

Fiscal Policy In A Stochastic Temporary Stabilization Model: Undiversifiable Devaluation Risk

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ABSTRACT

In this paper, we present a stochastic model of exchange-rate-based inflation stabilization with imperfect credibility that explicitly recognizes uncertainty in both the expected dynamics of the exchange rate and the expected behavior of tax policy. We assume that the exchange rate is driven by a mixed diffusion-jump process, and the tax rate on wealth follows a geometric Brownian motion. Under this setting, we suppose that the derivatives for hedging against future devaluation are not available, so financial markets are incomplete. We examine consumption and portfolio shares equilibrium dynamics when a stabilization plan is implemented and taxes on wealth are paid at an uncertain rate. We also assess the effects of exogenous shocks of both devaluation and taxes on welfare. Finally, we use the proposed model to carry out Monte Carlo simulation experiment that explains the observed orders of magnitude of consumption booms in the presence of taxes for the Mexican case between 1989 and 1994.

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Introduction

The impact of fiscal policy in exchange-rate-based inflation stabilization plans has been of great interest to policymakers for a long time. Most of the literature addressing this central concern does so within a deterministic setting. Our modeling assumes that agents have expectations of devaluation driven by a mixed diffusion-jump process. In this context, small diffusion movements of the exchange rate, which are always present, are modeled through a Brownian motion, and an extreme and sudden devaluation, which occasionally occurs, is governed by a Poisson process. Mixed diffusion-jump processes provide heavy tails and skewness in the exchange-rate distribution to rationalize inflation dynamics that cannot be generated by using only the Brownian motion. This fact is not just a theoretical sophistication but an important issue to be considered in further empirical research. The model assumes that contingent-claims markets to hedge against future devaluation are unavailable. In a still richer stochastic environment, we assume that an uncertain tax rate on wealth is driven by a geometric Brownian motion. By considering the whole distribution of the exchange rate and the tax rate on wealth, we might even examine those events that in spite of their small probability of

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occurrence, they could lead to significant impacts on temporary stabilization outcomes. In contrast with the deterministic setting, the presence of uncertainty in the tax rate on wealth may lead to significant quantitative and qualitative changes in the effects of fiscal policy.

In our approach, revenue raised by taxes, including seignorage, is wasted in unproductive government purchases. Assuming risk adverse agents, we examine the equilibrium dynamics of consumption and wealth when a stabilization program is implemented and fiscal policy is uncertain. Under this framework, we also address a number of specific policy issues. Specifically, we study the effects on consumption and economic welfare of once-and-for-all changes in the parameters that determine the expectations, namely the expected rate of devaluation, the exchange-rate volatility, the probability of devaluation, the expected size of a possible devaluation, the expected resident-based *ad valorem* tax on consumption, and the expected tax on wealth.

Inflation stabilization programs that took place in Argentina, Brazil, Chile, Uruguay, Israel, and Mexico between the 1970s and the 1990s have been widely documented.¹ There is a large literature reporting key empirical regularities associated with these programs: see *e.g.*, Helpman and Razin (1987), Kiguel and Liviatan (1992), and Végh (1992). There is also an increasing number of models providing explanations for such empirical regularities.

These models can be classified into several categories, such as: lack of credibility (Calvo 1986, Calvo and Végh 1993, and Reinhart and Végh 1993 and 1995); inflation inertia (Rodriguez 1982, and Calvo and Végh 1994); supply-side effects (Roldos 1995, Uribe 1997, Lahiri 2001, and Rebelo and Végh 1995); and durable goods (Matsuyama 1991, and De Gregorio, Guidotti and Végh 1998).

Although uncertainty is a key element when an exchange-rate-based inflation stabilization program is implemented, there are few studies concerning with stochastic settings. For example, Drazen and Helpman (1988) examined stabilization with exchange-rate management under uncertainty, Calvo and Drazen (1997) contemplated uncertainty in the permanence of economic reforms, Mendoza and Uribe (1996) and (1998) modeled exogenous and endogenous probabilities of devaluation, Venegas-Martínez (2005) and (2006) studied the role of uncertainty in the dynamics of the exchange rate examining quantitative implications. All of these models share important similarities: (1) markets of contingent-claims are unavailable, (2) the revenue raised by seignorage is not rebated back to the agents, and (3) policy variables are stochastic.

Our modeling has several distinctive features in studying the effects of uncertainty in exchange-rate-based inflation stabilization programs: (1) it takes into account all risk factors in the exchange-rate dynamics, providing a more realistic stochastic environment; (2) it derives tractable closed-form solutions, making easier the understanding of the key issues in the analysis of temporary stabilization; (3) it examines the effects on temporary stabilization plans of an uncertain tax on wealth; and (4) it explains the observed orders of magnitude of consumption booms by using Monte Carlo simulation methods.

The paper is organized as follows. In the next section, we work out a Ramsey-type, one-good, cash-in-advance stochastic model where agents have expectations of devaluation

driven by a mixed diffusion-jump process. We also assume that the agents pay taxes on wealth in accordance with a geometric Brownian motion. In section 3, we solve the consumer's choice problem. In section 4, we undertake policy experiments. In section 5, we examine welfare implications. In section 6, we study the dynamic behavior of wealth and consumption addressing a number of exchange-rate policy issues. In section 7, we carry out a Monte Carlo simulation of the response of consumption to permanent changes in the values of key parameters of the model when taxes on wealth are paid at a stochastic rate. Finally, in section 8, we draw conclusions, acknowledge limitations, and make suggestions for further research. Two appendices contain some technical details on the consumer's choice problem.

The Setting of the Model

In order to derive analytically tractable solutions in a stochastic Ramsey type model, the structure of the price-taking economy will be kept as simple as possible. The main assumptions of the model are stated in such a way that the key issues of temporary stabilization under uncertain fiscal policy are easier to understand.

