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Internal Model Control of MIMO Non-Square Discrete Systems: Design and Stability Analysis

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Abstract: This paper proposes an approach of Internal Model Control (IMC) for the linear discrete-time over-actuated systems with outputs more than inputs. The proposed IMC, which is based on a specific inversion principle, is the extension of to the case of square systems, in the controller design, a new method is presented to add a virtual outputs of the process to obtain a square transfer matrix and to eliminate the latter in the programming part. To guarantee the robust stability of the system and disturbance rejection, the controller design procedure is proposed to obtain the wish control.

A linear matrix inequality (LMI) approach is using to analyse stability and stabilization condition which is necessary and sufficient of linear discrete-time over-actuated systems. Then we introduce a new matrix variable (P) and we establish the necessary conditions of LMI for stabilizing controller design. Simulation illustrate the effectiveness of the proposed method for non-square multivariable processes.

Keywords: Internal Model Control; Discrete-time over-actuated systems; Stability; Linear Matrix Inequality (LMI); Disturbance rejection.

1. INTRODUCTION

The economic growth inherent to our nowadays society pushes the industries toward better performances. Thus, the multiple objectives to track become hard to achieve without compromises. In the context, Several important multivariable system control approaches have been developed such as the internal model control, which has the potential to ensure obtaining the desired performance in the presence of disturbance in the system and is characterized by its robustness and simplicity [6,10]. In the sense that, the considered system has more actuated degrees of freedom than control inputs.

In the last years, researches in the field of over-actuated systems control have led to a significant advance and several control methodologies have been proposed. Yao [12] proposed a decoupling internal model control for non-square systems by using the pseudo-inverse and approximation time delays by Taylor expansion diagrams, this method had good performance about dynamic decoupling but the control systems was sensitive to the

change of time delays. Chen [15] modified the internal model control of over-actuated systems by inserting compensated terms to remove the unrealizable factors from the obtained controller, but the controller parameters is the compromise between tracking performance and robustness.

In order to contribute to this researches, we propose in this paper to apply an interesting control approach called internal model control (IMC), to a class of linear discrete over-actuated systems.

The IMC structure was proposed by Garcia and Morari (1982) for Single-Input Single-Output systems (SISO) and then it was extended to Multi-Input Multi-Output (MIMO) systems (1985) [1,3,7,12]. The study was applied to the case of multivariable sampled systems having a number of inputs equal to number of output [4]. The obtained results are very encouraging in linear continuous and discrete cases [3,4], which lead us to extend the study to multivariable sampled systems having processes with more outputs than inputs or more inputs than outputs ,these systems are known as the non-square systems [14]. The problem of controlling a process with an unequal number of manipulated and controlled variables arises fairly often. The dominant control strategy for non-square systems is to transform them to corresponding square systems [5,9,13].

In this paper, in the hand, a new IMC controller approach is presented for non-square multivariable processes with outputs more than inputs, these systems are called: over-actuated systems. In the IMC controller design procedure a simple method designed to uses virtual outputs method [8]. In the other hand, the tracking, inversion and stabilization problems of discrete multivariable over-actuated systems are considered.

The analysis of the stability of elements of the IMC has been conducted in the literature by numerous fundamental researches that depend on the type of systems considered and the scope. There are many methods studying the stability of linear discrete multivariable systems. These stability criteria can be classified into two main categories namely the frequency criterion using the notion of the characteristic equations and the time criterion based on Lyapunov theory [19]. For linear discrete-time over-actuated systems, we consider stability and stabilization conditions using matrix inequality approach (LMI) [20].

This paper is organized as follows. In Section II we provide first an overview of the IMC basic structure. Secondly, we present the main problem and then we propose our solution by describing the modification we have introduced to this IMC structure so that it becomes applicable to discrete over-actuated systems. In section III, an example is employed to illustrate the effectiveness of the proposed method control. Some conclusions are drawn in section VI.

Case studies demonstrate the effectiveness of the proposed method for non-square multivariable processes. Before presenting the main results of this work, we first introduce some concepts that are important to their later development.

2. PRELIMINARIES

In this section, we present the main contribution of this work by showing the basic internal model control scheme of discrete fully-actuated systems. Secondly, we present the main problems and then we explain the design steps of the proposed IMC modified by describing the changes we have introduced to this IMC basic structure so that it becomes applicable to linear discrete over-actuated systems.

2.1. Basic Internal Model Control Structure

Given a $(m \times n)$ multivariable process with the matrix transfer function $G(z)$, where $Y(z)$ is a $(m \times 1)$ output vector, $U(z)$ a $(n \times 1)$ input vector, $v(z)$ a $(n \times 1)$ disturbance vector and $r(z)$ is a $(n \times 1)$ reference input vector, consider the standard structure of internal model control (IMC) as shown in Figure 1, where $M(z)$ is process model, $F(z)$ a filter which can be inserted to achieve a desired degree of robustness and $C(z)$ is the controller matrix transfer function.

