

Synthesis of Optimal Sum and Difference Patterns using Cuckoo Search Algorithm

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Abstract: Optimal synthesis problem plays a major role in designing the antenna arrays. It is highly essential to generate low side lobe narrow beams for high-resolution radars and point to point radars. A linear array can be designed by controlling amplitude and phase levels or element spacing functions of the array. A linear array is considered and it is optimally designed using cuckoo search algorithm. The aim of the work is to minimize the sidelobe levels with a constraint on the beamwidth and this results can be vastly used in the field of RADAR communications.

Keywords: Antenna Array, Sidelobe Level Synthesis, Cuckoo Search Algorithm (CSA), Particle Swarm Optimization (PSO)

I. INTRODUCTION

In array-pattern synthesis, the main focus is to find an appropriate weighting vector to yield the desired radiation pattern. Array holds its advantages mostly because we can control the major lobe direction and side lobe level of radiation pattern and are the potential advantages of the use of array, rather than of only single elements, is that the major lobe direction and side lobe level of radiation pattern are controllable and are function of the magnitude and the phase of the excitation current and the position of each array element [1]. Various analytical and numerical techniques have been developed to meet this challenge. Examples of analytical techniques include the well-known Taylor method and Chebyshev method [2].

Radars are highly effective in ascertaining the location of remote objects, but accuracy is greatly reduced when a radar is jammed. The difference pattern consists of a null in the boresight direction with two major lobes adjacent to null. The null in the difference pattern is suppressing the jamming source and tracking accuracy was improved [3]. The sum patterns are generated by several methods but the methods of generation of difference patterns are limited. In difference patterns, the depth of the null is significant and the difference slope is not enough for many applications. It is required to produce them for optimum performance. In this method, it is difficult to obtain deep null and high difference slope but for fine angular tracking, difference pattern should have a deep null at boresight and high difference slope for accurate angular tracking [4].

But the analytical techniques usually converge to local values rather than global optimum values. Hence there is need of evolutionary techniques that are applied to the antenna array synthesis to minimize the side lobe level of an array and these techniques produce best results than analytical techniques [5]. Optimization techniques like Cuckoo Search Algorithm (CSA) [6] and Accelerated Particle Swarm Optimization (APSO) [7] has been applied to different problems in antenna arrays. In the present work, these techniques are used to find suitable amplitude excitations to generate required sum patterns and difference patterns also. The patterns reported has sidelobe level around -35 dB. These mathematical techniques are adapted in the design of array to have the required side lobe level of an antenna.

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2. FORMULATION

2.1. Formulation for Sum Pattern

Synthesis of array antennas is very important to get the desired pattern and how to achieve low side lobe in the condition of a fixed main beam width has been considered since a long period of time [8]. The linear array is one of the commonly used arrays in many applications owing to its simplicity. The representation of such geometry is as shown in below Fig 1. Considering a linear array of N isotropic antennas [9], antenna elements are equally spaced at distance d apart from each other along the x -axis.

Mathematically, the array factor of a $2N$ element array is given by

$$E(\theta) = 2 \prod_{n=1}^N A_n e^{j\phi_n} \cos[k(n-0.5)d(u-u_0)] \quad (1)$$

Here, $k = \text{wave number} = 2\pi/\lambda$, $\lambda = \text{wave length}$

$\theta = \text{angle between the line of observer and broadside}$

$\theta_0 = \text{scan angle,}$

$u = \sin\theta$, $u_0 = \sin\theta_0$

$A_n = \text{excitation of current for the } n\text{th element on either side of the array.}$

$d = \text{spacing between the radiating elements.}$

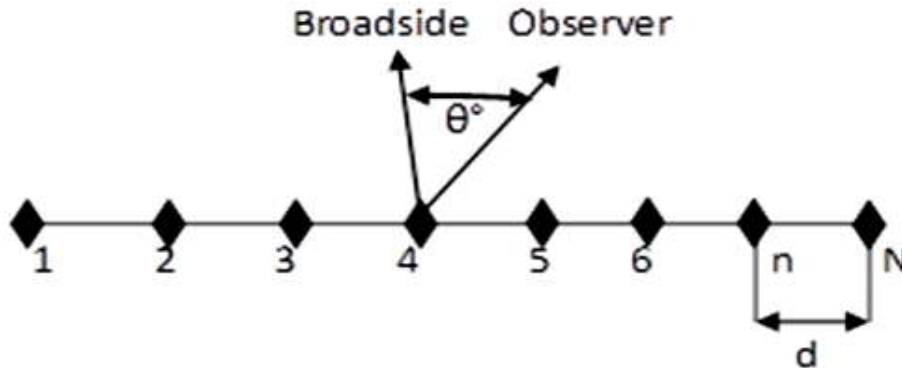


Figure 1: Linear antenna array.