Price Level Dynamics

Let us consider a small open economy with a single infinite-lived household in a world with a single perishable consumption good. We assume that the good is freely traded, and its domestic price level, P_t , is determined by the purchasing power parity condition, namely $P_t = P_t^* e_t$, where P_t^* is the foreign-currency price of the good in the rest of world, and e_t is the nominal exchange rate. We will assume, for the sake of simplicity, that P_t^* is equal to 1. We also assume that the initial value of the exchange-rate, e_0 , is known and equal to 1.

We suppose that the number of expected devaluations, *i.e.*, jumps in the exchange rate, per unit of time, follows a Poisson process N_t with intensity λ , so

$$\text{IP}^{(N)} \{ \text{one unit jump during } dt \} = \text{IP}^{(N)} \{ dN_t = 1 \} = \lambda dt + o(dt), \quad (1)$$

whereas

$$\text{IP}^{(N)} \{ \text{no jump during } dt \} = \text{IP}^{(N)} \{ dN_t = 0 \} = 1 - \lambda dt + o(dt). \quad (2)$$

Thus, $E^{(N)}[dN_t] = \text{Var}^{(N)}[dN_t] = \lambda dt$. We set the initial number of jumps identically equal to zero, that is, $N_0 = 0$.

Let us consider a Wiener process $(Z_t)_{t \geq 0}$ defined on some fixed filter probability space $(\Omega^{(z)}, \mathcal{F}^{(z)}, (\mathcal{F}_t^{(z)})_{t \geq 0}, \text{IP}^{(Z)})$. We assume that the consumer perceives that the

expected inflation rate, dP_t/P_t , and consequently the expected rate of devaluation, de_t/e_t , follows a geometric Brownian motion with Poisson jumps in accordance with²

$$\frac{dP_t}{P_t} = \frac{de_t}{e_t} = \pi dt + \sigma_p dZ_t + \eta dN_t, \quad (3)$$

where π is the mean expected rate of devaluation conditional on no jumps, σ_p is the instantaneous volatility of the expected price level, and η is the mean expected size of a exchange-rate jump. Process Z_t is supposed to be independent of N_t . In what follows, π , σ_p , λ and η are all supposed to be positive constants.

Real Money Holdings

The agent holds real cash balances, $m_t = M_t/P_t$, where M_t is the nominal stock of money. The stochastic rate of return of holding real cash balances, dR_m , is given by the percentage change in the price of money in the terms of goods. By applying Itô's lemma for diffusion-jump processes to the inverse of the price level, with (3) as the underlying process (see Appendix A, formula (A.2)), we obtain

$$dR_m = d\left(\frac{M_t}{P_t}\right) / \left(\frac{M_t}{P_t}\right) = (-\pi + \sigma_p^2)dt - \sigma_p dZ_t - \left(\frac{\eta}{1+\eta}\right)dN_t. \quad (4)$$

International Bonds

The agent also holds an international bond, b_t that plays a risk-free real interest rate, r , which is constant for all terms, satisfying

$$db_t = rb_t dt, \quad b_0 \text{ given.} \quad (5)$$

That is, the bond pays r units of the consumption good per unit of time. Equation (5) may be thought as the money market account, that is, as a security that is worth b_0 , at time zero, and earns the instantaneous risk-free interest rate, r , at any given time. The agent takes r as given.

Taxes on Wealth

Let us consider now a Wiener process $(U_t)_{t \geq 0}$ defined on a fixed filtered probability space $(\Omega^{(U)}, \mathcal{F}^{(U)}, (\mathcal{F}_t^{(U)})_{t \geq 0}, \mathbb{P}^{(U)})$. We assume that the representative consumer perceives that his/her wealth is taxed at an uncertain rate, τ_t , in accordance with the following stochastic differential equation:

$$\frac{d\tau_t}{\tau_t} = \bar{\tau}dt + \sigma_\tau d\tilde{Z}_t, \quad \tau_0 > 0, \quad (6)$$

with

$$\tilde{Z}_t = \rho Z_t + \sqrt{1-\rho^2}U_t \quad (7)$$

and

$$\text{Cov}\left(dZ_t, d\left(\rho Z_t + \sqrt{1-\rho^2}U_t\right)\right) = \rho dt, \quad (8)$$

where $\bar{\tau}$ is the mean expected growth rate of the taxes on wealth, σ_τ is the volatility of the tax rate on wealth, and $\rho \in (-1, 1)$ is the correlation between changes in the inflation and changes in wealth taxes. Notice that an increase in the rate of devaluation will produce a higher depreciation in real cash balances. This, in turn, will reduce real assets, which could lead to the fiscal authority to modify tax rates. Processes N_t , Z_t , and U_t are supposed to be pairwise independent.

A Cash-in-advance Constraint

Consider a cash-in-advance constraint of the Clower-Lucas-Feenstra form:

$$m_t = \alpha c_t, \quad (9)$$

where c_t is the consumption, and $\alpha > 0$ is the time that money must be held to finance consumption. Condition (9) is critical in linking exchange-rate policy and consumption. In such a case, devaluation acts as a stochastic tax rate on real cash balances.

The Consumer's Choice Problem

In this section, we characterize the household's optimal decisions on consumption and portfolio shares through the Hamilton-Jacobi-Bellman condition of the continuous-time stochastic dynamic programming.

