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### Sensitivity Study of Wavelet Selection in Structural Health Monitoring

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**Abstract:** The objective of this work focuses on damage detection in a beam structure using wavelet transform. The wavelet transform is a powerful tool for detecting the damages in beam and plate like structures. The data used for damage detection are transverse nodal displacement values. The different types of continuous wavelet transforms like Daubechies, Symlets, Coiflets, Gaussian, Dmeyer, Morlet, Biorsplines, and Reversebior are used. The rectangular beam is modeled numerically. The modal analysis is performed in a beam with a fixed- fixed boundary condition. The natural frequency and mode shapes of different damaged cases are examined. Damage is simulated by reducing young's modulus value percentage of the element. The difference data of damaged and undamaged mode shape data's are used. Mode sensitivity and wavelet selection procedure examined for beam structure. The absolute wavelet coefficient data's are plotted for damage localization and quantification. From this work it is proved the Symmetrical wavelet type is best for damage identification using translational spatial data.

**Keywords:** Structural condition monitoring, Damage identification, Mode shapes, Natural frequency, Wavelets, Absolute wavelet coefficient

#### 1. INTRODUCTION

Damage detection in the structure is complicated process before last two decades, now the Vibration based damage detection techniques is much easier than the traditional method, when the frequency based and mode shape based damage detection are not able to detect the small defect . Wavelet transform method is used to detect the small damage also for this process only spatial data is enough. Damage present in the structure will affect the structure performance. Damage may be occurring into following stages that are during the Manufacturing stage and another one is a working stage. Cawley and Adams <sup>[1]</sup> suggested utilizing the dynamic characteristics in damage detection based on natural frequency method. Hou et al <sup>[2]</sup> suggested wavelet based damage detection method is applicable online and offline. Chang and Chen <sup>[3]</sup> discuss the mode shapes of Timoshenko beam, only spatial data is sufficient for damage detection and it is high sensitivity to the crack depth. Loutridisa et al <sup>[4]</sup> examined a double-cracked cantilever beam by the symmlet wavelet and crack location identified even the noise present but it is unable to locate the depth of damage. Douka et al <sup>[5]</sup> proposed intensity factor for estimating the crack depth. Gentile and Messina <sup>[6]</sup> analyzed beam structure mathematically with Gaussian wavelet; results

explore some valuable suggestion like some mode shapes are not sensitive in damage detection but some other mode shapes have higher sensitivity and windows with high number of vanishing points preferred for damage detection, Hani and Hansang<sup>[7]</sup> explains FFT and CWT analyzes are performed in concrete beam and slab structures FFT is not able to locate damage slab but CWT is performed well enough to detect damage for both structures, Chang and Chen<sup>[8]</sup> extended Wavelet transform for rectangular plate structure, Gabor wavelets easily locate the damage but the size of the damage is shown higher sensitivity, Chang and Chen<sup>[9]</sup> examined rotating thick blades with damaged structure analyzed with the help of Gabor wavelets, results are agreeable, Geometric discontinues shows the peak for representing the crack, damage near by the boundaries are showing highest peak values because of local perturbations caused by the crack are larger, Li et al <sup>[10]</sup> implemented wavelet finite element method in the forced vibration analysis of beam structure with free-free boundary condition, results shows particularly overestimated the crack severity. Kim and Melham <sup>[11]</sup> shows discrete wavelets have less time consumption while comparing to the continuous wavelet transform, Ovanesova and Suarez<sup>[12]</sup> explained the selection of wavelet criteria for damage detection and properties of wavelets are discussed and crack detection in frame structure was examined, Chang and Chen<sup>[13]</sup> evaluated Timoshenko cantilever beam numerically and experimentally, and clearly suggests bending natural frequency is produced much better result and N number of natural frequency is necessary to detect the N number of cracks . Rucka and Wilde <sup>[14]</sup> utilized Gaussian wavelet for one dimensional beam structure and Reverse Biorthogonal wavelet applied for two dimensional plate structure for damage detection. Results shown symmetric wavelets are optimal for damage detection; FRF mode shapes data's are effective in wavelet based damage detection even the crack is near by the boundary. Castro et al <sup>[15]</sup> Highlights the two different types of defects in rod damage classified into two types that are Density type defect, Stiffness type defect, density type damage structures modal displacements having anti-node near the damage location. Xiang et al <sup>[16]</sup> proposed combination of wavelet based elements and genetic algorithm to minimize the error ratio in estimate the depth of the crack. Dong et al <sup>[17]</sup> explained a new method based on high-precision modal parameter identification method and wavelet finite element (WFE) model is presented to determine the depth and location of a transverse surface crack in a rotor system. Bouboulas and Anifantis <sup>[18]</sup> DWT can be used to identify the exact position of the crack. A qualitative analysis is required to understand the results precisely. It proves that FFT and CWT can be used as additional tools for crack detection techniques. Liu et al <sup>[19]</sup>, chosen Gabor and Morlet wavelets for wave signal processing due to their good time-frequency characteristics. Both wavelets perform well in the crack detection of composite beams. The results show that the crack location can be determined, and the crack depth can be estimated from the crack-reflection ratio. The Gabor wavelet can locate the cracks more accurately by virtue of its higher time resolution. Sandesh and Shankar <sup>[20]</sup> implemented novel hybrid Particle Swarm optimization algorithm combined with genetic algorithm for time domain damage identification. Hemanth et al <sup>[21]</sup> explained transfer function based on damage detection is effective in damage detection field. Lee and Lam <sup>[22]</sup> suggested a hybrid model of mixture of the general regression neural network model and fuzzy art FA (GRNFA) data's utilized neural network to diagnosis the structure. Reddy and Swarnamani <sup>[23]</sup> proposed FRF curvature energy damage index method for damage detection in plate like structures. Reddy and Swarnamani <sup>[24]</sup> used strain energy data for wavelet transform and show the effectiveness of the wavelet transform in the damage detection of plate structure.