2.2. Fitness Function of Sum Pattern

The fitness function associated with this array is the maximum Side Lobe Level of its associated radiation pattern to be minimized. The general form of the fitness function is given by

$$\text{Fitness} = \text{Max} \frac{\left(20 \log_{10}(|E(\theta)|)\right)}{\left(\text{max}|E(\theta_0)|\right)} \quad (2)$$

$$-\pi/2 \leq \theta \leq \pi/2, \theta \neq 0^\circ$$

2.3. Formulation for Difference Pattern

The difference pattern from a continuous line source is also obtained from

$$E_d(u) = \left[\int_{-1}^0 A(x) e^{j \frac{2\pi L}{\lambda} [ux + \alpha]} dx + \int_0^1 A(x) e^{j \frac{2\pi L}{\lambda} [ux + \alpha]} dx \right] \quad (3)$$

Here $A(x)$ is excitation function

$2L/\lambda =$ Normalized array length

$U = \sin \theta$

$x =$ position on line source

$\alpha =$ excitation phase

to generate a null in the boresight directions 180° phase shift is introduced to one-half of the array i.e.,

$$\alpha = 0 \text{ for } -1 \leq x \leq 0 \quad \alpha = \pi \text{ for } 0 \leq x \leq 1 \quad (4)$$

Substituting equation (5.2) in equation (5.1), then radiation pattern is now given by

$$E_d(u) = \int_{-1}^0 A(x) e^{j \frac{2\pi L}{\lambda} ux} dx + \int_{-1}^1 A(x) e^{j \frac{2\pi L}{\lambda} (ux + \pi)} dx \quad (5)$$

Let $a = 2\pi/L$ then

$$E(u) = \int_{-1}^U A(x) e^{j a u x} dx + \int_0^1 A(x) e^{j a (u x + \pi)} dx \quad (6)$$

$$E(\theta) = A(x) \left\{ \int_{-1}^U \cos(aux) + j \sin(aux) dx + A(x) \int_0^1 \cos(aux + \pi) + j \sin(aux) dx \right\} \quad (7)$$

As the line source is only a theoretical concept, a discrete array is used here. the radiation integral is replaced by a finite summation containing a number of terms equal to the number of elements in the array [9]. The radiation pattern is now given by

$$E(u) = \sum_{n=1}^N A(Xn) e^{j \frac{2\pi L}{\lambda} [u x_n + \alpha(x_n)]} dx \quad (8)$$

Where

$\alpha = 0$ for $-1 \leq x_n \leq 0$

$\alpha = \pi$ for $0 \leq x_n \leq 1$.

$A(x) = 1$ indicates that the source is uniformly excited and corresponding radiation pattern is a uniform array, deep null is at the boresight but the first sidelobe level of the difference pattern is found to be -11 dB. These are considered to be inadequate for fine angular tracking radars.

2.4. Fitness Function of Difference Pattern

The fitness function associated with this array is the maximum sidelobe Level of its associated radiation pattern to be minimized. The general form of the fitness function is given by

$$Fitness = \text{Max} \frac{\left(20 \log_{10} \left(|E(\theta)| \right) \right)}{\left(\max |E(\theta_0)| \right)} \quad (9)$$

$$-\pi/2 \leq \theta \leq \pi/2, \theta \neq 0^\circ$$

3. ALGORITHM

3.1. Cuckoo Search Algorithm

Cuckoo search is a new metaheuristic search algorithm, based on cuckoo bird's behaviour. Cuckoo search is a very new population heuristic algorithm for global optimization and it is one of the evolutionary technique, inspired by the reproduction strategy of cuckoos [10].

At the most basic level, if a host bird discovers the eggs are not their own, it will either throw these alien eggs away or simply abandon its nest and build a new nest elsewhere. Each egg in a nest represents a solution, and a cuckoo egg represents a new solution.

The aim is to use the new and potentially better solutions (cuckoos) to replace a not-so-good solution in the nests. In the simplest form, each nest has one egg [11].

The algorithm can be extended to more complicated cases in which each nest has multiple eggs representing a set of solutions for simplicity in describing the cuckoo search, the 3 idealized assumptions:

1. Each cuckoo lays one egg at a time, and dumps its egg in a randomly chosen nest.
2. The best nests with the high quality of eggs will carry over to the next generation.
3. The number of available hosts nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability lies in between (0 and 1).