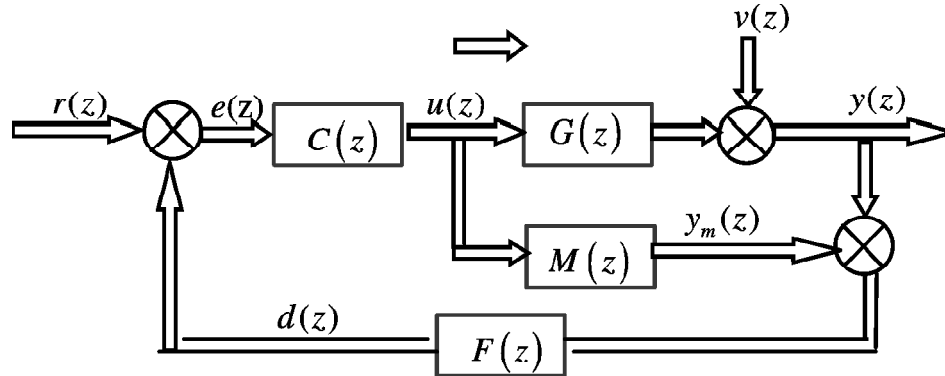


Figure 1: IMC structure of multivariable discrete fully-actuated systems

This control structure defined in Figure. 1 leads us to the following equations:

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} GC & I_m - CM \\ C & -C \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} \tag{1}$$

2.2. The Problem of Inversion of Over-actuated Systems

$C(z)$ is the proposed controller in this structure; it is the inverse of the chosen model $M(z)$, if is realizable in order to ensure perfect set-point tracking. Achieving this inversion is the basic problem associated to the IMC approach. In fact, this difficulty of direct model inversion for several physical systems is due to the denominator order which is usually greater than the numerator order on the model expression or the presence of unstable zeros or/and time delays [3,4,5] and too in the case of over-actuated systems.

The realization of the direct model’s inverse is difficult or not possible in the case of the over-actuated systems, because the model must provide an accurate description of the process dynamic and characteristic. For the over-actuated systems such that the number of control inputs is equal to n and the number of outputs is equal to m , the transfer matrix of the process is of dimension $(m \times n)$ making it a rectangular matrix, so the model can’t be invertible.

In order to remedy this problem of inversion of model $M(z)$, it is necessary to use inversion techniques, we quote for example methods, virtual outputs [8,12], non-square effective relative gain (NERGA) [16], Moore-Penrose pseudo-inverse technique [17,18].

We design an approximate inverse of the model plant by using the virtual outputs method and we show after the modification of the basic IMC structure.

2.3. Proposed Controller Design of Discrete Over-actuated Systems

In this internal model control structure, the controller can be obtained by the inverse method proposed in [5,11].

The IMC controller $C(z)$ is showing in Figure 2 :

The inversion method is based on the gain matrix A_1 , in order to realize an inverse for the system model. A_1 is a diagonal matrix; its coefficients are selected to satisfy the conditions of stability [8, 1, 5].

The expression of the controller $C(z)$ is given by the following equation :

$$C(z) = A_1(I_n + A_1M(z))^{-1} \tag{2}$$

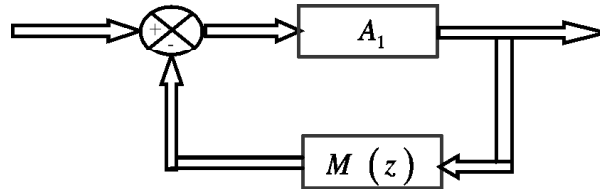


Figure 2: Structure for model inversion

A_1 is chosen as the following expression $A_1 = \alpha \times I_n$, to ensure stability for the controller, where I_n is the identity matrix and α is the chosen coefficients.

It is necessary to analyze the internal stability and robust stability of the proposed control system to derive the tuning region of parameter A_1 . When there exists the largest uncertainty in the control system, we need tune the control gain matrix A_1 to be the largest values to hold the system robust stability. For the stable linear discrete over-actuated process $G(z)$, the IMC structure as shown in Figure 1 is internally stable if and only if $C(z)$ is stable.

2.4. The New Imc Structure of Over-actuated Systems

The suggested solution of controlling them consists of the utilization of a square model and then eliminating the excess outputs which are applied to the system.

In fact, the transfer matrix $G(z)$ of over-actuated system expressed by (3) is of dimension $(m \times n)$, $(n \times m)$, with outputs more than inputs, where m is the system's outputs number and n is the system's inputs number.

$$G(z) = \begin{pmatrix} G_{11}(z) & G_{12}(z) & \dots & G_{1n}(z) \\ G_{21}(z) & G_{22}(z) & \dots & G_{2n}(z) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(z) & G_{m2}(z) & \dots & G_{mn}(z) \end{pmatrix} \quad (3)$$

The new IMC structure has the specificity of insertion of the two blocks as shown in Figure 3. The first block (added virtual outputs) use a virtual outputs by adding $((n - m) \times m)$ lines to the transfer matrix of the over-actuated system, up to have a square transfer matrix of dimension $(m \times m)$ and that can be reversed [7]. The model process $M(z)$ is chosen close to $G(z)$, for this reason, the added $((n - m) \times m)$ transfer functions can be first-order systems in order to simplify the study and to avoid the inversion problems.

