Additive White Gaussian Noise (AWGN) channel. The below figure 6 shows the performance estimation results of QC-LDPC codes at various code rates.

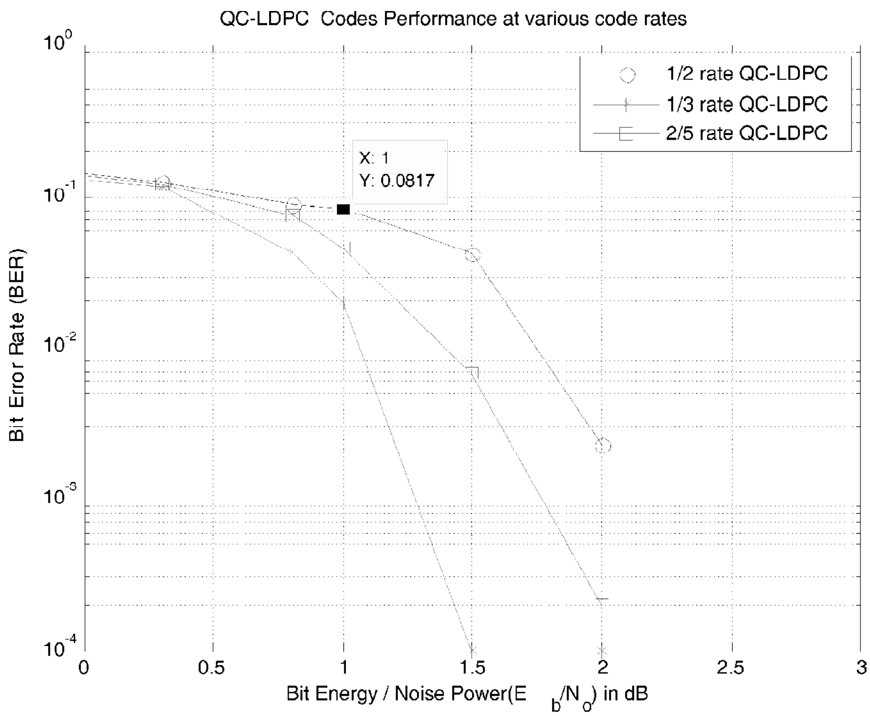

Figure 6: QC-LDPC performance at various data rates

In figure 6, three distinct code rates 1/2 , 2/5 and 1/3 performance results and comparison graph shown in between BER vs. Eb/No. For each code rate amount of redundancy is varies and hence the performance evaluation results also varies in accordance with the code rate. For Example for code rate 1/2 , for each data bit added here two redundancy bits similarly for code rate 1/3, for each data bit added here three redundancy bits and same for remaining code rates redundancy bits are added.

Figure 7 shows, comparison of three code rates 1/2, 2/5, and 1/3. 1/3 code rate have better BER compare to other two rates (1/2 and 2/5) because of its low code rate (higher redundancy). In this code rate we are adding three redundancy bits for each data bit i.e. redundancy increased hence, its BER performance improved. On observation of above two figures (6 & 7) we can say that, higher redundancy code have better performance compared to lower redundancy code. The main advantage of QC-LDPC code is that its BER results at very low ranges of Eb/Noi.e., with in 3 dB range.