Katunin and Holewik<sup>[24]</sup> proposed a Novel method of damage identification based on wavelet decomposition and reconstruction. Khorram et al <sup>[25]</sup> applied Continuous Wavelet Transform (CWT) combined with factorial design on simply supported beam subjected to a moving load along the beam. Applying factorial design, it is shown that the damage index (DI) of one crack does not depend on the size and location of other cracks in a multiple cracked beam. Xiang et al <sup>[26]</sup> proposed a hybrid damage detection approach for conical shell, analyzed wavelet-based method have higher efficiency and precision than traditional finite element methods. Katunin <sup>[27]</sup> proposed 2D signal decomposition using 2D wavelet transform (WT) with B-spline wavelets of fractional order based two steps damage detection algorithm and analyzed honey comb- core sandwich structure. Mehrjoo et al <sup>[28]</sup> proposed inverse approach to find transverse crack in structure based on genetic algorithm. Feng and Li <sup>[29]</sup>

proposed novel supposed waveform method to measure vibrating signals. Katunin et al [30] proposed a novel method for automated damage identification and classification from the 3D CT scans using 3D wavelet-based algorithm. Xu et al utilized SLV measurement system to observe for 1D and 2D mode shapes of beam and plate damage detection. Results explained it is better in traditional measurement based on sensor <sup>[31]</sup>

From the studies on vibration analysis of damaged beams, only few useful and practical techniques are found for very small damage identification. In this paper a study is carried out to investigate the influence of using the translational mode shapes are input for the wavelets on damage identification for different scenarios, damaged beam subjected to fixed-fixed boundaries. This paper presents a method for beam structure damage detection. The different wavelets transforms are used to analyze spatially distributed signals of translation mode shapes of the structure. The crack location can be effectively detected by the proposed method even though the crack size is very small.

## 2. FUNDAMENTAL OF WAVELET ANALYSIS

In employing the wavelet technique, two important mathematical entities must be introduced wavelet functions and wavelet transform. A complex-valued  $\varphi(x)$  that is localized in both time and frequency domains is used to create a family of wavelets  $\varphi_{a,b}(x)$ , where  $a$  and  $b$  are real numbers that dilate (scale) and translate the function  $\varphi(x)$ , respectively these  $\varphi(x)$  functions are known as mother wavelets and can be continuous or discrete. For the continuous case, the complex -valued wavelets generated from the mother wavelet are given by

$$\varphi_{a,b}(x) = |a|^{-1/2} \varphi\left(\frac{x-a}{a}\right) \quad (1)$$

Where  $a$  is the real-valued dilation parameter and  $b$  the real -valued translation parameter. For the discrete case, the wavelets take the form.

$$\varphi_{j,k}(x) = 2^{\frac{j}{2}} \varphi(2^j x - k) \quad (2)$$

Where  $\varphi$  is the mother wavelet,  $j$  the integer dilation and integer translation index, for given integrated signal, the wavelet transform obtained by integrating the product of signal and the complex conjugate wavelet function. The resulting wavelet is the set of wavelet coefficients is the measure of the correlation between wavelet and corresponding wavelet signal. Mathematically, continuous wavelet transform,  $wf_{a,b}$  of a  $f(x)$  is defined as

$$\begin{aligned} wf_{a,b} &= \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \varphi\left(\frac{x-b}{a}\right) dx \\ &= \int_{-\infty}^{\infty} f(x) \varphi_{ab}(x) dx \end{aligned} \quad (3)$$

Whereas discrete wavelet transform  $wf_{j,k}$ , is defined

$$wf_{a,b} = 2^{\frac{j}{2}} \int_{-\infty}^{\infty} f(x) \varphi(2^j x - k) \quad (4)$$

$$= \int_{-\infty}^{\infty} f(x) \varphi_{j,k}(x) dx \quad (4)$$

The Gaussian family built function by taking the  $p^{th}$  derivative of  $f$ .

$$f(x) = c_{pe} - x^2 \quad (5)$$

Wavelet coefficients are performed with various scale indexes, local perturbations in the mode shapes will be found in the fine scale wavelets that are positioned at the locations of the perturbations.

The procedure of the damage detection is as follows [24] :

1. Find the mode shapes of the structure.
2. Calculate the spatial wavelet coefficients of the mode shapes.
3. Plot the value of Absolute Coefficient of different mother wavelets along the length of the beams.
4. Examine the distributions of wavelet coefficients at each scale. A sudden change in the distributions of the wavelet coefficients identifies the damage position.
5. Maximum wavelet coefficient present in the damage location values shows the damage severity

### 3. SIMULATION RESULT AND DISCUSSION

#### 3.1. Beam model and damage scenarios

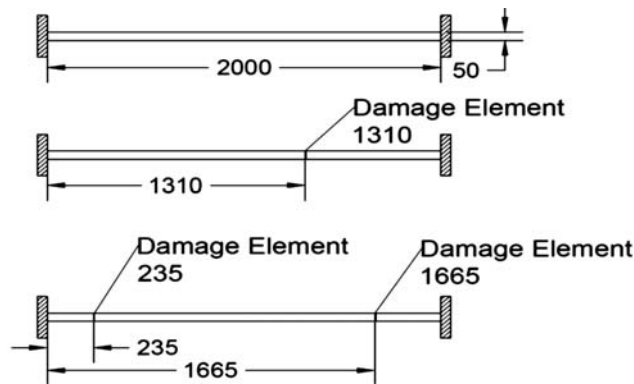


Figure 1: Fixed- fixed beam without and with damage (dimensions: 2000 X50X5) in mm

Figure 1 shows the undamaged rectangular beam structure and single and double damaged beam structure and damaged element location is mentioned with element number. In this current work simply supported beam with rectangular cross section  $h = 50\text{mm}$ ,  $b = 5\text{mm}$ , and length  $2000\text{mm}$  modeled in ANSYS 12. The Young's modulus ( $69\text{Gpa}$ ), Poisson's ratio ( $0.35$ ), density ( $2710\text{ kg/m}^3$ ). The 2 node BEAM 3 elastic is taken as element type. The length of the beam is divided into 2000 divisions and width of the beam is considered a single division. The total numbers of elements obtained are 2000. The damage is simulated by reducing Young's modulus value of the element in beam structure. Different damage cases and corresponding reducing in Young's modulus values are tabulated as shown in Table 1. The modal analysis is performed on the beam structure with fixed-fixed boundary conditions. Natural frequency, mode shapes and nodal displacements (translational) values of damaged and undamaged and all cases calculated.