Intertemporal Budget Constraint

The stochastic consumer's wealth accumulation in terms of the portfolio shares, $w_t = m_t/a_t$, $1 - w_t = b_t/a_t$, and consumption, c_t , is given by the following system:

$$\begin{cases} da_t = a_t w_t dR_m + a_t (1 - w_t) dR_b - (\tau_t a_t + (1 + \hat{\tau}) c_t) dt, & a_0 = m_0 + b_0 > 0, \\ d\tau_t = \bar{\tau} \tau_t dt + \sigma_\tau \tau_t \left(\rho dZ_t + \sqrt{1-\rho^2} dU_t \right), & \tau_0 > 0, \end{cases} \quad (10)$$

where $dR_b = db_t/b_t$, and $\hat{\tau}$ is a resident-based *ad valorem* tax rate on consumption. By substituting (4), (5) and (9) into the first equation of (10), we get

$$da_t = a_t \left[(r - \beta w_t + \tau_t) dt - w_t \sigma_p dZ_t - w_t \left(\frac{\eta}{1 - \eta} \right) dN_t \right], \quad (11)$$

where $\beta = (1 + \hat{\tau})\alpha^{-1} + r + \pi - \sigma_p^2$.

The Satisfaction Index

The von Neumann-Morgenstern utility at time t , V_t , of the competitive risk-averse consumer is assumed to have the time-separable form:

$$V_t = E \left\{ \int_t^\infty \log(c_s) e^{-rs} ds \middle| \mathcal{F}_t \right\}, \quad (12)$$

where $\mathcal{F}_t = \mathcal{F}_t^{(Z)} \otimes \mathcal{F}_t^{(U)}$ stands for all available information at t . Notice that the agent's subjective discount rate has been set equal to the interest rate, r , to avoid unnecessary technical difficulties. We consider the logarithmic utility function in order to derive closed form solutions and make the analysis easy to manage.

The Hamilton-Jacobi-Bellman Equation

In this case, the Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing the agent's life-time expected utility subject to the intertemporal budget constraint is:

$$\begin{aligned} & \lambda I(a_t, \tau_t, t) - I_t(a_t, \tau_t, t) - I_\tau(a_t, \tau_t, t) \bar{\tau}_t - \frac{1}{2} I_{\tau\tau}(a_t, \tau_t, t) \tau_t^2 \sigma_\tau^2 - I_a(a_t, \tau_t, t) a_t (r - \tau_t) \\ & = \max_w \left\{ \log(\alpha^{-1} a_t w_t) e^{-rt} - I_a(a_t, \tau_t, t) a_t \beta w_t + \frac{1}{2} I_{aa}(a_t, \tau_t, t) a_t^2 w_t^2 \sigma_p^2 \right. \\ & \quad \left. - I_{a\tau}(a_t, \tau_t, t) a_t \tau_t w_t \sigma_p \sigma_\tau \rho + \lambda I \left(a_t \left(\frac{1 + \eta(1 - w_t)}{1 + \eta} \right), \tau_t, t \right) \right\}, \end{aligned} \quad (13)$$

where

$$I(a_t, \tau_t, t) = \max_w E_t \left\{ \int_t^\infty \log(\alpha^{-1} a_s w_s) e^{-rs} ds \middle| \mathcal{F}_t \right\}$$

is the agent's indirect utility function (or welfare function) and $I_a(a_t, \tau_t, t)$ is the co-state variable.

Reduction in the Dimension of the Problem

Given the exponential time discounting in (14), we specify $I(a_t, \tau_t, t)$ in a time-separable form as

$$I(a_t, \tau_t, t) \equiv F(a_t, \tau_t) e^{-rt}. \quad (14)$$

Hence, (14) is transformed into

$$\begin{aligned} & (\lambda + r)F(a_t, \tau_t) - F_\tau(a_t, \tau_t)\bar{\tau}\tau_t - \frac{1}{2}F_{\tau\tau}(a_t, \tau_t)\tau_t^2\sigma_\tau^2 - F_a(a_t, \tau_t)a_t(r - \tau_t) \\ &= \max_w \left\{ \log(\alpha^{-1}a_t w_t) - F_a(a_t, \tau_t)a_t\beta w_t + \frac{1}{2}F_{aa}(a_t, \tau_t)a_t^2 w_t^2 \sigma_P^2 \right. \\ & \quad \left. - F_{a\tau}(a_t, \tau_t)a_t\tau_t w_t \sigma_P \sigma_\tau \rho + \lambda F\left(a_t \left(\frac{1 + \eta(1 - w_t)}{1 + \mu}\right), \tau_t\right) \right\}. \end{aligned} \quad (15)$$

We postulate

$$F(a_t, \tau_t) = \delta_0 + \delta_1 \log\left(\frac{a_t}{\tau_t}\right) + H(\tau_t; \delta_2, \delta_3), \quad (16)$$

where δ_0 , δ_1 and $H(\tau_t; \delta_2, \delta_3)$ are to be determined from equation (15). Coefficients δ_2 and δ_3 must satisfy $H(\tau_0) = 0$ and $H'(\tau_0) = 0$. Substituting (16) into (15), we have

$$\begin{aligned} & r(\delta_0 + \delta_1 \log(a_t)) + \delta_1(\bar{\tau} - r - \frac{1}{2}\sigma_\tau^2) \\ & + rH(\tau_t) - H'(\tau_t)\tau_t\bar{\tau} - \frac{1}{2}H''(\tau_t)\tau_t^2\sigma_\tau^2 - r\delta_1 \log(\tau_t) + \delta_1\tau_t \\ &= \max_w \left\{ \log(\alpha^{-1}a_t w_t) - \delta_1\beta w_t - \frac{1}{2}\delta_1 w_t^2 \sigma_P^2 + \lambda\delta_1 \log\left(\frac{1 + \eta(1 - w_t)}{1 + \eta}\right) \right\}. \end{aligned} \quad (17)$$

First order Conditions and Determination of Coefficients

The first order conditions of the intertemporal optimization of the risk averse representative agent lead to a time-invariant $w_t \equiv w$, and

$$\frac{1}{\delta_1 w} - \frac{\lambda\eta}{1 + \eta(1 - w)} = (1 + \hat{\tau})\alpha^{-1} + r + \pi - \sigma_P^2 + w\sigma_P^2. \quad (18)$$

