IJMS, *Vol. 15, Nos. 3-4, (July-December 2016)*
ISSN: 0972-754X

A STUDY OF ANTI FUZZY SU-ALGEBRAS

Utsanee Leerawat[1] and Rattana Sukklin[2]

Abstract: In this paper, the notion of anti fuzzy SU-subalgebra and anti fuzzy SU-ideal in SU-algebra are established and some of their properties are investigated. Moreover, the relations between fuzzy SU-subalgebra(SU-ideal) and anti fuzzy SU-subalgebra(SU-ideal) of SU-algebras are studied.

Keywords: SU-algebras, anti fuzzy SU-subalgebra, anti fuzzy SU-ideal.

Mathematics Subject Classification: 03G25; 06F35; 08A72.

1. INTRODUCTION

R. Biswas introduced the concept of anti fuzzy subgroups of a group [1] and studied the basic properties of a group in terms of anti fuzzy subgroups. Hong and Jun [2] modified Biswas idea and applied it into BCK algebra. M. Mostafa, A.Abd-Elnaby and M.M. Yousef [3] introduced the notion of a fuzzy KU-ideals of KU-algebras and they investigated several basic properties which are related to fuzzy KU-ideals. Recently, a new algebraic structure was presented as SU-algebra and a concept of ideal in SU-algebra [4]. R. Sukklin and U. Leerawat introduced the concept of fuzzy SU-subalgebra and fuzzy SU-ideal in SU-algebra [5]. In this paper, we introduce the concept of anti fuzzy SU-subalgebras and anti fuzzy SU-ideals in SU-algebras. Moreover, we investigate some of their properties.

2. PRELIMINARIES

We give some definitions and results which are essential for the subsequent sections.

Definition 2.1: [4] A SU-algebra is a nonempty set X with a constant 0 and a binary operation "$*$" satisfying the following conditions:

(1) $((x * y)\ (x * z))\ (y * z) = 0$,

(2) $x * = 0\ x$,

(3) if $x * y = 0$ imply $x = y$

for all $x, y, z, \in X1$.

From now on, a binary operation "$*$" will denoted by juxtaposition.

Theorem 2.2: [4] Let X be a SU-algebra. Then the following results hold for all $x, y \in X$

(1) $xx = 0$

(2) $xy = yx$,

(3) $0x = x$,

Definition 2.3: [4] Let X be a SU-algebra. A nonempty subset I of X is called a SU-subalgebra of X if $xy \in I$ for all $x, y \in X$.

Definition 2.4: [4] Let X be a SU-algebra. A nonempty subset I of X is called an ideal of X if it satisfies the following properties:

(1) $0 \in I$,

(2) if $(xy)\, z \in I$ and $y \in I$ imply $xz \in I$ for all $x, y, z, \in X$.

Theorem 2.5: [4] Let X be a SU-algebra. Then X be a BCI-algebra.

Definition 2.6: [6] Let X be a Set. A fuzzy set μ in X is a function $\mu : X \to [0, 1]$.

Definition 2.7: [7] Let X be a SU-algebra and μ be a fuzzy set in X. Then the complement of μ is denoted by μ^c and defined as $\mu^c(x) = 1 - \mu(x)$ for all.

Definition 2.8: [2] Let X be a BCI-algebra. A fuzzy set μ in X is called fuzzy BCI-ideal X in.

(1) If $\mu(0) \leq \mu(x)$,

(2) $\mu(x) \leq \max\, (\mu(xy), \mu(y)\}$ for all $x, y \in X$

Definition 2.9: [5] Let X be a SU-algebra. A fuzzy set μ in X is called fuzzy SU-subalgebra in X.

If $\mu(xy) \geq \min\, \{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Definition 2.10: [5] Let X be a SU-algebra**.** A fuzzy set μ of X is called a fuzzy SU-ideal in X if it satisfies the following conditions:

$$(F_1)\ \mu(0) \geq \mu(x)$$

$$(F_2)\ \mu(xz) \geq \min\{\mu((xy)z),\ \mu(y)\}$$

for all $x, y, z \in X$

Definition 2.11: Let X be a SU-algebra and μ be a fuzzy set of X. For $t \in [0, 1]$, the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called a upper level subset of μ and the set $\mu^t = (x \in X \mid \mu(x) \leq t\}$ is called a lower level subset of μ. Clearly $\mu_t \cup \mu^t = X$.