Table 1  
Represents different damage cases with element number

Damage Cases	% of Young's modulus reduction	Damage element number
1	90	1310
2	60	1310
3	30	1310
4	10	1310
5	1	1310
6	60	235,1665

### 3.2. Effect of damage on modal parameter

For all damage cases dynamic characteristics (Natural frequency and Mode shapes) of damaged and undamaged beam structure are numerically calculated. Table 2 shows the first five natural frequency of the damaged and undamaged beam structure. The natural frequencies are varying in the range of first mode, 0.0001 to 0.0094% for second mode, 0.001 to 0.064% for mode 3, 0.002 to 0.017% for mode 4 0.001 to 0.17 % for the last mode 0.004 to 0.31 % .

**Table 2**  
First five natural frequencies (Hz) for undamaged and damaged cases

Damage Cases	Frequency(Hz) for the mode number				
	1	2	3	4	5
UD	6.4897	17.888	35.065	57.959	86.570
Case 1	6.4803	17.824	35.063	57.786	86.265
Case 2	6.4881	17.877	35.065	57.930	86.518
Case 3	6.4892	17.885	35.065	57.950	86.555
Case4	6.4896	17.887	35.065	57.957	86.566
Case 5	6.4897	17.888	35.065	57.958	86.570
Case 6	6.4871	17.886	35.048	57.903	86.479

Changes in natural frequencies highlight the damage present in the structure but it doesn't provide information about the location of structural damage.

### 3.3. Damage Detection using Mode Shapes

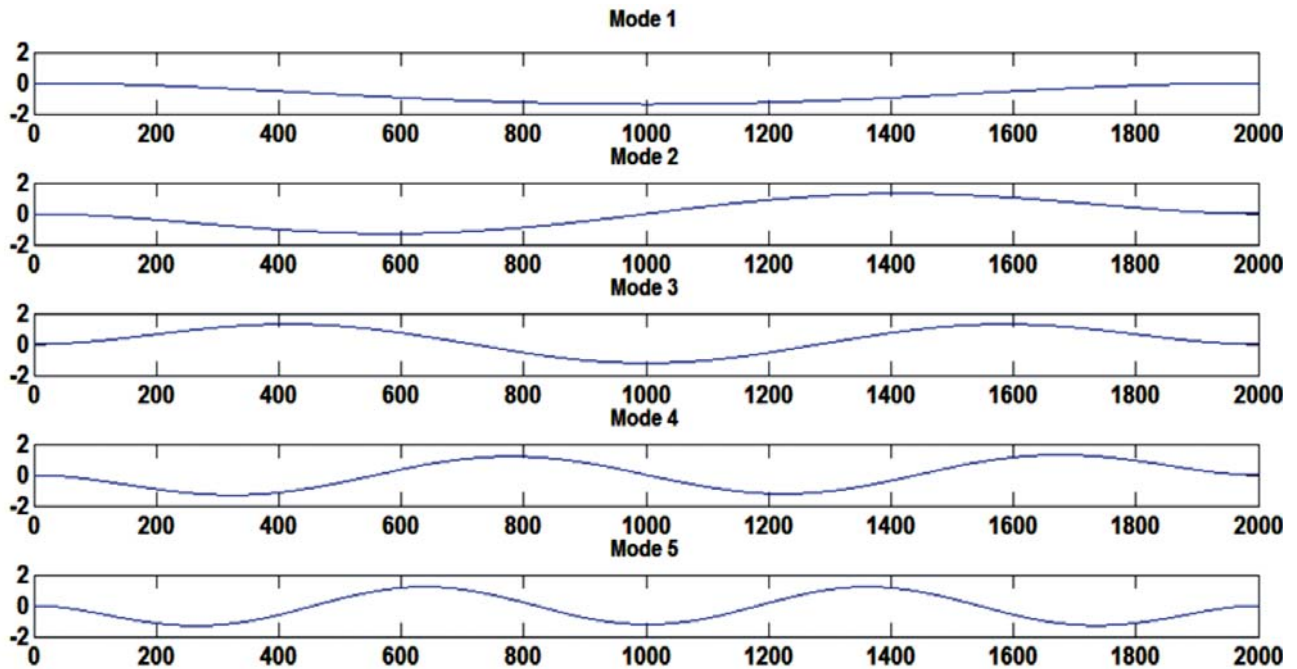


Figure 2: First five mode shapes for Damage Case 1

Figure 2 shows the first five mode shapes of the damaged (Case 1) beam structure. From the mode shapes damage location is not able to detect because the damage size is very small.

### 3.4. Optimal Mode selection & Wavelet selection Criteria

Wavelet based damage detection technique is an effective method in damage detection even the damage size is small. Sensitivity of this method depends upon the modal data because some modes are very effective but some other modes are not much effective, consider this drawback initially the modes are analyzed with different wavelets. Difference data of undamaged and (Case 1) Damaged beam structure's transverse displacements data's are processed into different types of wavelets, Absolute wavelet coefficient data's are plotted along the length of the beam, Abs. Coefficient peak values shows the damage location. The Absolute Wavelet coefficient value is used to measure the damage severity. Maximum Absolute wavelet coefficient and its location observed to predict the damage location and severity of the damage.

