International Journal of Control Theory and Applications

ISSN : 0974-5572

Volume 10 • Number 35 • 2017

Frequency Estimation of Seismic Signal Using Eigen Vector Method

V. Gopi Tilak[a] and S. Koteswara Rao[a]

[a]*Corresponding author, Department of ECE, K L University, Green Fields, Guntur DT, AP, 522502, India. Email: gopitilak7@gmail.com*
[b]*Department of ECE, K L University, Green Fields, Guntur DT, AP, 522502, India. Email: rao.sk9@gmail.com*

Abstract: Earthquake data recorded over multiple traces, each trace of signal model contains complex exponentials in noise. The noise in seismic signal is ground roll noise. The estimation of power spectrum using non-parametric methods has some limitations and they need to go for Eigen decomposition methods. Estimation of frequencies where peaks occur in spectrum is frequency estimation method which is one of the parametric estimation methods. It uses Eigen decomposition of signal that follows the separation of noise and complex exponentials is formulated as a generalized Eigen value decomposition assuming the noise is white noise with uniformly distributed amplitudes.

Keywords: Seismology, Stochastic signal processing, Adaptive Signal processing, Applied statistics.

1. INTRODUCTION

A phenomenon having certain behaviour and attributes and the information about those parameters can be conveyed as a function that is known as a signal. In real world, signal is any quantity showing fluctuations in time or fluctuations in space. A signal is useful in analysing different aspects like the position of a physical system which carry some data between observers and between supplementary possibilities.

Signals representation in mathematical form are known as deterministic signals [8] and based upon the characteristics and properties of signals, mathematically represented signals are again divided into different types. The random systems which can process signal without any priori information and produce required outputs need spectral estimation.

Earthquake [1, 7] is a physical phenomenon that occurs in different layers of earth. The occurrence of earthquake follows both natural and artificial reasons. Naturally, they happen due to geological movement of tectonic plates and artificially by the explosions in underground layers due to volcano and man-made explosions. Earthquake origin is determined by the seismic data of that earthquake recorded from at least three diverse receiver positions [5, 7]. The time span of earthquake is very small that lasts up to some seconds only.

1.1. Seismic Waves

Energy evolved from earthquake illuminates isotropically all over the surroundings [7]. The energy illumination is studied in form of waves known as seismic waves. Seismic waves characterization and analysis gives the details about different layers of earth. Body waves and surface waves are two types of seismic waves, P-waves and S-waves comes under category of body waves [1]. Body waves having low magnitude and propagate with high velocity. P waves are first seismic waves to be recorded in seismogram so named as primary waves as they can propagate through solid and underground water bodies. P waves travel longitudinally in propagation of energy and S waves travel in transverse manner. S waves cannot propagate through fluid medium and are shown to be following waves of P waves. P waves travel 60 times faster than S waves.

Surface waves are classified into two ways based on the names of scientists who realized them mathematically which are Rayleigh waves and Love waves. Love waves originate from horizontally polarized S waves, where first surface waves are love waves which also exist in the sub surface layers. Love waves travel left and right in direction of propagation where Rayleigh waves travel along up and down ways. Rayleigh waves are also named as ground roll noise. Surface waves disturb and shown to be noise in seismic signal.

1.2. Seismic Signal Processing

Seismic signal processing follows so many techniques to characterize the type of earthquake and also predicts the happening of earthquake based on previous data available. Seismic data recorded from seismograph that records relevant data about earthquake and other disturbances that occur randomly due to natural and man- made reasons. The noise is of coherent or non-coherent in nature. Signal processing algorithms analyses the characteristics of seismic signal and that allows for prediction of earthquakes in future. Statistical estimation [5,7] of signal follows power spectral estimation and frequency or harmonic estimation.

Parametric and non-parametric methods follows power density spectrum of seismic signal which improves the rejection of side lobes and increases main lobe width in smart antenna systems [10]. Statistical estimation methods provide better results for estimation of direction of arrival in smart antenna systems [13,10]. The frequency estimation of multiple sinusoidal signals is equates to liner prediction algorithms like maximum likelihood estimation [14]. The parameters in frequency estimation are frequencies of signal. A band pass filter gives better resolution on pre-processed seismic signal.