Theorem 2.12: [5] Let X be a SU-algebra and μ be a fuzzy set in X. Then μ_t is a SU-subalgebra in X for any $t \in [0, 1]$ and $\mu_1 \neq \phi$ if and only if μ is a fuzzy SU-subalgebra in X.

Theorem 2.13: [5] Let X be a SU-algebra and μ be a fuzzy set in X. Then μ is a fuzzy SU-subalgebra in X if and only if μ is a fuzzy SU-ideal in X.

3. ANTI FUZZY SU-SUBALGEBRAS

We first give the definition of anti fuzzy SU-subalgebra and provide some its properties.

Definition 3.1: Let X be a SU-algebra. A fuzzy set μ in X is called anti fuzzy SU-subalgebra in X if $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$ for $x, y \in X$ all .

The set $Im(\mu) = (t \in [0, 1] \mid \mu(x) = t$, for some $x \in X\}$ is called the image set of μ.

Example 3.2: Let $X = (0, 1, 2, 3\}$ be a set in which operation $*$ is defined by the following:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then X is a SU-algebra[4]. Define a fuzzy set $\mu : X \to [0, 1]$ by, $\mu(0) = 0.3$, $\mu(1) = 0.6$ and $\mu(2) = \mu(3) = 0.7$. Then μ is an anti fuzzy SU-subalgebra in X.

Theorem 3.3: Let X be a SU-algebra and μ be a fuzzy set in X. Then

(i) If μ is an anti fuzzy SU-subalgebra in X, then $\mu(0) \leq \mu(x)$ for all $x \in X$.

(ii) If μ is an anti fuzzy SU-subalgebra in X if and only if μ^c is an fuzzy SU-subalgebra in X.

Proof: (i) Let $x \in X$ and μ be an anti fuzzy SU-subalgebra in X. Since $xx = 0$, then $\mu(0) \leq \max\{\mu(x), \mu(x)\} = \mu(x)$. Thus $\mu(0) \leq \mu(x)$.

(ii) Assume μ is an anti fuzzy SU-subalgebra in X. Let $x, y, z, \in X$.

So $\mu^c(xy) = 1 - \mu(xy) \geq 1 - \max\{\mu(x), \mu(y)\} = \min\{1 - \mu(y)\} = \min\{\mu^c(x), \mu^c(y)\}$. Hence $\mu^c(xy) \geq \min\{\mu^c(x), \mu^c(y)\}$. Therefore μ^c is a fuzzy SU-subalgebra in X.

Conversely, let μ^c be a fuzzy SU-subalgebra in X. So

$\mu(xy) = 1 - \mu^c(xy) \leq 1 - \min\{\mu^c(x), \mu^c(y)\} = \max\{1 - \mu^c(x), 1 - \mu^c(y) = \max\{\mu(x), \mu(y)\}$.

Hence $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$. Therefore μ is an anti fuzzy SU-subalgebra in X.

Remark: If μ is a fuzzy set in SU-algebra X, then $\mu^t = \mu^c_{1-t}$ for all $t \in [0, 1]$.

Theorem 3.4: Let X be a SU-algebra and μ be a fuzzy set in X. Then μ^t is a SU-subalgebra in X for any $t \in [0, 1]$ and $\mu^t \neq \phi$ if and only if μ is an anti fuzzy SU-subalgebra in X.

Proof: Let μ^t be a SU-subalgebra in X. We have $1 - t \in [0, 1]$ and $\mu^t = \mu^c_{1-t}$. Since μ^t is a SU-subalgebra in X, then μ^c_{1-t} is a SU-subalgebra in X and $\mu^c_{1-t} = \phi$. Hence μ^c is a fuzzy SU-subalgebra in X. By theorem 3.3, we have μ is an anti fuzzy SU-subalgebra in X.