**First mode:**

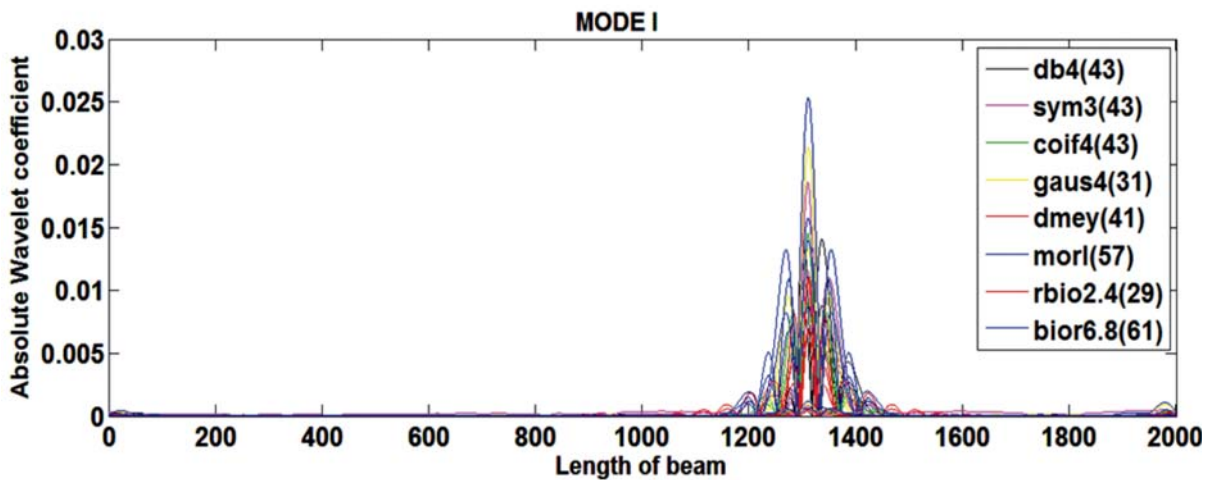


Figure 3: Damage Case 1. Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

Table 3  
Damage detection sensitivity of different wavelets for Mode 1

Different Wavelet	Damaged element	Predicted element	Max. Abs coefficient
Db4	1310	1340	0.00066
Sym3	1310	1311	0.00083
Coif4	1310	1313	0.00064
Gaus	1310	1313	0.00098
Dmey	1310	1315	0.00048
Morl	1310	1312	0.00068
Rbio	1310	1312	0.00048
bior	1310	1312	0.0012



Figure 3 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. Table 3 highlight all the wavelets are detecting the damage location with small error but Db4 wavelet is having relatively large error. Absolute wavelet coefficient values are not identical for the same level of damage, so selecting the wavelet type is a challenging task, the value of Absolute wavelet coefficients are consider to be thickness of damage, but it always show slightly higher than actual, because of its fundamental shape fits with discontinuity location present in the geometric shape. In this mode damage localization is sensitive but the Absolute wavelet coefficients values are poor comparing to original damage level so this mode is not suitable to estimate the thickness of the damage.

**Second mode:**

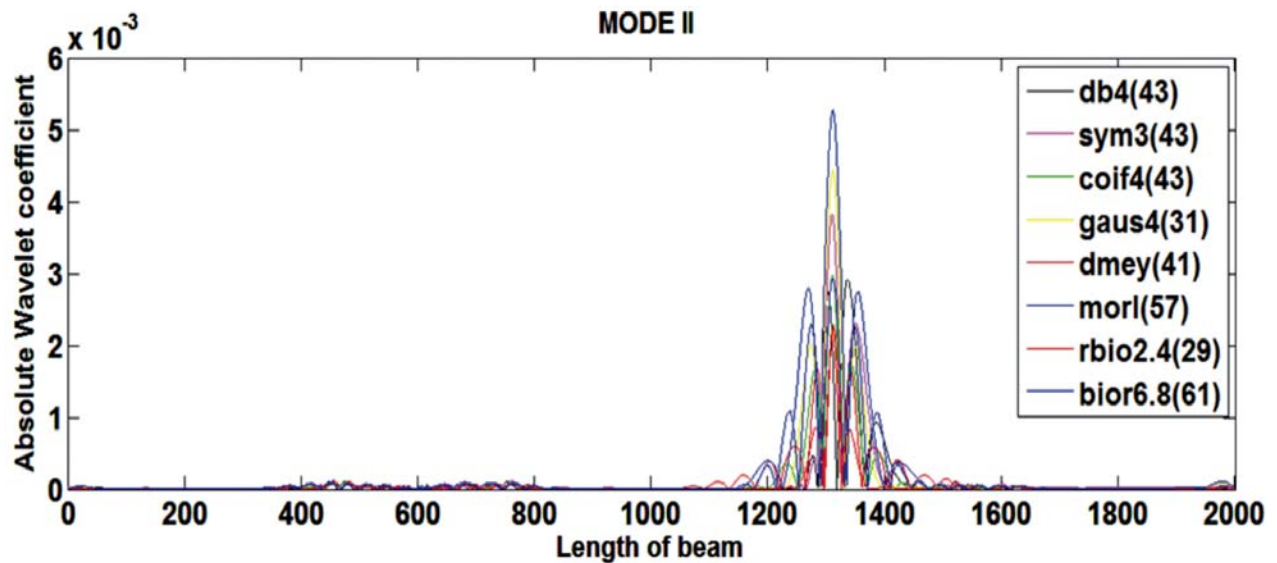


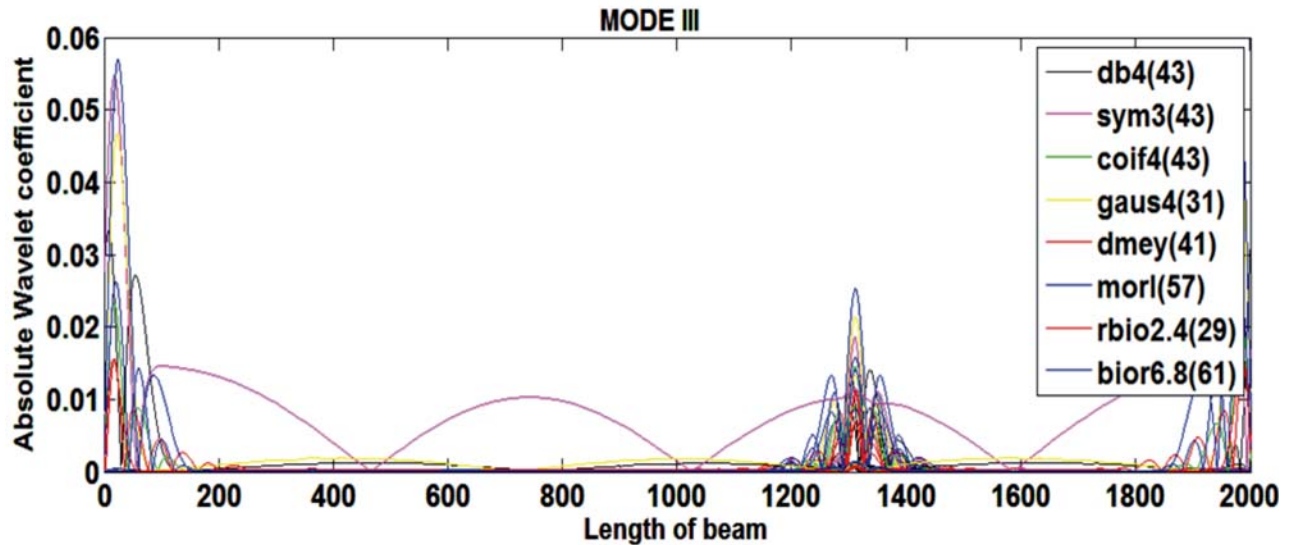
Figure 4: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