We choose now $H(\tau_t)$ as a solution of

$$rH(\tau_t) - H'(\tau_t)\tau_t\bar{\tau} - \frac{1}{2}H''(\tau_t)\tau_t^2\sigma_\tau^2 - r\delta_1 \log(\tau_t) + \delta_1\tau_t = 0. \quad (19)$$

Coefficients δ_0 and δ_1 are determined from (15) after substituting optimal w^* . Thus, $\delta_1 = r^{-1}$, so the coefficient of $\log(a_t)$ in (17) becomes zero, and

$$\delta_0 = \frac{1}{r} \log(\alpha^{-1} w^*) - \frac{1}{r^2} \left[\left((1 + \hat{\tau}) \alpha^{-1} + r + \pi - \sigma_p^2 \right) w^* + \frac{1}{2} (w^* \sigma_p)^2 + \bar{\tau} - r - \frac{1}{2} \sigma_\tau^2 - \lambda \log \left(\frac{1 + \eta(1 - w^*)}{1 + \eta} \right) \right]. \quad (20)$$

Logarithmic utility implies that w depends only upon the parameters determining the stochastic characteristics of the economy, and hence w is a constant. In other words, the consumer's attitude toward currency risk is independent of his/her wealth, *i.e.*, the resulting level of wealth at any instant has no relevance for portfolio decisions. Moreover, due to the logarithmic utility, the correlation coefficient, ρ , plays no role in the consumer's optimal portfolio, only matters the trend and volatility components of the stochastic processes driving the dynamics of the exchange rate and the tax policy. Finally, it is important to point out that equation (18) is cubic, therefore it has at least one real root.

Notice also that from $\delta_1 = r^{-1}$, we have that the solution of (19) is (see Appendix B)

$$H(\tau_t) = \delta_2 \tau_t^{\gamma_1} + \delta_3 \tau_t^{\gamma_2} + \frac{1}{\tau} \log(\tau_t) \left(1 + \frac{2}{(\sigma_\tau^2 + 2\bar{\tau})} \tau_t \right) + \frac{1}{\tau} \left(1 - \frac{\sigma_\tau^2}{2\bar{\tau}} \right), \quad (21)$$

where

$$\gamma_1 = \frac{4r}{(2\bar{\tau} - \sigma_\tau^2) + \sqrt{(2\bar{\tau} - \sigma_\tau^2)^2 + 8r\sigma_\tau^2}}$$

and

$$\gamma_2 = \frac{4r}{(2\bar{\tau} - \sigma_\tau^2) - \sqrt{(2\bar{\tau} - \sigma_\tau^2)^2 + 8r\sigma_\tau^2}}.$$

Coefficients δ_2 and δ_3 are determined in such a way that $H(\tau_0) = 0$ and $H'(\tau_0) = 0$ (see Appendix B). The first initial condition, $H(\tau_0) = 0$, assures that economic welfare,

$$W \equiv I(a_0, \tau_0, 0) = F(a_0, \tau_0) = \delta_0 + \frac{1}{r} \log \left(\frac{a_0}{\tau_0} \right),$$

is independent of the choice of H . The second initial condition, $H'(\tau_0) = 0$, simply says that taxing wealth reduces welfare, that is,

$$\left. \frac{\partial I}{\partial \tau} \right|_{\tau=\tau_0} = -\frac{1}{r\tau_0} < 0,$$

and also assures that H is the unique solution of (19).

A Viable Allocation of Portfolio Shares

Equation (18) has one negative and two positive roots. This can be seen by intersecting the straight line defined by the right-hand side of (18) with the graph defined by the left-hand side of (18). In such a case, there is only one intersection defining a unique, perfectly viable, steady-state share of wealth set apart for consumption such that $w^* \in (0,1)$.

Policy Experiments, Comparative Statics

We are now in a position to derive the first result: a once-and-for-all increase in the rate of devaluation, which results in an increase in the future opportunity cost of purchasing goods, leads to a permanent decrease in the proportion of wealth devoted to future consumption. To see this, we may differentiate (18) to find that

$$\frac{\partial w^*}{\partial \pi} = -\Psi^{-1} < 0, \quad (22)$$

where

$$\Psi = \left[\frac{r}{(w^*)^2} + \frac{\lambda \eta^2}{[1 + \eta(1 - w^*)]^2} + \sigma_p^2 \right]. \quad (23)$$

A second result is the response of the equilibrium share of real monetary balances, w^* , to once-and-for-all changes in the intensity parameter, λ . A once-and-for-all increase in the expected number of devaluations per unit of time causes an increase in the future opportunity cost of purchasing goods. This, in turn, permanently decreases the proportion of wealth set aside for future consumption. Indeed, after differentiating (18), we get

$$\frac{\partial w^*}{\partial \lambda} = -\frac{\eta}{\psi[1 + \eta(1 - w^*)]} < 0. \quad (24)$$

A similar effect is obtained for a once-and-for-all change in the mean expected size of a jump:

$$\frac{\partial w^*}{\partial \eta} = -\frac{\lambda}{\psi[1 + \eta(1 - w^*)]^2} < 0. \quad (25)$$

Finally, an increase in the *ad valorem* tax on consumption will produce a permanent reduction in the proportion of wealth devoted to future consumption.

$$\frac{\partial w^*}{\partial \hat{\tau}} = -\frac{1}{\alpha\Psi} < 0. \quad (26)$$