In this case, the host bird can either throw the egg away or abandon the nest to build a completely new nest in a new location. Random-walk style search is better performed by Lévy flights rather than simple random walk[12].

An important advantage of this algorithm is its simplicity. In fact, comparing with other population or agent based metaheuristic algorithms such as particle swarm optimization, there is essentially only a single parameter in pa CSA (apart from the population size n. Therefore, it is very easy to implement.

The best nest is chosen as the optimum variable and in this case it refers to the new current amplitude of linear antenna array elements $x_i^{(t+1)}$ for, say cuckoo i, a Lévy flight is performed.

$$x_i^{(t+1)} = X_i^{(t)} + \alpha \oplus \text{Lévy}(\lambda), \quad (10)$$

The Levi flight equation represents the stochastic equation for a random walk as it depends on the current position and the transition probability (second term in the equation).

Where α is the step size related to the problem of interest. In most cases, we can use $\alpha = (1)$. The product \oplus means entry-wise multiplications. Lévy flights essentially provide a random walk while their random steps are drawn from a Lévy distribution for large steps

$$\text{Lévy} \sim u = t^{-\lambda}, (1 < \lambda \leq 3) \quad (11)$$

Where, in the above equation, $t^{-\lambda}$ is a parameter dealing with fractal dimension and t being the step size. The value of probability Pa used in this paper is 0.25 as used by Deb.

The implementation of the algorithm is explained in the form Pseudo code for CSA in Figure 2.

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Begin
    Objective function f (X), X =(x1, x2, xd)
    Generate initial population of n host nests Xi (i =1, 2... n)
While (t < Max Generation) or (stop criterion)
    Get a cuckoo randomly by Levy flights evaluate its quality/ fitness Fi
    Choose a nest among n (say, j) randomly
If (Fi > Fj),
    Replace j by the new solution;
End
    A fraction (pa) of worse nests are abandoned and new ones are built;
    Keep the best solutions
    (Or nests with quality solutions);
    Rank the solutions and find the current best
End while
    Post process results and visualization

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4. RESULTS

The Cuckoo search algorithm is applied to evaluate amplitude distribution required to maintain sum patterns and difference pattern with Sidelobe level at -35dB. The patterns are numerically computed for different arrays of elements

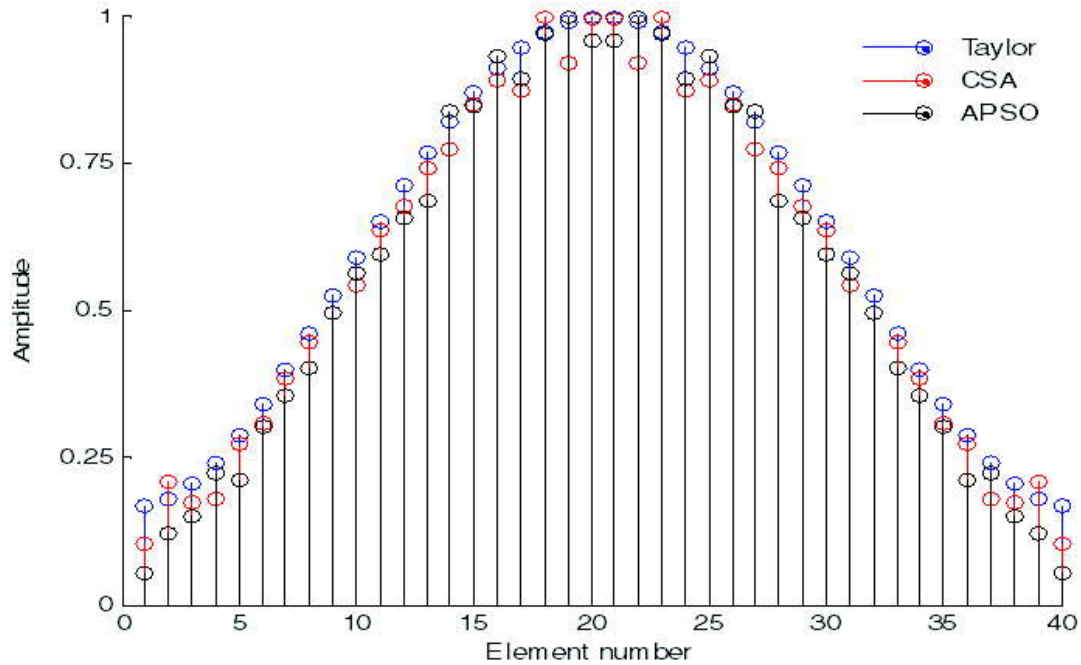


Figure 2: Amplitude distribution for N=40 element array using Taylor, CSA & APSO.