Table 4  
Damage detection sensitivity of different wavelets for Mode 2

Different Wavelet	Damaged element	Predicted element	Max. Abs coefficient
Db4	1310	1337	0.0029
Sym3	1310	1311	0.0038
Coif4	1310	1311	0.0030
Gaus	1310	1312	0.0044
Dmey	1310	1313	0.0023
Morl	1310	1312	0.0029
Rbio	1310	1312	0.0022
bior	1310	1312	0.0053

Figure 4 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. Table 4 highlight all the wavelets are detecting the damage location with small error but Db4 wavelet is having relatively large error. In this mode damage localization is sensitive and also the Absolute wavelet coefficients values of few wavelets like Sym3, Gaus4, and Bior6.8 are matching with the original thickness reduction. So these wavelets are optimal for these kind of problems, Predicted element severity is always high comparing with the original thickness of the damage.

**Third mode:**



**Figure 5: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets**

Figure 5 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. Table 5 highlight all the wavelets are not detecting the damage location. In this mode damage localization is not sensitive. If the mode is having less sensitivity the Absolute wavelet coefficients values are not similar with actual damage depth, so this mode is not preferable for damage detection.

**Table 5  
Damage detection sensitivity of different wavelets for Mode 3**

Different Wavelet	Damaged element	Predicted element	Max. Abs coefficient
Db4	1310	8	0.0333
Sym3	1310	18	0.0546
Coif4	1310	18	0.0232
Gaus	1310	23	0.0467
Dmey	1310	1988	0.0156
Morl	1310	21	0.0262
Rbio	1310	17	0.0155
bior	1310	25	0.0569



**Fourth mode:**

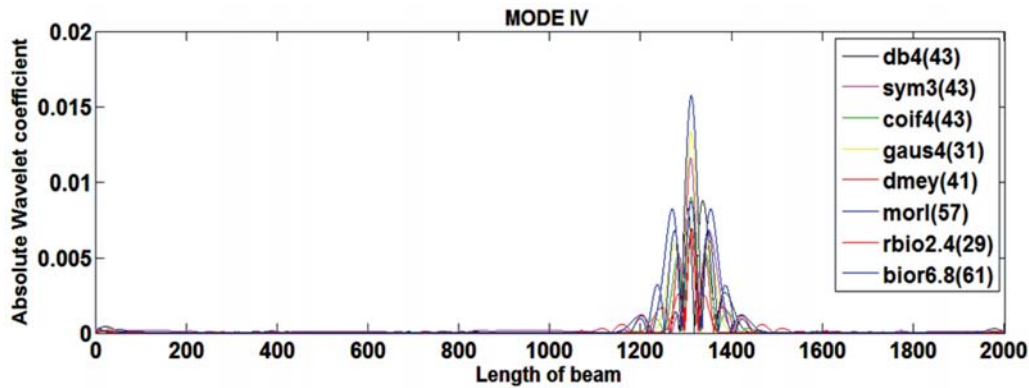


Figure 6: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

**Table 6**  
Damage detection sensitivity of different wavelets for Mode 1

Different Wavelet	Damaged element	Predicted element	Max. Abs coefficient
Db4	1310	1337	0.0088
Sym3	1310	1311	0.0116
Coif4	1310	1311	0.0090
Gaus	1310	1312	0.0133
Dmey	1310	1313	0.0069
Morl	1310	1312	0.0087
Rbio	1310	1312	0.0067
bior	1310	1312	0.0157

Figure 6 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. Table 6 highlight all the wavelets are detecting the damage location with small error but Db4 wavelet is having relatively large error. In this mode damage localization is sensitive and also the Absolute wavelet coefficients values of few wavelets like Sym3, Gaus4, and Bior6.8 are matching with the original thickness reduction. So these wavelets are optimal for these kind of problems, Predicted element severity is always high comparing with the original thickness of the damage. Comparing to the entire modes fourth mode is suitable for damage detection

**Fifth mode:**

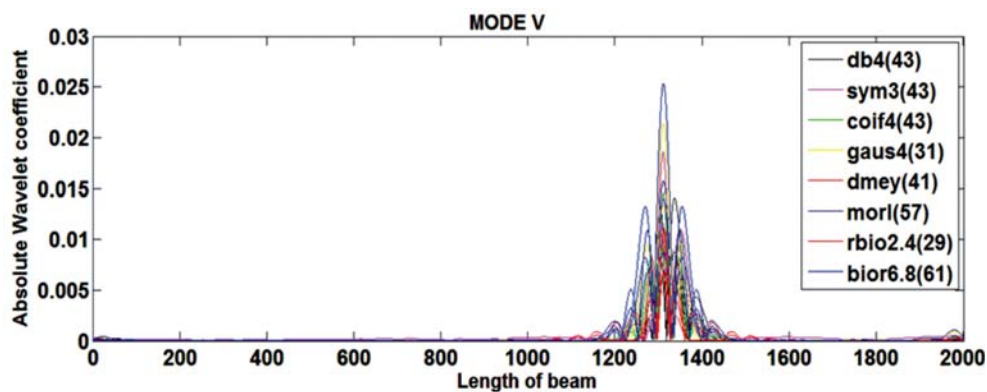


Figure 7: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

Figure 7 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location.