2. EIGEN VECTOR DECOMPOSITION

Eigen vector decomposition [2, 9] is one of the frequency estimation methods to estimate the signal by dividing the received signal into signal of interest and noise signal subspaces [6,11]. The received signal is the combination of signal of interest plus noise with amplitudes of complex exponentials. In many applications sinusoidal or harmonic model is more appropriate for signals of interest that are complex exponentials contained in white noise. The signals of complex frequencies are observed in array processing, the formant frequencies in speech processing, moving targets in radars and spatially propagating signals. Eigen decomposition method implies other applications over signal and image processing. In the applications of sky wave estimation [12], Eigen decomposition plays major role on estimating delays in sky wave propagation. The prediction of signal characteristics can also implemented using Eigen decomposition [15].

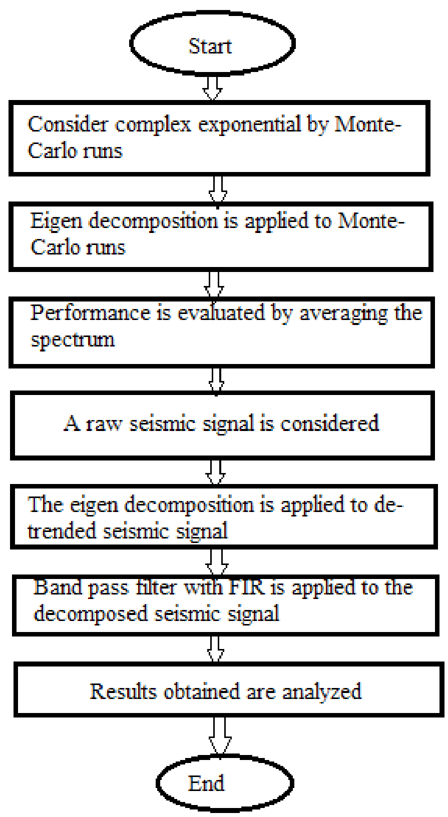
Complex conjugate pair (sinusoids) made up by complex exponentials for real signals and for complex signals, they may occur at single frequency. Frequencies of signals are the parameter of interest for complex exponentials in noise. The frequencies at which peaks occur in spectrum are the frequency estimates. Non-parametric methods are appropriate for complex exponential signals because they do not consider underlying process. All-pole model [3] or maximum entropy method is considered for spectral estimation.

This method follows the vector properties of signal. For an array of seismic sensors, which records multiple traces of seismic signals, where the amplitude of signal in white noise varies in complex exponential nature. One of the trace is considered such that it has time window length K and the count of complex exponentials is D. The auto correlation of noise is product variance of white noise and identity matrix. The eigenvalues are equal to powers of complex exponentials of signal. The consideration is that the time interval length is always greater than count of complex exponentials. The range up to complex exponentials is taken as signal and other than those are noise.

The eigenvalue decomposition technique includes these subsequent steps as follows:

1. Define the raw seismic signal in form of autocorrelation matrix of sum of eigenvectors of signal and noise in time window length K.
2. Define autocorrelation matrices of signal and noise based on condition between time window length and number of complex exponentials.
3. Apply a finite impulse response band pass filter to Eigen decomposed signal that gives better resolution about occurrence of peaks in the spectrum.

Flow chart



3. MATHEMATICAL MODELLING

The signal model of '*d*' complex exponentials in noise is

$$x(n) = \sum_{d=1}^{D} \alpha_d e^{i2\pi n f_d} + \omega(n) \qquad (1)$$

The normalized, discrete-time frequency of the d^{th} component is

$$f_d = \frac{\omega_d}{2\pi} = F_d / F_s \tag{2}$$

The frequency of discrete time is denoted by ω_d in radians, the d^{th} complex exponential actual frequency is denoted by F_d, and sampling frequency F_s. In the condition of real signals, the complex exponentials can result either independently or in pairs of complex conjugates. In general, possibly to estimate the frequencies and also the amplitudes of these signals. The phase of each complex exponential is contained in the amplitude, that is

$$\propto_d = |\propto_d| \, e^{j\varphi_d} \tag{3}$$

where, the phases φ_d are uncorrelated random variables uniformly distribute over $[0, 2\pi]$. The magnitude $|\propto_d|$ and frequency f_p are deterministic quantities. Take into account, the harmonic process's spectrum contains impulses set which has a steady background level at the white noise's power $\sigma_\omega^2 = E\{|\omega(n)|^2\}$. The line spectrum is generally considered as complex exponentials power spectrum.