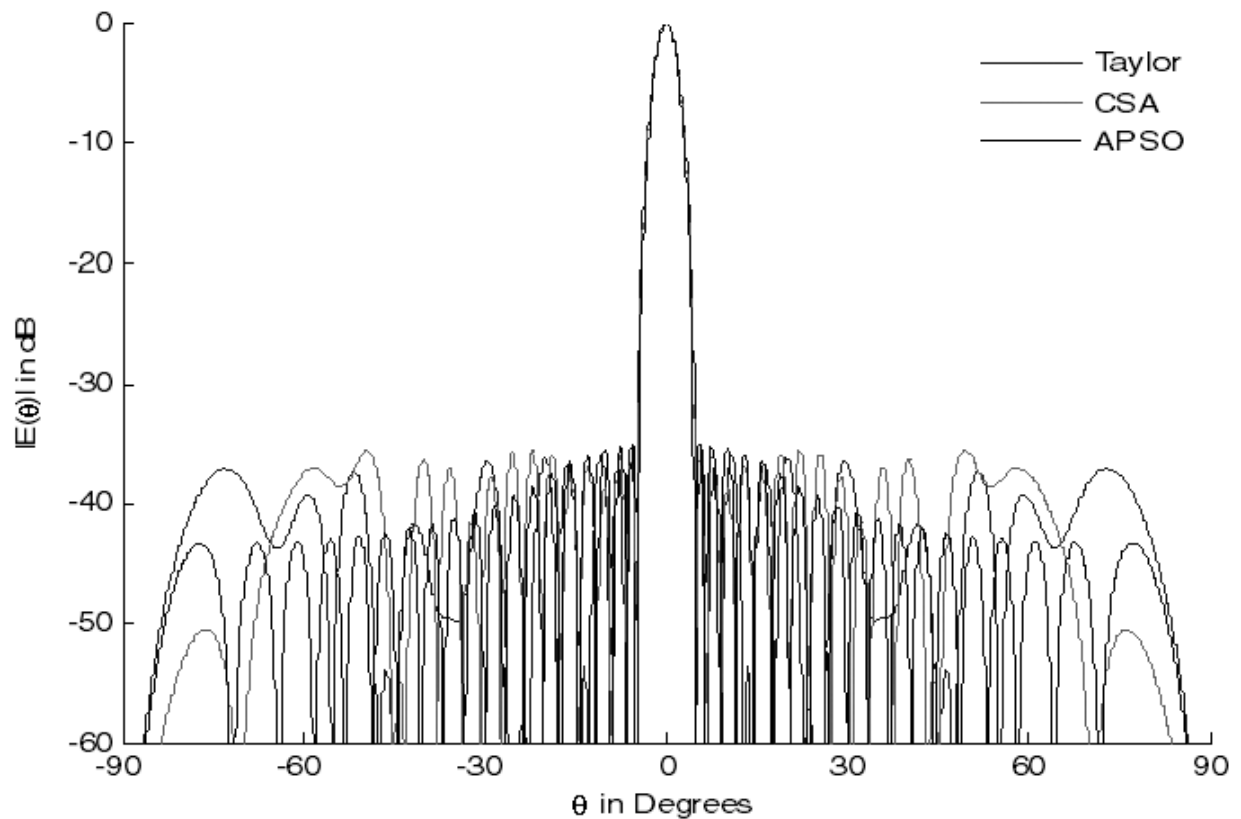


Figure 3: Sum pattern optimized for N=40 element array with $d=0.45$ and $\bar{n}=6$ using Taylor, CSA & APSO.