**Table 7**  
**Damage detection sensitivity of different wavelets for Mode 1**

Different Wavelet	Damaged element	Predicted element	Max. Abs coefficient
Db4	1310	1337	0.0141
Sym3	1310	1311	0.0186
Coif4	1310	1312	0.0144
Gaus	1310	1312	0.0214
Dmey	1310	1313	0.0111
Morl	1310	1312	0.0140
Rbio	1310	1312	0.0108
bior	1310	1312	0.0253

Table 7 highlight all the wavelets are detecting the damage location with small error but Db4 wavelet is having relatively large error. In this mode damage localization is sensitive and also the Absolute wavelet coefficients values of few wavelets like Sym3, Gaus4, and Bior6.8 are matching nearby with the original thickness reduction. So these wavelets are optimal for these kind of problems, Predicted element severity is always high comparing with the original thickness of the damage.

Comparing all the five modes the fourth mode is better in damage detection the mode selection criteria is based on not only locating the damage and also consider to provide the damage thickness data's, in this aspect the fourth mode is optimal. Wavelet selection is one of the important criteria for the wavelet based damage detection method because each and every wavelets having the different fundamental shape, mostly symmetrical wavelets are most effective in wavelet method, the Absolute wavelet coefficients values of few wavelets like Sym3, Gaus4, and Bior6.8 are matching nearby with the original thickness reduction, so all other damage cases are analyzed with these wavelets in particularly fourth mode, because these two factor is dominant in the entire research, once optimal mode and optimal wavelet selection. Further these wavelets are tested for all damage cases for fourth mode spatial data.

**Damage Case 2:**

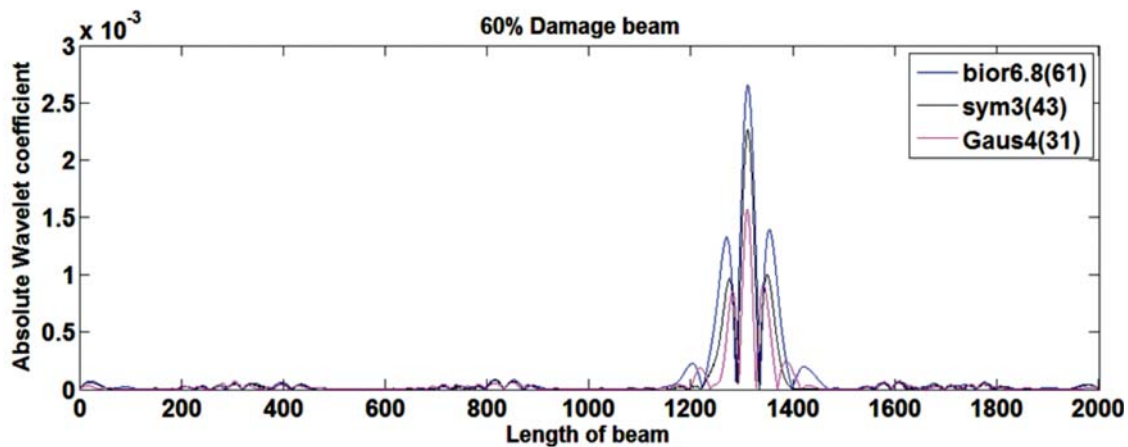


Figure 8: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

**Table 8**  
**Damage case 2 sensitivity of different wavelets for Mode 4**

<i>Different Wavelet</i>	<i>Damaged element</i>	<i>Predicted element</i>	<i>Max. Abs coefficient</i>
Sym3	1310	1311	0.0016
Gaus4	1310	1312	0.0023
Bior6.8	1310	1312	0.0027

Figure 8 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. Table 8 highlight Sym3, Gaus4, and Bior6.8 wavelets are detecting the damage location with small error. When the damage size is reduced means the value of Absolute wavelet coefficient is reducing comparing to the Damage case 2. Sym3, Gaus4, and Bior6.8 are matching nearby with the original thickness reduction. Biorthogonal 6.8 always provide the maximum Absolute wavelet coefficient, Symmetrical 3 provide minimum absolute wavelet coefficient, Gaussian 4 produce the medium absolute wavelet coefficient comparing to other two types, Symmetrical and near form of symmetrical wavelets are all effective in damage detection.

**Damage Case 3:**

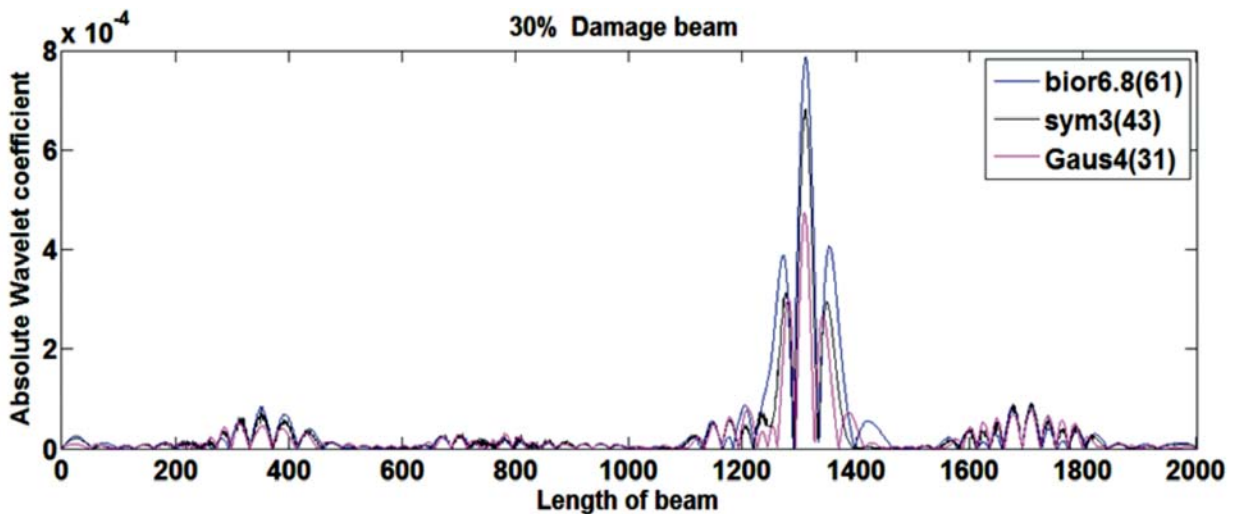


Figure 9: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

**Table 9**  
**Damage case 3 sensitivity of different wavelets for Mode 4**

<i>Different Wavelet</i>	<i>Damaged element</i>	<i>Predicted element</i>	<i>Max. Abs coefficient</i>
Sym3	1310	1310	0.00047
Gaus4	1310	1312	0.00068
Bior6.8	1310	1312	0.00079

Figure 9 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. Table 9 highlight Sym3, Gaus4, and Bior6.8 wavelets are detecting the damage location with small error. When the damage size is reduced means the value of Absolute wavelet coefficient is reducing comparing to the Damage case 3. Sym3, Gaus4, and Bior6.8 are matching nearby with the original thickness reduction. Biorthogonal 6.8 always provide the maximum Absolute wavelet coefficient, Symmetrical 3 provide minimum absolute wavelet coefficient, Gaussian 4 produce the medium absolute wavelet coefficient comparing to other two wavelets, Symmetrical and near form of symmetrical wavelets are all effective in damage detection.