Making use of matrix techniques on the basis of a few intervals of length K, the formation of vector is useful to describe the signal model over this time interval which consists of the sample delays of signal.

Time window of signal from (1) for its present and future K − 1 values represented as

$$x(n) = [x(n)\; x(n+1) \;\ldots\; x(n+K-1)]^T \tag{4}$$

The signal model complex exponentials included in noise for length K time interval vector is

$$x(n) = \sum_{d=1}^{d} \alpha_d v(f_d) e^{i2\pi n f_d} + \omega(n) = s(n) + \omega(n) \tag{5}$$

White noise with time window, $\omega(n) = [\omega(n)\omega(n+1) \;\ldots\; \omega(n+K-1)]^T$.

$$v(f) = [1 e^{j2\pi f} \ldots e^{j2\pi(K-1)f}]^T \tag{6}$$

$v(f)$ is frequency vector of time interval. The differentiation between signal $s(n)$, combination of noise content $\omega(n)$ and complex exponentials respectively.

From (5), time window vector model contains noise with which includes complex exponentials sum. The representation of signal and noise autocorrelation matrices and its sum as autocorrelation matrix of this model as

$$R_x = E\{x(n)\, x^h(n)\} = R_s + R_\omega \tag{7}$$

$$= \sum_{d=1}^{D} \alpha_d v(f_d) v^H(f_d) + \sigma_\omega^2 I = VAV^H + \sigma_\omega^2 I$$

where,

$$V = [v(f_1) v(f_2) \;\ldots\; v(f_d)] \tag{8}$$

Equation (6) is frequency vectors of time window is a column matrix of a K × D matrix of complex exponentials

$$A = \begin{bmatrix} |\alpha_1|^2 & 0 & \cdots & 0 \\ 0 & |\alpha_2|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & |\alpha_d|^2 \end{bmatrix} \tag{9}$$

The complex exponentials power is represented as diagonal matrix in (9) and the white noise autocorrelation matrix is

$$R_\omega = \sigma_\omega^2 I \tag{10}$$

For D < K it is rank deficient as opposed to R_s, full rank of ω. In practice, the assumption is that the number of complex exponentials D always less than length of time window K.

The Eigen decomposition of autocorrelation matrix is

$$R_k = \sum_{k=1}^{K} \lambda_k q_k q_k^H = Q\Lambda Q^H \tag{11}$$

where, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$ are descending eigen values of λ_k, their corresponding eigenvectors are q_k, the diagonal matrix of descending Eigen values is Λ, the eigenvectors due to signals is sum of time windowed signal power and noise.

$$\lambda_k = K\,|\propto_d|^2 + \sigma_\omega^2 \quad \text{for } k \leq D \tag{12}$$

The noise given by remaining eigenvalues as

$$\lambda_k = \sigma_\omega^2 \quad \text{for } k > D \tag{13}$$

The complex exponentials of the signal correspond by P largest eigenvalues and the noise corresponded by remaining eigenvalues having equal variance. Thus the autocorrelation matrix can be portioned into spaces of eigenvectors of signal and noise as

$$R_x = \sum_{k=1}^{d} q_k q_k^H \left(K\left|\propto_d\right|^2 + \sigma_\omega^2\right) + \sum_{k=d+1}^{K} \sigma_\omega^2 q_k q_k^H \tag{14}$$

$$= Q_s \Lambda_s Q_s^H + \sigma_\omega^2 Q_\omega Q_\omega^H$$

where,

$$Q_s = [q_1 \quad q_2 \quad \ldots \quad q_d] \qquad Q_\omega = [q_{d+1} \quad \ldots \quad q_K] \tag{15}$$

The eigenvectors of signals represented in columns of matrices respectively, from (12) Λ_s is D × D diagonal matrix consists signal eigenvalues. The signal and noise eigenvectors helps to span the time window signal vector from (5) which is K-dimensional signal subspace. The property of Hermitian symmetry of autocorrelation represents orthogonality of signal and noise subspaces.

The band pass filter with finite impulse response, The response of band pass filtered de-trended raw seismic signal which is Eigen decomposed is

$$y(i) = \sum_{n=0}^{M} h_d(n) R_x(i-n) \tag{16}$$

4. SIMULATION AND RESULTS

Step 1: Representation of complex exponential noisy signal in form of Monte-Carlo runs with respect to Eigen decomposition method. Figure 1 shows the power spectral density of complex exponential signal shows that peaks occur at 0.2, 0.3 and 0.5 radians that shows code is functioning well. The number of Monte-Carlo runs is 10.