Conversely, let μ be an anti fuzzy SU-subalgebra in X. Since μ is an anti fuzzy SU-subalgebra in X, then μ^c is a fuzzy SU-subalgebra in X. We have $1 - t \in [0, 1]$ and $\mu^t = \mu^c_{1-t}$. Hence $\mu^c_{1-t} \neq \phi$ and μ^c_{1-t} is a SU-subalgebra in X. Thus is a SU-subalgebra in X.

Theorem 3.5: Let X be a SU-algebra and A be a SU-subalgebra in X. Then for any $t \in [0, 1)$, there exists an anti fuzzy SU-subalgebra μ of X, such that $\mu^t = A$.

Proof: Let A be a SU-subalgebra in X and $t \in [0, 1]$.

Define a fuzzy set $\mu : X \to [0, 1]$ by

$$\mu(x) = \begin{cases} t \text{ if } x \in A; \\ 1 \text{ if } x \notin A. \end{cases}$$

Then μ is an anti fuzzy subalgebra of X. Obviously, $\mu^t = A$.

Theorem 3.6: Let X be a SU-algebra and μ be an anti fuzzy SU-subalgebra in X. If μ^s, μ^t for some $0 \leq s < t \leq 1$, are lower level subsets of μ, then $\mu^s = \mu^t$ if and only if $\{x \in X \mid s < \mu(x) \leq t\} = \phi$.

Proof: Let μ^s, μ^t be lower level SU-subalgebra of μ such that $\mu^s = \mu^t$ for some $0 \leq s < t \leq 1$.

Suppose $\{x \in X \mid s < \mu(x) \leq t\} = \phi$. There exists $y \in X$ such that $s < \mu(y) \leq t$, then $y \in \mu^t$ but $y \notin \mu^s$. Hence $\mu^s \neq \mu^t$, which is contradiction. Conversely, let $\{x \in X \mid s < \mu(x) \leq t\} = \phi$. It is obvious that $\mu^s \subseteq \mu^t$. If $x \in \mu^t$ we have $\mu(x) \leq t$. Since $\{x \in X \mid s < \mu(x) \leq t\} = \phi$, we have $\mu(x) \leq s$, then $x \in \mu^s$ thus $\mu^t \subseteq \mu^s$. Therefore $\mu^s = \mu^t$.

Remark: If $t_1 = \mu(0)$, then μ^{t_1} is the smallest lower level subset of μ. Hence we have $\mu^{t_1} \subset \mu^{t_2} \subset \mu^{t_3} \subset \ldots\ \mu^{t_n} = X$, where $\text{Im}(\mu) = (t, t_2, \ldots, t_n)$ with $t_1 < t_2 < t_3 < \ldots\ t_n$, and n is positive integers. Note that in Example 3.2, if $t_1 = 0.3$, $t_2 = 0.6$, and $t_3 = 0.7$, then

$\mu^{t_1} \subset \mu^{t_2} \subset \mu^{t_3} = X$. Let $\beta \in [0, 1]$ be such that $\mu(0) \leq \beta$ and $\beta \notin \text{Im}(\mu)$. If $t_1 < \beta\ t_{i+1}$, then $\{x \in X \mid t_i < \mu(x) \leq \beta\}$. By Theorem 3.6, then $\mu^{t_i} = \mu^{\beta}$. If $t_n < \beta$, then $\mu^{t_n} \subseteq \mu^{\beta}$. Since $\mu^{t_n} = X$, we have mb = X. Hence $\mu^{t_n} = \mu^{\beta}$. Therefore for any $\beta \in [0,1]$with $\mu(0) \leq \beta$, the lower level subset μ of is one of $\mu^{t_i} \mid 1 < i \leq n\}$.

Corollary 3.7: Let X be a SU-algebra and μ be an anti fuzzy SU-subalgebra in X.

If $\text{Im}(\mu) = (t_1, t_2, ..., t_n)$ with $t_1 < t_2 < t_3 < ... < t_n$, then the set $\mu^{t_i} \mid 1 < i \leq n\}$ is the set of all lower level subsets of μ.

4. ANTI FUZZY IDEALS OF SU-ALGEBRAS

In this section, we introduce the notions of anti fuzzy SU-ideal and disscuss some related properties.