Welfare Implications

We will now assess the effects of exogenous shocks on economic welfare. As usual, the welfare criterion, W , of the representative individual is the maximized utility starting from the initial real wealth, a_0 , and the initial tax rate on wealth, τ_0 . In virtue of (14), welfare is given by:

$$W(\pi, \lambda, \eta, \bar{\tau}, \hat{\tau}; a_0, \tau_0) \equiv I(a_0, \tau_0, 0) = F(a_0, \tau_0) = \frac{1}{r} \left[1 + \log\left(\frac{a_0}{\tau_0}\right) + \log(\alpha^{-1}w^*) \right] \\ - \frac{1}{r^2} \left[\left((1 + \hat{\tau})\alpha^{-1} + r + \pi - \sigma_p^2 \right) w^* + \frac{1}{2} (w^* \sigma_p)^2 + \bar{\tau} - \frac{1}{2} \sigma_\tau^2 - \lambda \log\left(\frac{1 + \eta(1 - w^*)}{1 + \eta}\right) \right], \quad (27)$$

where we have used the fact that $H(\tau_0) = 0$.

Effects of Exchange-rate Shocks on Welfare

We now compute the impact on welfare of once-and-for-all changes in the mean expected rate of devaluation, the probability of devaluation, and the expected size of a devaluation. First, notice that under the assumption of logarithmic utility, an increase in the stochastic tax coming from devaluation reduces welfare. Indeed, differentiating (27) with respect to π , we find

$$\frac{\partial W}{\partial \pi} = -\frac{w^*}{r^2} < 0, \quad (28)$$

Similarly, exogenous shocks on the probability of devaluation will produce a reduction in economic welfare. To see this, it is enough to differentiate (27) with respect to λ

$$\frac{\partial W}{\partial \lambda} = \frac{1}{r^2} \log\left(\frac{1 + \eta(1 - w^*)}{1 + \eta}\right) < 0, \quad (29)$$

A once-and-for-all increase in the expected size of a devaluation decreases welfare, as

$$\frac{\partial W}{\partial \eta} = -\frac{1}{r^2} \left(\frac{\lambda w^*}{(1 + \eta)(1 + \eta(1 - w^*))} \right) < 0, \quad (30)$$

Effects of Fiscal Shocks on Welfare

Let us now compute the impact on welfare of once-and-for-all changes in the mean expected tax rate on wealth and the expected *ad valorem* tax on consumption. In this case, we have that

$$\frac{\partial W}{\partial \tau} = -\frac{1}{r^2} < 0, \quad (31)$$

and

$$\frac{\partial W}{\partial \hat{\tau}} = -\frac{1}{r^2} \alpha^{-1} w^* < 0. \quad (32)$$

Hence, increasing the mean expected tax rate on wealth and the tax rate on consumption will lead to a reduction in economic welfare.

Wealth and Consumption

We now derive the stochastic process that generates wealth when the optimal rule is applied. After substituting the optimal share w^* into (11), we get

$$da_t = a_t \left[\left(\frac{\lambda \eta w^*}{1 + \eta(1 - w^*)} + (w^* \sigma_p)^2 - \tau_t \right) dt - w^* \sigma_p dz_t + \left(\frac{1 + \eta(1 - w^*)}{1 + \eta} - 1 \right) dN_t \right], \quad (33)$$

where

$$\tau_t = \tau_0 \exp \left\{ \left(\bar{\tau} - \frac{1}{2} \sigma_\tau^2 \right) t + \varepsilon \sigma \sqrt{t} \right\}, \quad (34)$$

and $\varepsilon \sim N(0,1)$. The density of τ_t , given τ_0 , satisfies

$$f_{\tau_t | \tau_0}(x | \tau_0) = \frac{1}{\sqrt{2\pi t \sigma_\tau x}} \exp \left\{ -\frac{1}{2} \left(\frac{\log(x/\tau_0) - \left(\bar{\tau} - \frac{1}{2} \sigma_\tau^2 \right) t}{\sigma_\tau \sqrt{t}} \right)^2 \right\}. \quad (35)$$

We also have that

$$E[\tau_t | \tau_0] = \tau_0 e^{\bar{\tau} t} \quad (36)$$

and

$$Var[\tau_t | \tau_0] = \tau_0^2 e^{2\bar{\tau} t} \left(e^{\sigma_\tau^2 t} - 1 \right). \quad (37)$$

The solution to the stochastic differential equation in (33), conditional on a_0 , is (see Appendix A, formula (A.3))

$$a_t = a_0 e^{\xi_t}, \quad (38)$$

where

$$\xi_t = \theta_t + \phi_t, \quad \theta_t | \tau_t \sim \mathcal{N} \left[\left[F(w^*) - \tau_t \right] t, G(w^*) t \right], \quad (39)$$

$$\phi_t = L(w^*) N_t, \quad (40)$$

and³

$$N_t \sim \mathcal{P}(\lambda t). \quad (41)$$

The stationary components of the parameters of the above distributions are:

$$F(w^*) = \frac{\lambda \eta w^*}{1 + \eta(1 - w^*)} + \frac{(w^* \sigma_P)^2}{2},$$

$$G(w^*) = (w^* \sigma_P)^2,$$

and

$$L(w^*) = \log \left(\frac{1 + \eta(1 - w^*)}{1 + \eta} \right).$$

Notice also that

$$\mathbb{E}[\xi_t | \tau_t] = \left[F(w^*) - \tau_t + L(w^*) \lambda \right] t \quad (42)$$

and

$$\text{Var}[\xi_t | \tau_t] = \left[G(w^*) + \left[L(w^*) \right]^2 \lambda \right] t. \quad (43)$$

Moreover, it readily follows that

$$\mathbb{E}[\xi_t] = \mathbb{E} \left\{ \mathbb{E}[\xi_t | \tau_t] \right\} = \left[F(w^*) - \tau_0 e^{\bar{\tau}} + L(w^*) \lambda \right] t \quad (44)$$

and

$$\text{Var}[\xi_t] = \text{Var} \left\{ \mathbb{E}[\xi_t | \tau_t] \right\} + \mathbb{E} \left\{ \text{Var}[\xi_t | \tau_t] \right\} = t^2 \tau_0^2 e^{2\bar{\tau}} \left(e^{\sigma^2 t} - 1 \right) + \left[G(w^*) + \left[L(w^*) \right]^2 \lambda \right] t. \quad (45)$$

Finally, according to (38), notice that the last two equations determine the mean and variance of the growth rate of real assets.