Step 2: Average of Monte Carlo spectrum using Eigen vector by normalizing spectrum with total number of samples which shows peaks of single normalized signal in Figure 2 taking as reference for explaining Eigen decomposition for further results.

Step 3: The seismic signals used in this paper are attained from MATLAB file present in [5], Book_Seismic_Data.mat through a geophone array in southern United States. This reference data is recorded from the man-made seismic waves to idealistically represent the real-time earthquake scenario. Here source is observed to be a high explosive material filled in about 100 feet below the earth surface by making holes. There are 33 traces, each divided individually into 1500 samples with sampling interval of 0.002 s. To analyse this spectrum one of the traces is considered among the supplied traces. In Figure 3, trace 5 of the seismic signal data is shown.

Step 4: Figure 4 shows the de-trended raw seismic signal which is observed from bias of raw seismic signal. De-trending makes the signal to get rid of small fluctuations by taking the mean of the signal.

Step 5: The prescribed Eigen vector algorithm is applied on the de-trended seismic signal and the power spectrum density achieved out of it is represented in Figure 5. From this figure, the maximum peak is determined to be at 0.08594π normalized frequency. The sampling frequency is calculated by using the sampling interval 0.002s as,

$$fs = 1/0.002 = 500 \text{ Hz}$$

$$w = \frac{2\pi f}{fs} = 0.08594\pi$$

$$= \frac{2\pi f}{500}$$

$$= \frac{2\pi}{fs} f = 0.\ 08594\pi$$

where, 'f' is the frequency of the signal and it is given by,

$$\text{Tonal Frequency } f = \frac{500}{2} \times 0.08594$$

$$= 250 \times 0.08594$$

$$= 25 \times 0.8594$$

$$= 21.485 \text{ Hz}$$

Step 6: The frequency range of earthquake signal varies between 15 to 60 Hz. Figure 6 shows the frequency spectrum of finite impulse response of a band pass filter of order 8.

Step 7: Figure 7 shows the convolved output of de-trended raw seismic signal with Fir response of ban pass filter.

Step 8: Applying fast Fourier transform to obtain spectrum of band pass filtered raw seismic signal and taking normalization over frequency which gives better analysis about the signal, represented in Figure 8.

Step 9: The power spectral density of band pass filtered, Eigen decomposed seismic signal specifies the occurrence of peak shown in Figure 9. The maximum peak is obtained at 0.08398π. By repeating the process like step 5, the frequency of the signal is 20.995 Hz.