Definition 4.1: Let X be a SU-algebra**.** A fuzzy set μ in X is called anti fuzzy SU-ideal of X if it satisfies the following conditions:

$$(AF_1)\ \mu(0) \leq m(x)$$

$$(AF_2)\ \mu(xz) \leq \max\{\mu((xy)z), \mu(y)\}, \text{ for all } x, y, z, \in X.$$

Example 4.2: A fuzzy set μ is defined as Example 3.2, then μ be an anti fuzzy SU-ideal in X.

Theorem 4.3: Let X be a SU-algebra. If μ_1, μ_2 are anti fuzzy SU-ideals in X, then $\bar{u}$ is an anti fuzzy SU-ideal in X, where $\bar{u}(x) = \max\{\mu_1(x), \mu_2(x)\}$ for all $x \in X$.

Proof: Let μ_1, μ_2 be anti fuzzy SU-ideals in X. Obviously, $\bar{u}$ is fuzzy set in X.

So $\bar{u}(0) = \max\{\mu_1(0), \mu_2(0)\} \leq \max\{\mu_1(x), \mu_2(x)\} = \bar{u}(x)$. Thus $\bar{u}(0) \leq \bar{\mu}(x)$ for all $x \in X$.

Now we will show that $\bar{\mu}(xz) \leq \max\{\bar{\mu}((xy)z), \bar{\mu}(y)\}$ for all $x, y, z, \in X$.

We have

$$\bar{\mu}(xz) = \max\{\mu_1(xz), \mu_2(xz)\} \leq \max\{\max\{\mu_1((xy)z), \mu_1(y)\}, \max\{\mu_2((xy)z), \mu_2(y)\}\}$$

$$= \max\{\max\{\mu_1((xy)z), \mu_2((xy)z)\}, \max\{\mu_1(y), \mu_2(y)\}\} = \max\{\bar{\mu}((xy)z), \bar{\mu}(y)\}.$$

Thus $\bar{\mu}(xz) \leq \max\{\bar{\mu}((xy)z), \bar{\mu}(y)\}$ for all $x, y, z \in X$. Therefore is an anti fuzzy SU-ideal in.

Corollary 4.4: Let X be a SU-algebra. If μ_1, μ_2, μ_2,...μ_n are anti fuzzy SU-ideals in X, then $\overline{\mu}$ is an anti fuzzy SU-ideal in X, where $\overline{\mu}(x) = \max\{\mu_1(x), \mu_2(x), \mu_2(x), ..., \mu_n(x)\}$ for all $x \in X$ and n is positive integers.

Theorem 4.5: Let X be a SU-algebra and μ be a fuzzy set in X. Then μ is an anti fuzzy SU-ideal in X if and only if μ^c is a fuzzy SU-ideal in X.

Proof: Assume μ is an anti fuzzy SU-ideal in X. Let $x, y, z \in X$.

(i) $\mu^c(0) = 1 - \mu(0) \geq 1 - \mu(x) = \mu^c(x)$

(ii) $\mu^c(xz) = 1 - \mu(xz) \geq 1 - \max\{\mu((xy)z), \mu(y)\} = \min\{1 - \mu((xy)z), 1 - \mu(y)\}$

$= \min\{\mu^c((xy)z), \mu^c(y)\}$ Hence $\mu^c(xz) \geq \min\{\mu^c((xy)z), \mu^c(y)\}$. From (*i*) and (*ii*), we have μ^c is a fuzzy SU-ideal in X. Conversely, let μ^c be a fuzzy SU-ideal of X. Let $x, y, z, \in X$.

(iii) $\mu(0) = 1 - \mu^c(0) \leq 1 - \mu^c(x) = \mu(x)$

(iv) $\mu(xz) = 1 - \mu^c(xz) \leq 1 - \min\{\mu^c((xy)z), \mu^c(y)\} = \max\{1 - \mu^c((xy)z), 1 - \mu^c(y)\}$

$= \max\{\mu((xy)z), \mu(y)\}$. Hence $\mu(xz) \leq \max\{\mu((xy)z), \mu(y)\}$. From (*iii*) and (*iv*), we have μ is an anti fuzzy SU-ideal in X.