Consumption Dynamics

In virtue of (9) and (38), the stochastic process for consumption can be written as

$$c_t^* = \alpha^{-1} w^* a_0 e^{\xi_t}. \quad (46)$$

This indicates that, in the absence of contingent-claims markets, the devaluation risk has an effect on wealth through the uncertainty in ξ_t , that is, uncertainty changes the opportunity set faced by the consumer. On the other hand, the devaluation risk also affects the composition of portfolio shares *via* its effects on w^* . Thus, a policy change will be accompanied by both wealth and substitution effects. From (46), we can compute the probability that, in a given time interval, certain levels of consumption occur. It is also important to note, regarding (46) and (12), that the assumption that the agent's time-preference rate is equal to the world's interest rate does not ensure a steady-state level of consumption.

However, we do have a steady-state share of wealth set aside for consumption. We may conclude that uncertainty is the clue to rationalize richer consumption dynamics that could not be obtained from deterministic models. Finally, in virtue of (46), equations (44) and (45) determine the mean and variance of the growth rate of consumption.

Consumption Booms

We will analyze now a policy of the form:

$$\pi_t = \begin{cases} \pi_1 & \text{for } 0 \leq t \leq T, \\ \pi_2 & \text{for } t > T, \end{cases} \quad (47)$$

where T is exogenously determined and $\pi_1 < \pi_2$, as in Calvo (1986). Notice that in our stochastic setting, there is a lack of credibility even if we do not change the four parameters since agents always assign some probability to the event of currency devaluation. Let us examine the response of consumption to (47). From (46), we may write

$$\frac{c_{T+\Delta}^*}{c_T^*} = \frac{w_2^*}{w_1^*} \exp\{-(\xi_t(\pi_1) - \xi_{T+\Delta}(\pi_2))\}$$

The exponential above tends to 1 as $\Delta \rightarrow 0^+$ a.s. (almost surely). This means that although the stationary components of the random variable ξ_t are different before and after time T , such a difference becomes negligible when $\Delta \rightarrow 0^+$. Consequently,

$$\lim_{\Delta \rightarrow 0^+} c_{T+\Delta}^* = c_T^* \frac{w_2^*}{w_1^*} \text{ a.s.} \quad (48)$$

We also notice that $w_2^*/w_1^* < 1$, together with (48), imply $c_T^* > \lim_{\Delta \rightarrow 0^+} c_{T+\Delta}^*$ a.s., indicating a jump (boom) in consumption at time T . In other words, if the plan is expected to be temporary, then there is a jump in consumption at T , as we have shown above. Therefore, Calvo's (1986) deterministic result on the response of consumption to temporary stabilization is locally maintained (around T a. s.) in our stochastic setting. Notice that the findings are related to those of Calvo and Drazen (1997) with no contingent assets. A similar analysis can be applied to any of the remaining parameters determining the expectations of devaluation, namely λ and η .

Simulation Exercise

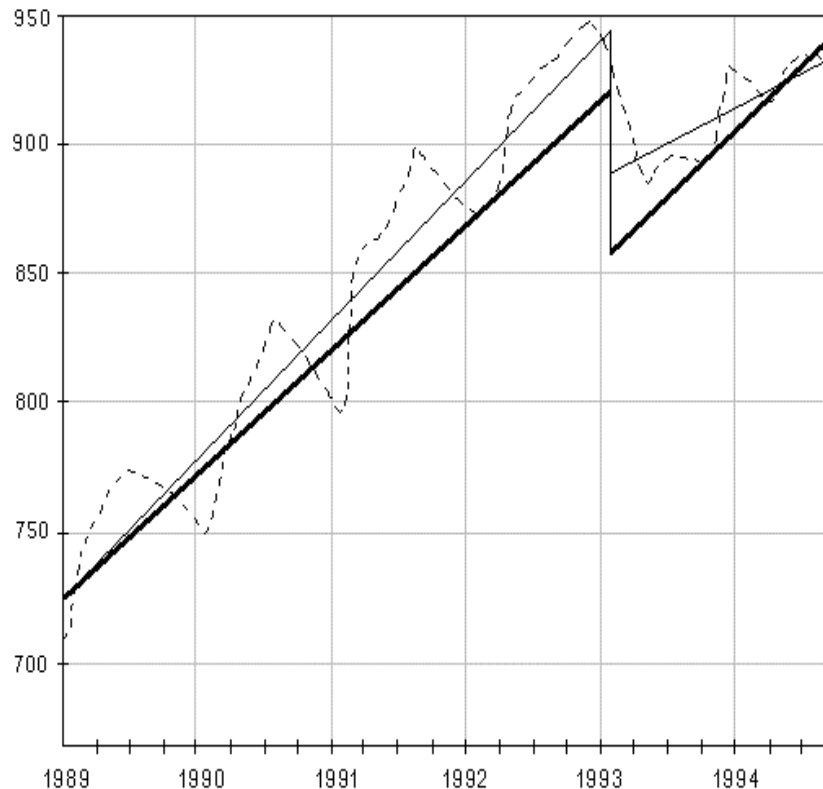
The following experiment is intended to simulate, via Monte Carlo methods, the response of consumption to permanent changes in the values of the parameters that determine the expectations of devaluation. Table 1 presents two vectors of parameter values, $(\pi_j, \sigma_{p_j}^{-1}, \lambda_j, \eta_j)$, $j = 1, 2$, that reproduce the trend and jump of the observed consumption in Mexico between 1989 and 1994.