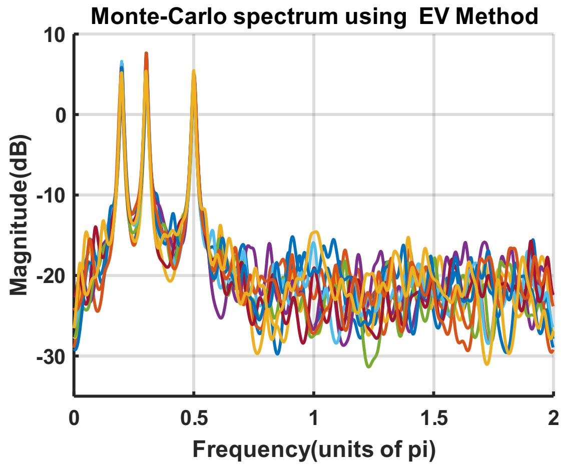


Figure 1: Monte-Carlo spectrum using EV method

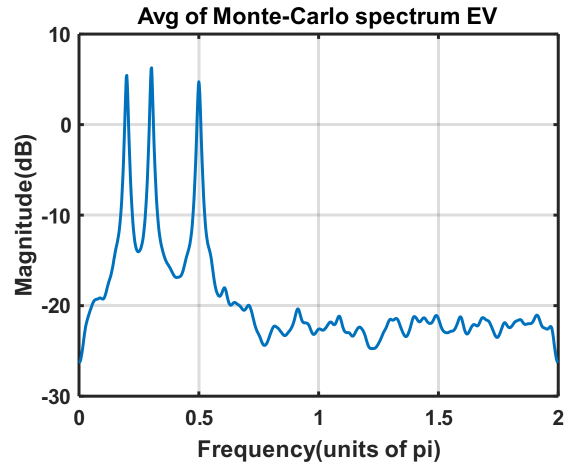


Figure 2: Average of Monte-Carlo spectrum EV

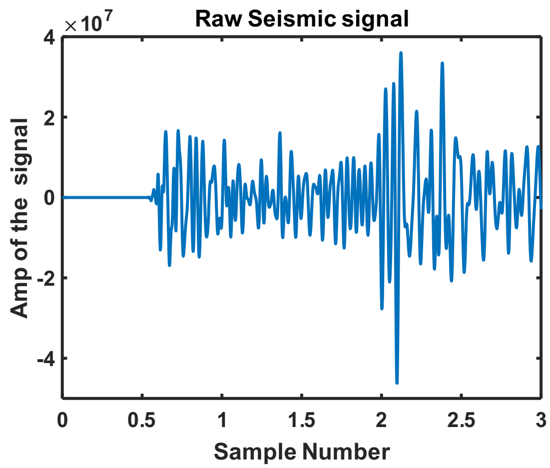


Figure 3: Raw seismic signal

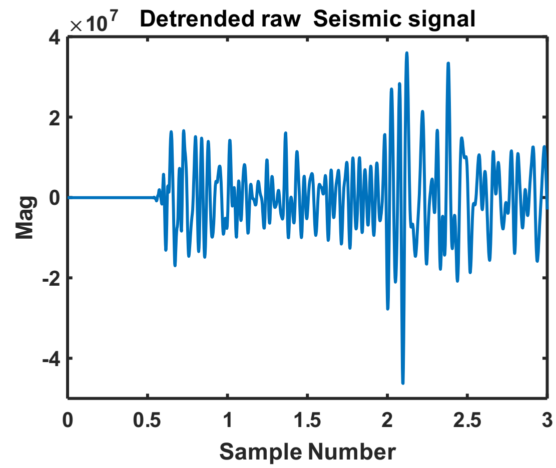


Figure 4: De-trended seismic signal

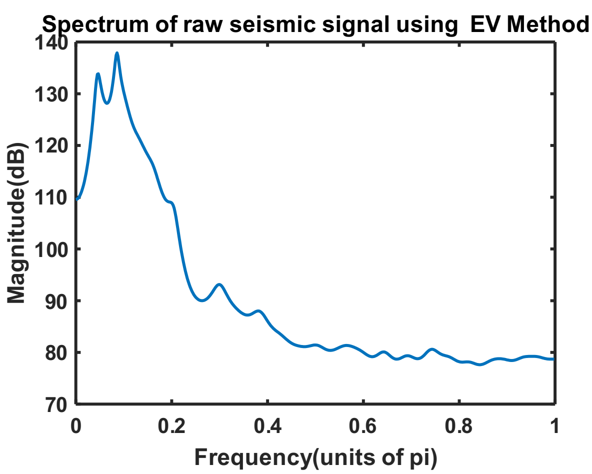


Figure 5: spectrum of raw seismic signal Using EV method

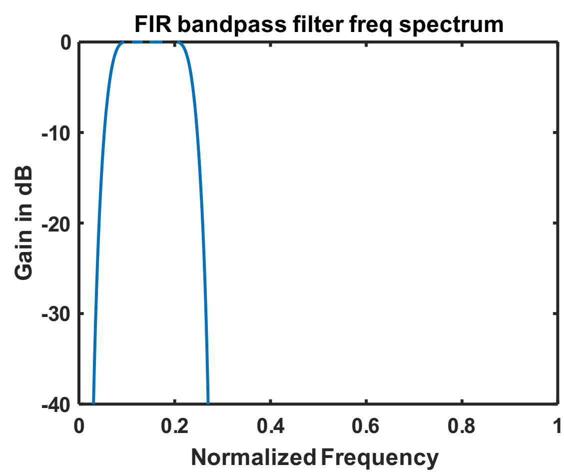


Figure 6: FIR Band pass filtered frequency spectrum

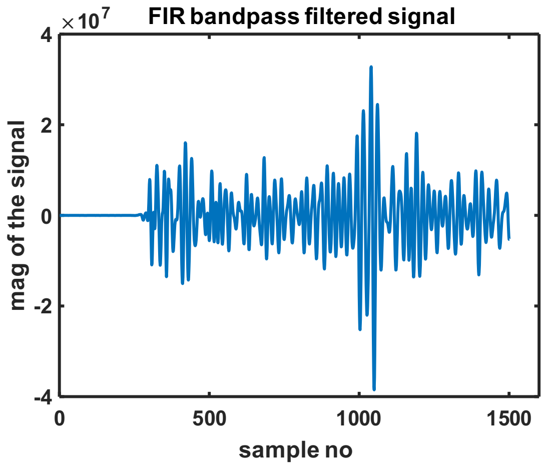


Figure 7: FIR band pass filtered signal

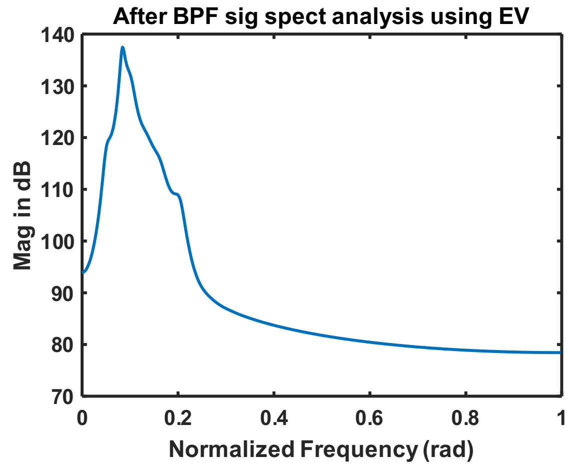


Figure 8: After BPF signal spectral analysis using EV

5. CONCLUSION

The maximum resolution can be obtained by spectral estimation method and eigenvector. This method attains good response by separating complex exponentials in noise from seismic signal by separating them with their respective subspaces. The band pass filtered eigenvector decomposed signal gives better resolution over peaks occurrence at complex exponentials. This Eigen decomposition method provides easier way for other frequency estimation techniques in order to attain better resolution.

REFERENCES

[1] Peter M. Shearer, "Introduction to Seismology", vol.2, © P. Shearer 2009

[2] V U Reddy and K Maheswara Reddy, "Eigen structure based spatial spectrum estimation: Problems encountered in practice and signal processing solutions", July and September 2001, pp.405-438.

[3] Jae s.Lim, V. Oppenheim, "All Pole Modeling of Degraded Speech", IEEE Transactions on Acoustics, Speech,And Signal Processing, Vol. Assp-26, No. 3, June 1978.