Theorem 4.6: Let X be a SU-algebra and μ be a fuzzy set in X. Then μ^t is a SU-ideal in X for $t \in [0, 1]$ and $\mu^t \neq \phi$ any and if and only if μ is an anti fuzzy SU-ideal in X.

Proof: The proof is similar to the proof of Theorem 3.4.

Theorem 4.7: Let X be a SU-algebra and A be a SU-ideal in X. Then for any $t \in [0, 1)$, there exists an anti fuzzy SU-ideal of μ of X, such that $\mu^t = A$.

Proof: The proof is similar to the proof of Theorem 3.5.

Theorem 4.8: Let X be a SU-algebra and μ be an anti fuzzy SU-ideal in X. If $\mu^s = \mu^t$, for some $0 \leq s < t \leq 1$, are lower level subsets of μ, then $\mu^s = \mu^t$ if and only if $\{x \in X \mid s < \mu(x) \leq t\} = \phi$.

Proof: The proof is similar to the proof of Theorem 3.6.

Theorem 4.9: Let X be a SU-algebra and μ, be a fuzzy set in X. Then μ is an anti fuzzy SU-subalgebra in X if and only if μ is an anti fuzzy SU-ideal in X.

Proof: If μ is an anti fuzzy SU-subalgebra in X, then μ^c is a fuzzy SU-subalgebra in X.

We have μ^c is a fuzzy SU-ideal in X. Thus μ is an anti fuzzy SU-ideal in X.

Conversely, let μ be an anti fuzzy SU-ideal in X. Then μ^c is a fuzzy SU-ideal in X.

We have μ^c is a fuzzy SU-subalgebra in X. Thus μ is an anti fuzzy SU-subalgebra in X.

Theorem 4.10: Let X be a SU-algebra and μ be a fuzzy set in X. If μ is an anti fuzzy SU-ideal in X, then μ is an anti fuzzy BCI-ideal in X.

Proof: Assume μ is an anti fuzzy SU-ideal in X. Let $x, y, \in X$. We have $\mu(0) \leq \mu(x)$. We put $z = 0$ in (AF_2), we have $\mu(x) \leq \max\{\mu(xy), \mu(y)\}$. Thus μ is an anti fuzzy BCI-ideal in X.

Corollary 4.11: Let X be a SU-algebra and μ be a fuzzy set in X. If μ is an anti fuzzy SU-subalgebra in X, then μ is an anti fuzzy BCI-ideal in X.

Acknowledgements

The author are highly grateful to the referees for several useful suggestions and valuable comments. Moreover, this work was supported by a grant from Kasetsart University.

REFERENCES

[1] Biwas. R. Fuzzy Subgroups and Anti Fuzzy Subgroups. 1990. *Fuzzy Sets and System*. **35:** pp. 121-124.

[2] Hong S.M. and Y.B. Jun. 1998. Anti Fuzzy Ideals in BCK-Algebras. *Kyung Pook Math J*. **38:** pp. 145-150.

[3] Samy M. Mostafa Mokhtar, A.Abd-Elnaby and Mouustafa M.M. Yousef. 2011. Fuzzy KU-ideals of KU-Algebras. *International Mathematical Forum*. **6:** pp. 3139-3149.

[4] Keawrahun. S.and U. Leerawat. 2011. On Classification of a Structure Algebra : SU-Algebras. *Scientia Magna Journal*. **7:** pp. 69-76.

[5] Sukklin.R and U.Leerawat. Fuzzy SU-Subalgebras and Fuzzy SU-ideals. 2013. *Far East Journal of Mathematical Sciences*. **74:** pp. 191-200.

[6] Zadeh. L.A. 1965. Fuzzy Sets. *Inform and Control*. **8:** pp. 338-353.

[7] Kim. K.M.,Y.B. Jun and Y.H.Yon. 2005. On Anti Fuzzy Ideals in Near-Rings. *Iranian Journal of Fuzzy Systems*. **2:** pp. 71-80.

Utsanee Leerawat[1] and Rattana Sukklin[2]

Department of Mathematics
Kasetsart University, Bangkok, Thailand
E-mails: fsciutl@ku.ac.th