Table 1
Optimal consumption shares and parameter values

| | |
|--|---------------------------|
| $w_1^* = 0.455299$ | $w_2^* = 0.430004$ |
| $\pi_1 = 0.200000$ | $\pi_2 = 0.300000$ |
| $\sigma_{p_1} = 0.0999$ | $\sigma_{p_2} = 0.009999$ |
| $\lambda_1 = 0.010000$ | $\lambda_2 = 0.100000$ |
| $\eta_1 = 0.200000$ | $\eta_2 = 0.300000$ |
| $F(w_1^*) = 0.001858$ | $F(w_2^*) = 0.011026$ |
| $G(w_1^*) = 0.002073$ | $G(w_2^*) = 0.000019$ |
| $L(w_1^*) = -0.000343$ | $L(w_2^*) = -0.004539$ |
| $\alpha = 0.980000$ | |
| $r = 0.085000$ | |
| $\bar{\tau} = 0.060000$ | |
| $\sigma_\tau = 0.180000$ | |
| $a_0 = 1.84900 \times 10^{12}$ (pesos of 1993) | |

In Figure 1, the light solid line represents the simulated trend of consumption before wealth and consumption taxes, and the heavy solid line the simulated trend of

Figure 1
Simulated Trends: The Light Solid Line Represents before-tax Consumption, and the Heavy Solid
Line After-Tax Consumption (Thousands of Millions of Pesos of 1993).
The Dashed line Corresponds to Observed Consumption



consumption after wealth and consumption taxes. The dashed line corresponds to observed consumption. Notice that, with the above parameter values, the stochastic simulation, with taxes, mimics the order of magnitude of the consumption jump observed in the first quarter of 1993: a jump about 60 thousands of millions of pesos of 1993. Without the presence of taxes the magnitude of the consumption jump is almost preserved but the simulated trend is steeper.

Conclusions

The “credibility literature” has by now exhausted a class of deterministic models aimed at explaining tax effects. Most of the existing models ignore uncertainty providing elaborate theoretical justification. After all, what produces expected temporariness is uncertainty. We have presented a stochastic model of exchange-rate-based stabilization with imperfect credibility. An important feature of our formulation is that there is a lack of credibility even if we do not change the parameters determining the expectations of devaluation.

Various forms of distortionary taxes have been considered, a stochastic tax on welfare and an *ad valorem* tax on consumption. We have shown that an uncertain tax rate on wealth may lead to significant quantitative changes in the effects of fiscal policy, in contrast with the deterministic setting. The consideration of taxes have led to more complex transitional dynamics, but results were certainly richer. In our proposal, uncertainty has been the clue to rationalize richer consumption dynamics in temporary stabilization.

Our stochastic framework, in which a Brownian motion and a Poisson process drive the expectations of devaluation, and a geometric Brownian motion guides a tax rate on wealth, has provided new elements to carry out simulation experiments and empirical research on some observed regularities that still need to be explained. In particular, our stochastic model was capable of explaining the observed orders of magnitude of consumption booms, in the presence of an uncertain tax policy, for the Mexican case of 1989-1994.

APPENDIX A

In this appendix we state without proof⁷ two useful results in the development of this paper:

1) The Itô's lemma for mixed diffusion-jump processes, which can be stated as follows. Given the homogeneous linear stochastic differential equation

$$dx_t = x_t (\mu dt + \sigma dz_t + \eta dq_t), \quad z_t \sim \mathcal{N}(0, t), \quad q_t \sim \mathcal{P}(\lambda t). \quad (\text{A.1})$$

and $g(x)$ twice continuously differentiable, then the "stochastic" differential of $g(x_t)$ is given by

$$dg(x_t) = \left[g_x(x_t) \mu x_t + \frac{1}{2} g_{xx}(x_t) \sigma^2 x_t^2 \right] dt + g_x(x_t) \sigma x_t dz_t + [g(x_t(1+\eta)) - g(x_t)] dq_t. \quad (\text{A.2})$$

2) The solution to (A.1) is given by

$$x_t = x_0 \exp \left\{ \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma \int_0^t dz_u + \log(1+\eta) \int_0^t dq_u \right\}. \quad (\text{A.3})$$

It is also worthwhile to keep in mind, when using (A.3), that for $t \geq 0$ the following properties for z_t and q_t hold:

$$\mathbb{E} \left[\int_0^t dz_u \right] = 0, \quad \mathbb{E} \left[\left(\int_0^t dz_u \right)^2 \right] = \int_0^t du = t, \quad \text{and} \quad \mathbb{E} \left[\int_0^t dq_u \right] = \lambda t.$$

APPENDIX B

In this appendix, we solve the nonhomogeneous linear second-order ordinary differential equation stated in (20). Let $H = H(\tau)$ and consider the nonhomogeneous Euler-Cauchy type equation

$$\tau^2 H'' + \frac{2\tau}{\sigma^2} \tau H' - \frac{2r}{\sigma^2} H = -\frac{2}{\sigma^2} \log(\tau) + \frac{2}{r\sigma^2} \tau, \quad (\text{B.1})$$

where r and σ are positive constants. In order to transform (B.1) into a differential equation with constant coefficients, we apply Euler's method by using the change of variable $\tau = e^t$. Hence $t = \log(\tau)$,

$$\frac{\partial H}{\partial \tau} = \frac{1}{\tau} \frac{\partial H}{\partial t}, \quad (\text{B.2})$$

and

$$\frac{\partial^2 H}{\partial \tau^2} = \frac{1}{\tau^2} \left(\frac{\partial^2 H}{\partial t^2} - \frac{\partial H}{\partial t} \right). \quad (\text{B.3})$$