[4] Yanwei Wang, Jian Li and Petre Stoica, "Spectral analysis of signals", Morgan and Claypool publishers.

[5] Wail A. Mousa and Abdullatif A. Al-Shuhail, "Processing seismic reflection data using MATLAB", Morgan and Claypool publishers.

[6] Monson H. Hayes, "Statistical digital signal processing and modeling", John Wiley and sons Inc., publishers.

[7] Dr. N. Purnachandra Rao, "Earthquakes", Andhra Pradesh Akademi of Sciences (APAS) publishers.

[8] Alan V. Oppenheim, Allan S. Wilskey, S. Hamid Nawab, "Signals and Systems", Prentice hall international, lnc.

[9] Lucas Parra,Paul Sajda, "Blind Source Seperation Via Generalized Eigenvalue Decomposition", Journal of Machine Learning Research 4(2003) 1261-1269,

[10] Umar Mujahid, Jameel Ahmed, Abdur Rehman, Umair Shahid, Abbass and Mudassir, "Performance Analysis of Spectral Estimation for Smart Antenna System", 2013, pp.1695-1705.

[11] Dimitris G Manolakis, Vinay K Ingle and Stephen M Kogon, "Statistical and adaptive signal processing", Artech house, Inc.

[12] Yi Bian, David Last, "Eigen decomposition technique for Loran-C sky wave estimation", IEEE transactions on Aerospace and electronic systems, Vol. 33, no.1, jan-1997.

[13] P. Stoica, K. C. Sharman, "Novel Eigenanalysis method for direction estimation", IEEE Proceedings, Vol. 137, Pt. F, No. I, feb 1990.

[14] Donald W. Tuffs, Ramdas Kumaresam, "Estimation of frequencies of multiple sinusoids: Making linear prediction perform like maximum likelihood", Proceedings of the IEEE, Vol. 70, No. 9, sep-1982.

[15] D.J. Jeffries, D.R. Farrier, "Asymptotic results for Eigenvector methods", IEEE Proceedings, Vol. 132, Pt. F. No.7, Dec-1985.