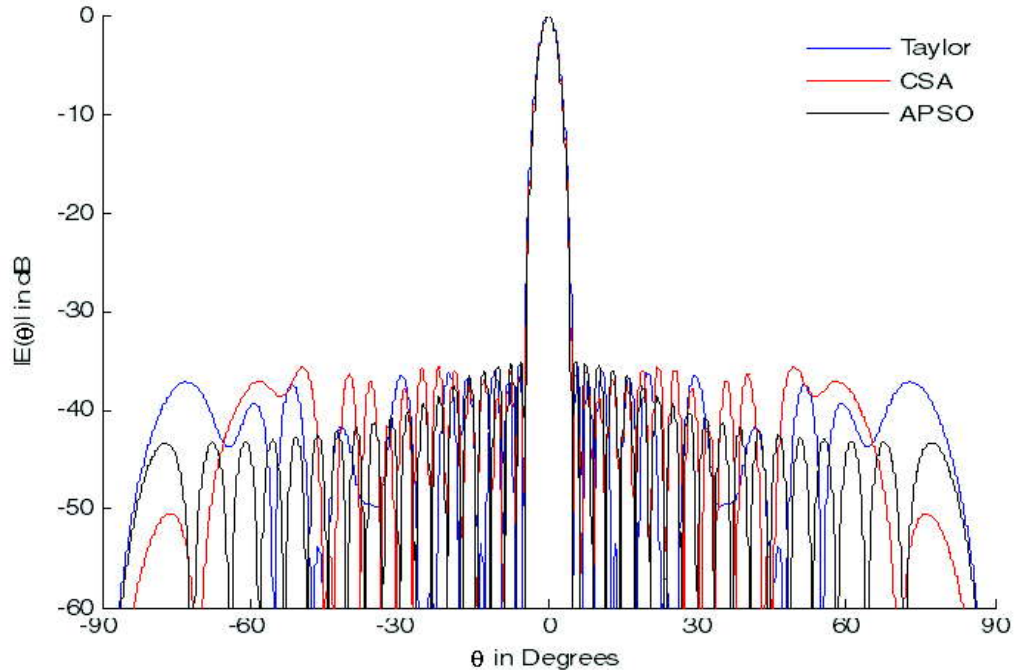


Figure 4: N=40 Elements sum and difference pattern using APSO and CSA.

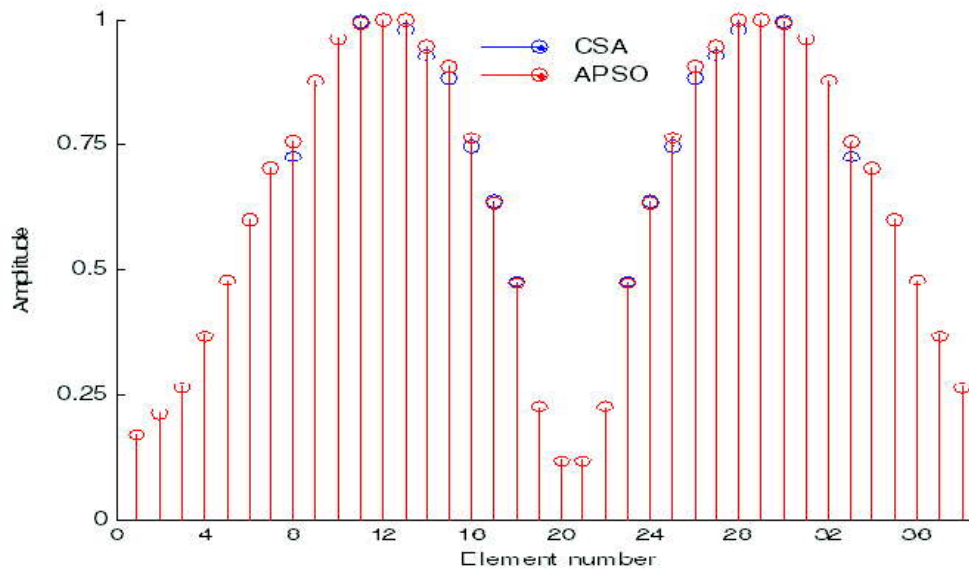


Figure 5: Amplitude distribution for N=40 element array using Taylor, CSA & APSO

N=20 and 40 elements by varying the spacing between elements as $d=0.50$. As the number of elements increased in the array, the Null to Null beamwidth is found to vary. The results are compared with Accelerated Particle Swarm Optimization. Fig 3&4 are for N=40 elements to generate sum pattern whereas Fig 5&6 are to generate difference pattern for N=40.

5. CONCLUSION

The synthesis of equally spaced linear arrays for sidelobe level reduction purpose has been considered in the present work. The Algorithms has shown up to be useful to desired radiation pattern. The method is useful to solve multi objective array problem involving any number of constraints. The side lobe level decreased up -35 dB down the main lobe which represents the high reduction in the side-lobe level which is very much desirable. The side lobe

reduction is more useful in radar, wireless, and sonar applications. Particularly in radar application, it is very much useful to reduce the side lobe level to avoid interference from other sources. The beam width remains unaltered even after reducing the first side lobe level. However, it has become necessary to compromise the rise in other side lobe levels. The difference patterns obtained by optimizing amplitude distributions using CSA are found superior to its other counterparts. The sidelobe level for CSA is around -35 dB whereas for APSO it is around -33 dB. It is evident from the above results that the difference patterns have ultra-low sidelobe levels and deep null in their boresight makes them suitable for EMC applications. These patterns are very useful in high resolution tracking radars which are applicable to defense applications for identifying the targets.

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