After substituting (B.2) and (B.3) in (B.1), we obtain

$$\frac{\partial^2 H}{\partial t^2} + \left(\frac{2\bar{\tau}}{\sigma^2} - 1 \right) \frac{\partial H}{\partial t} - \frac{2r}{\sigma^2} H = -\frac{2}{\sigma^2} t + \frac{2}{r\sigma^2} e^t. \quad (\text{B.4})$$

The general solution of this equation is of the form:

$$H(t) = H_C(t) + H_P(t), \quad (\text{B.5})$$

where H_C is the complementary function associated with the homogeneous equation, and H_P is a single particular solution of the nonhomogeneous equation. To find H_C we need to solve first the following characteristic equation:

$$\gamma^2 + \left(\frac{2\bar{\tau}}{\sigma^2} - 1 \right) \gamma - \frac{2r}{\sigma^2} = 0.$$

Hence, the complementary function is

$$H_C(t) = \delta_2 e^{\gamma_1 t} + \delta_3 e^{\gamma_2 t}, \quad (\text{B.6})$$

where the two roots are given by

$$\gamma_1 = \frac{4r}{(2\bar{\tau} - \sigma^2) + \sqrt{(2\bar{\tau} - \sigma^2)^2 + 8r\sigma^2}}$$

and

$$\gamma_2 = \frac{4r}{(2\bar{\tau} - \sigma^2) - \sqrt{(2\bar{\tau} - \sigma^2)^2 + 8r\sigma^2}}$$

To find now H_P , we may use the method of undetermined coefficients. Let us try the guess

$$H_P(t) = At + B + Cte^t, \quad (\text{B.7})$$

so $H'_P(t) = A + C(t+1)e^t$ and $H''_P(t) = C(t+2)e^t$. After substituting (B.7) in equation (B.4), we get

$$\left(\frac{2\bar{\tau}}{\sigma^2} - \frac{2r}{\sigma^2}\right)Cte^t + \left(1 + \frac{2\bar{\tau}}{\sigma^2}\right)Ce^t - \frac{2r}{\sigma^2}At + \left(\frac{2\bar{\tau}}{\sigma^2} - 1\right)A - \frac{2r}{\sigma^2}B = -\frac{2}{\sigma^2}t + \frac{2}{r\sigma^2}e^t.$$

Solving for the coefficients A , B and C , we obtain

$$A = \frac{1}{r}, \quad B = \frac{1}{2r^2}(2\bar{\tau} - \sigma^2), \quad \text{and} \quad C = \frac{2}{r(\sigma^2 + 2\bar{\tau})},$$

and find that for a particular solution we must have $\bar{\tau} = r$. Therefore,

$$H_P(t) = \frac{1}{\tau}t - \frac{\sigma^2}{2\tau^2} + \frac{1}{\tau} + \frac{2}{\tau(\sigma^2 + 2\bar{\tau})}te^t. \quad (\text{B.8})$$

Substituting (B.6) and (B.8) into (B.5) leads to

$$H(t) = \delta_2 e^{\gamma_1 t} + \delta_3 e^{\gamma_2 t} + \frac{1}{\tau}t - \frac{\sigma^2}{2\tau^2} + \frac{1}{\tau} + \frac{2}{\tau(\sigma^2 + 2\bar{\tau})}te^t.$$

Since $\tau = e^t$, the general solution of (B.1), in terms of τ , is given by

$$H(\tau) = \delta_2 \tau^{\gamma_1} + \delta_3 \tau^{\gamma_2} + \frac{1}{\tau} \log(\tau) \left(1 + \frac{2}{(\sigma^2 + 2\bar{\tau})} \tau \right) + \frac{1}{\tau} \left(1 - \frac{\sigma^2}{2\bar{\tau}} \right). \quad (\text{B.9})$$

The values of δ_2 and δ_3 satisfying the initial conditions $H(\tau_0) = H'(\tau_0) = 0$ are

$$\delta_2 = \frac{\tau_0^{-\gamma_1}}{\tau(\gamma_1 - \gamma_2)} \left[\gamma_2 \left(\log(\tau_0) + 1 - \frac{\sigma^2}{2\bar{\tau}} \right) - \frac{2\tau_0}{\sigma^2 + 2\bar{\tau}} (1 + \log(\tau_0)(1 - \gamma_2)) + 1 \right]$$

and

$$\delta_3 = \frac{\tau_0^{-\gamma_2}}{\tau(\gamma_1 - \gamma_2)} \left[-\gamma_1 \left(\log(\tau_0) + 1 - \frac{\sigma^2}{2\bar{\tau}} \right) + \frac{2\tau_0}{\sigma^2 + 2\bar{\tau}} (1 + \log(\tau_0)(1 - \gamma_1)) + 1 \right].$$

Notes

1. We direct the reader to the references contained in Calvo and Végh (1999).
2. An alternative stochastic volatility approach can be found in Venegas-Martínez (2006).
3. $x \sim \mathcal{P}(a)$ denotes a Poisson random variable x with mean a .
4. In order to choose a pair of vectors replicating stylized facts, we tried about 800 different feasible combinations of parameter values.
5. For simulation purposes, we have used a standard discrete-time version of (46) with an appropriate unit of time, see, for instance, Ripley (1985) and Press *et al.* (1992). Here, the critical part of Monte Carlo simulation is the simulation of a Brownian motion combined with a jump process by generating independent random numbers drawn from both the normal

- and the Poisson distributions. The consumption trend is estimated as an average of simulated paths using (46) repeatedly. Results are based on 10,000 iterations.
6. Data Source: INEGI.
 7. For the proofs, we refer the reader to Gihman and Skorohod (1972, chapter 2).

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