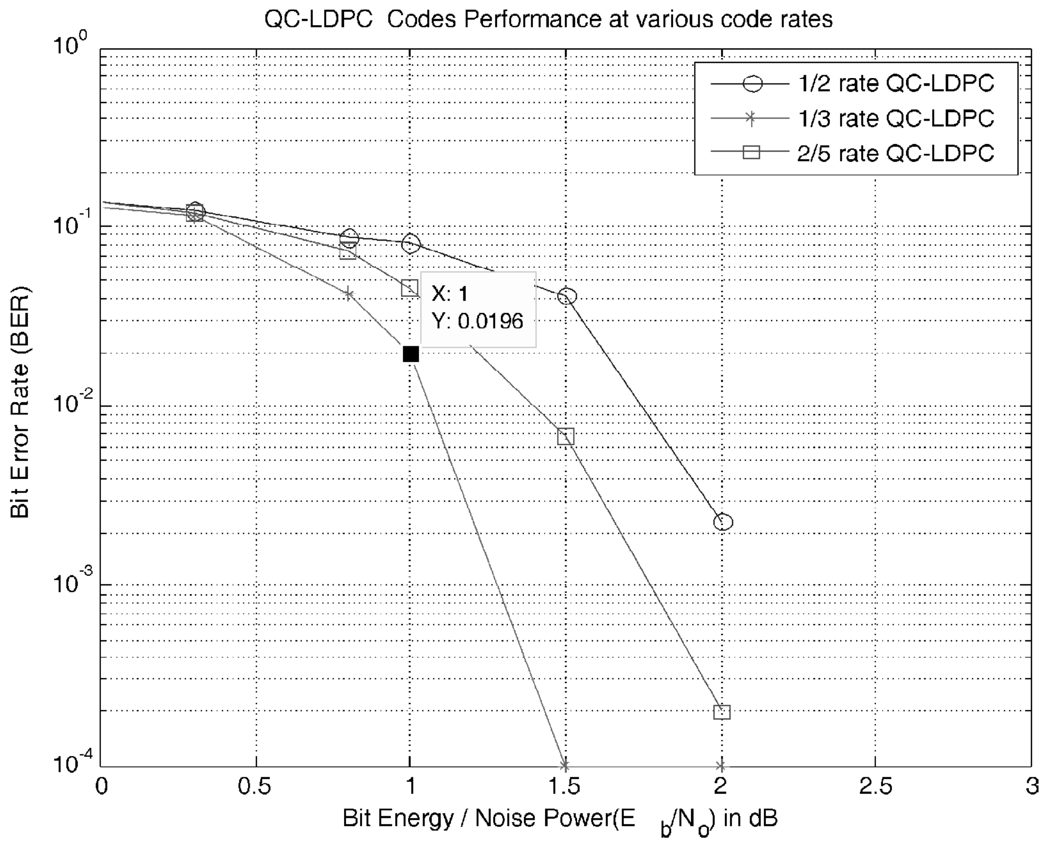

Figure 7: QC-LDPC performance at various data rates

Table 1
Comparison table for QC-LDPC Code at various code rates

S.No.	*Code Rate (k/n)*	*Bit Energy/ Noise Power (E_b/N_o) in dB*	*Bit Error Rate (BER)*
1	½	1	0.0817
2	2/5	1	0.0458
3	1/3	1	0.0196

Table 1 above shows the at constantEb/Novalue = 1, BER values are reported for three distinct code rates (1/2, 2/5 and 1/3). Among three code rates, 1/3 code rate have the low BER value (0.0196) due to it is having increased added redundancy. This will shows the low code has the better BER compared to other higher code rate i.e., mentioned above table 1.

Figure 8 shows, the comparison of Turbo code and QC-LDPC code at rate 1/2.

On observing the above Figure 8 at code rate 1/2, QC-LDPC code has low Bit Error Rate (BER) curve compared to Turbo code. Hence we can say that QC-LDPC have better performance compared to Turbo code and there is no further improvement in BER graph after ten iterations in turbo codes but in case of QC-LDPC code there is a further improvement in BER graph even after hundred iterations this is the major key advantage of the QC-LDPC code.

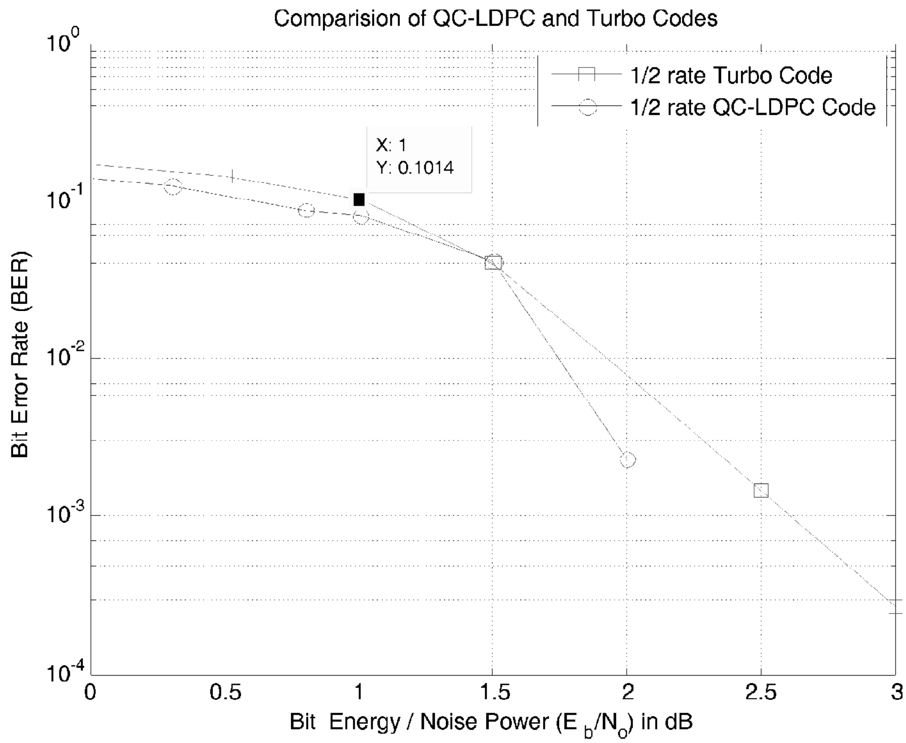

Figure 8: Comparison of QC-LDPC and Turbo Codes

Table 2
Comparison table for QC-LDPC Codes and Turbo codes

S.No.	*Code Rate (k/n)*	*Bit Energy / Noise Power (E_b/N_o) in dB*	*Bit Error Rate (BER)*
1	LDPC Code at ½ rate	1	0.0817
2	Turbo Code at ½ rate	1	0.1014

Table 2 shows the both QC-LDPC and Turbo codes performances at 1/2 code rate, at Eb/No value = 1. BER performance of both code rates hasvarious values. QC-LDPC has the best BER value (0.0817) compared to Turbo code. Only two states needed for QC-LDPC codes for decoding where as in turbo code sixteen states are require for decoding.

4. CONCLUSION

In this paper, proposes a Quasi-Cyclic Low-Density Parity-Check code for reducing encoding and decoding complication. Encoding complexity is reduced by means of circulant permutation (Quasi-Cyclic) matrices, and decoding complexity reduced by using iterative Min-Sum decoding algorithm. A thorough study at distinct code rates is tested and obtained the Bit Error Rate (BER) versus Bit Energy to the Noise Power Spectral Density ratio (Eb/No) graphs is reported and also reported the best comparison results of QC-LDPC than turbo codes.

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