**Damage Case 4:**

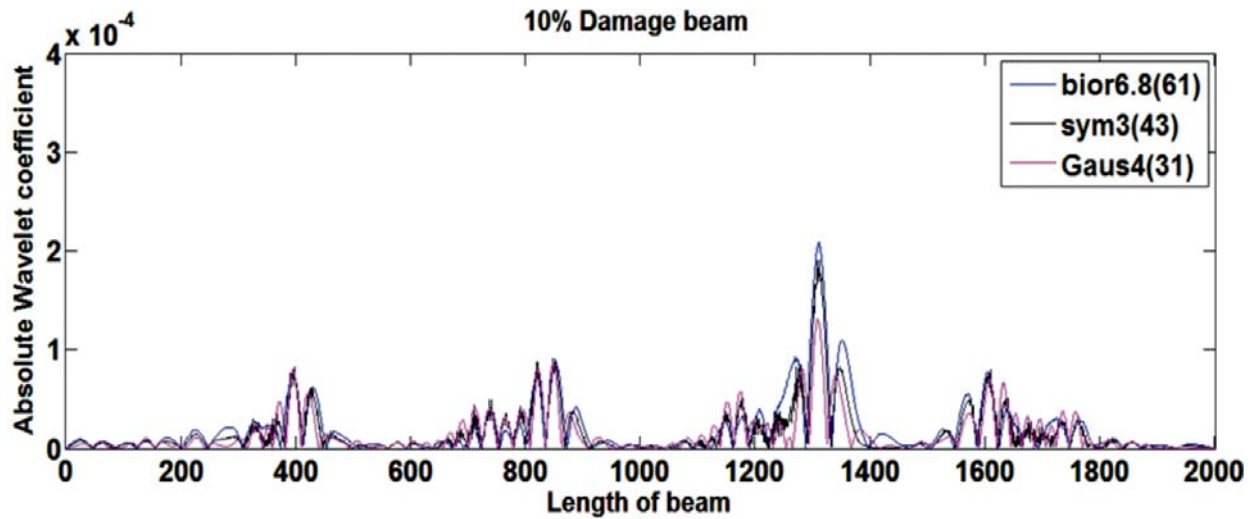


Figure 10: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

Figure 10 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. In this figure few peaks are available because of its sudden varying stiffness, even the damage level is 10% also the maximum absolute wavelet coefficient are locate the defect.

**Table 10**  
Damage case 4 sensitivity of different wavelets for Mode 4

Different Wavelet	Damaged element	Predicted element	Max. Abs coefficient
Sym3	1310	1309	0.00013
Gaus4	1310	1312	0.00019
Bior6.8	1310	1312	0.00021

Table 10 highlight Sym3, Gaus4, and Bior6.8 wavelets are detecting the damage location with small error. When the damage size is reduced means the value of Absolute wavelet coefficient is reducing comparing to the Damage case 4. Sym3, Gaus4, and Bior6.8 are matching nearby with the original thickness reduction. Biorthogonal 6.8 always provide the maximum Absolute wavelet coefficient, Symmetrical 3 provide minimum absolute wavelet coefficient, Gaussian 4 produce the medium absolute wavelet coefficient comparing to other two wavelets, Symmetrical and near form of symmetrical wavelets are all effective in damage detection.

**Damage Case 5:**

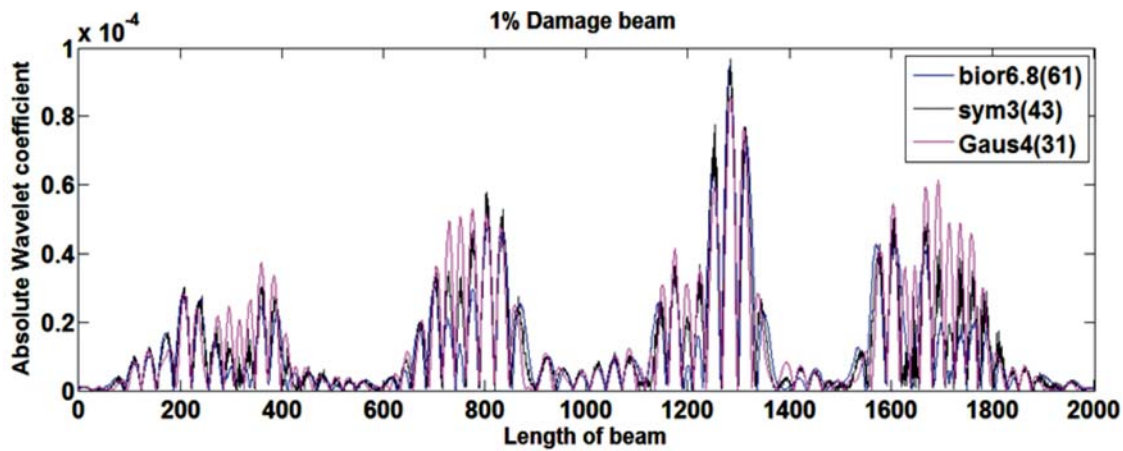


Figure 11: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

Figure 11 shows the translation mode shapes data’s absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. In this figure many peaks are available because of its damage level is almost 1% , still the Wavelets showing peaks at the damage level but the analyzing this kind of small damage higher level of knowledge is required analysis.

**Table 11**  
Damage case 5 sensitivity of different wavelets for Mode 4

<i>Different Wavelet</i>	<i>Damaged element</i>	<i>Predicted element</i>	<i>Max. Abs coefficient</i>
Sym3	1310	1283	0.000086
Gaus4	1310	1284	0.000097
Bior6.8	1310	1282	0.000094

Table 11 highlight Sym3, Gaus4, and Bior6.8 wavelets are detecting the damage location with small error. When the damage size is reduced means the value of Absolute wavelet coefficient is reducing comparing to the Damage case 5. Sym3, Gaus4, and Gaussian 4 are matching nearby with the original thickness reduction. Biorthogonal 6.8 always provide the maximum Absolute wavelet coefficient, Symmetrical 3 provide minimum absolute wavelet coefficient, Bior6.8 produce the medium absolute wavelet coefficient comparing to other two wavelets, Symmetrical and near form of symmetrical wavelets are all effective in damage detection.

**Damage case 6:**

**Table 12**  
Damage case 4 sensitivity of different wavelets for Mode 4

<i>Different Wavelet</i>	<i>Damaged element</i>	<i>Predicted element</i>	<i>Max. Abs coefficient</i>
Sym3	235,1665	234,1667	0.0017
Gaus4	235,1665	233,1668	0.0025
Bior6.8	235,1665	234,1667	0.0030



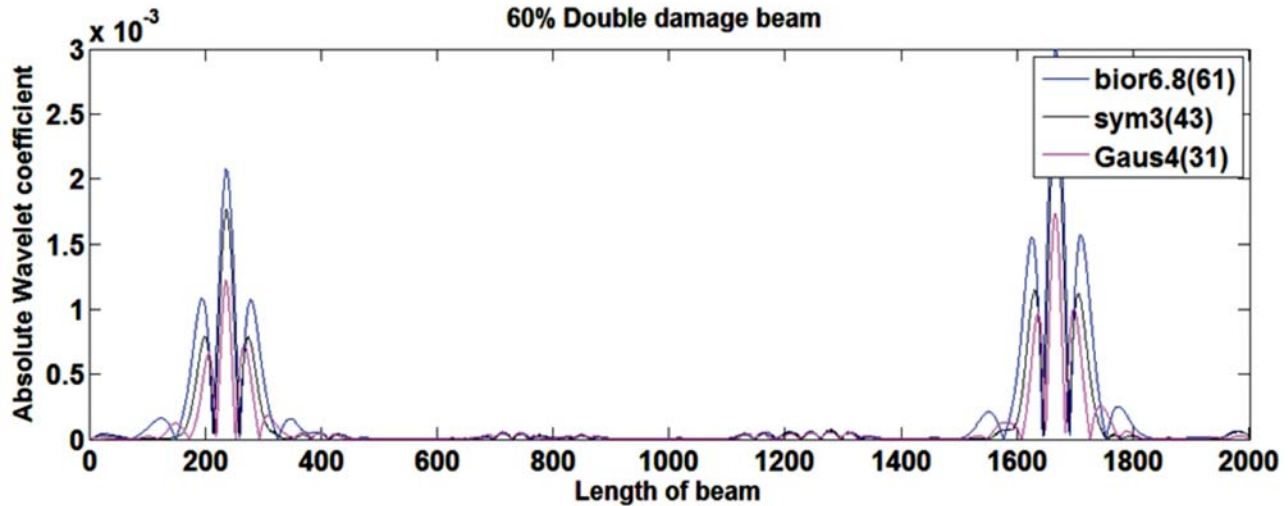


Figure 12: Two dimensional plot of Abs. Coefficient along the length of beam for different mother wavelets

Figure 12 shows the translation mode shapes data's absolute wavelet coefficient for different wavelets are plotted along the length of beam to locate the damage, peak values represent the damage location. In this figure two peaks are visible because of its structure's stiffness affected in both location, the maximum absolute wavelet coefficient are used to locate the defect. Table 12 highlight Sym3, Gaus4, and Bior6.8 wavelets are detecting the damage location with small error. When the damage size is reduced means the value of Absolute wavelet coefficient is reducing comparing to the Damage case 6. Sym3, Gaus4, and Bior6.8 are matching nearby with the original thickness reduction. Biorthogonal 6.8 always provide the maximum Absolute wavelet coefficient, Symmetrical 3 provide minimum absolute wavelet coefficient, Gaussian 4 produce the medium absolute wavelet coefficient comparing to other two wavelets, Symmetrical and near form of symmetrical wavelets are all effective in damage detection. Results explain this technique is able to identify the multi damage locations, here damage case is same but the peaks value is different because of its mode shape.

#### 4. CONCLUSION

The objective of this paper is to apply spatial wavelet transform in damage detection and evaluating the sensitivity for detection and localization of damage in a beam structure with both ends are fixed, using transverse displacements are input. It was observed translational mode shapes are effective in particular mode only and also it is helpful when the damage severe is up to 10%. Changes in displacements of mode shapes due to the presence of structural damage, represented here in a numerical finite element model, have been investigated. The results of the beam model demonstrate the usefulness of the changes in the translation of mode shape as a diagnostic parameter in detecting and locating damage with regard to the beam with fixed-fixed boundary conditions. In 90% damaged beam structured nodal data's are evaluated from first mode to fifth mode all modes are efficient but the third mode is not effective, it is clearly known because of its frequencies are not much change even in high damage cases also, Mode selection and wavelet selection procedures are studied and analyzed. Maximum Absolute wavelet coefficient values and location of the nodal points are observed, from based on these criteria fourth mode is effective in damage detection, and Symmetrical, Gaussian and Biorthogonal wavelets are optimal. Continuous wavelet transform is useful in multi crack detection but calculating the crack depth is slightly varies even the same level of damage present in the structure because of its mode shapes. Thus, boundary damages are best identified when the difference data of damaged and undamaged translation mode shape is given as input to wavelet transform and the absolute coefficients plotted in a length of a beam. The reflection of absolute wavelet coefficient are identified from the damage location is representing the damage severity.



Mode selection is depends upon the damage location and optimal wavelet selection is based on Absolute wavelet coefficient value in damage location observed and compared with actual less error percentage wavelets adopted for best results. In order to avoid the disturbances near fixed position difference data's of damaged and undamaged beam structure mode shapes are used to find the Absolute Coefficient of different wavelets. This procedure is applicable for any complicating structure.

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