The obtained matrix $M(z)$ is expressed by

$$M(z) = \left. \begin{matrix} \begin{bmatrix} M_{11}(z) & M_{12}(z) & \dots & M_{1n}(z) \\ M_{21}(z) & M_{22}(z) & \dots & M_{2n}(z) \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1}(z) & M_{m2}(z) & \dots & M_{mn}(z) \end{bmatrix} \\ \begin{bmatrix} M_{m+1,1} & M_{m+2,2} & \dots & M_{m+1,n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \dots & M_{nn} \end{bmatrix} \end{matrix} \right\} \begin{matrix} \text{initial bloc} \\ \text{added bloc} \end{matrix} \quad (4)$$

The Secondblock (elimination of (n-m) outputs) is added to the basic IMC structure, is used to eliminate the excess virtual outputs and the programming part we will remove them.

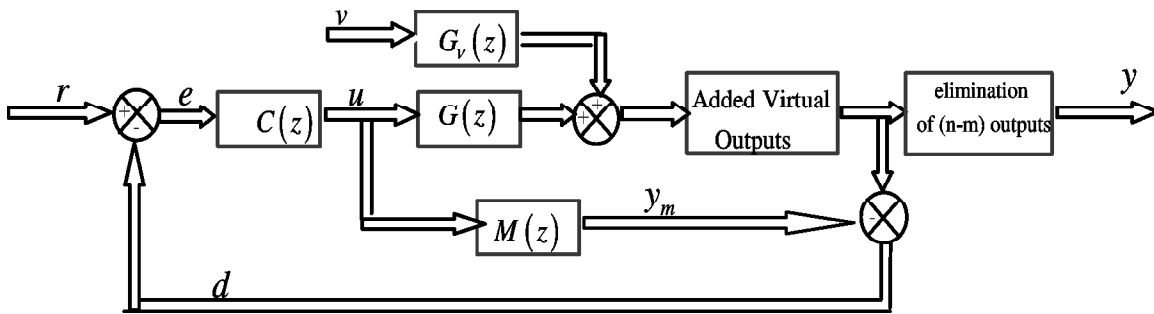


Figure 3: IMC structure for multivariable over-actuated discrete system

With the changes we have introduced to the basic IMC structure so that it becomes applicable to discrete over-actuated systems.

2.5. Study of the Stability of the Corrector

The IMC controller $C(z)$ is represented by the following state space equation:

$$\{c(k+1) = Ax(k) \tag{5}$$

Where $x(z) \in \mathbb{R}^n$ is the descriptor variable, $A \in \mathbb{R}^{n \times n}$ is constant. The system (5) is stable (i.e., all trajectories converge to zero) if and only if there exists a positive definite matrix $P = P^T \succ 0$ such that

$$P \succ 0, (A^T P A - P) \prec 0 \tag{6}$$

The requirement $P \succ 0, (A^T P A - P) \prec 0$ is what we now call a Lyapunov inequality on P, which is a special form of an LMI. Lyapunov also showed that this first LMI could be explicitly solved.

Where A is given matrices of appropriate sizes, and $p = p^T$ is the variable.

3. APPLICATION

In order to show the simulation results to validate the proposed internal model control of multivariable over-actuated systems; let's consider the following systems with three inputs and two outputs.

With $u(z) = [u_1 \ u_2 \ u_3]^T$ is the system input vector, $y(z) = [y_1 \ y_2]^T$ is the system output system and the reference vector r is chosen step of amplitude 1.

The system can be described by:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{s+3}{s^2+2s+1} & \frac{1}{2s+3} & \frac{2s+2}{3s^2+2s+1} \\ \frac{2s+1}{s^2+3s+2} & \frac{3}{s+4} & \frac{s+1}{2s^2+2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{7}$$

The model can be described by :

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{s+3}{s^2+2s+1} & \frac{1}{2s+3} & \frac{2s+2}{3s^2+2s+1} \\ \frac{2s+1}{s^2+3s+2} & \frac{3}{s+4} & \frac{s+1}{2s^2+2s+1} \\ \frac{s+3}{s^2+2s+1} & \frac{1}{2s+3} & \frac{2s+2}{3s^2+2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (8)$$

We can apply the study developed in [4], therefore the bilinear method of discretization are applied for the plant $G(s)$ and the model $M(s)$. Using LMI approach, guaranteeing quadratic stability of the internal model controller $C(z)$ each is described by equation (5). LMIs has been performed in MATLAB environment.

Solving the LMI in equation (6), we obtain a matrix P of dimension (45×45) , This matrix ensures the stability of our system.

Using LMI approach, The interval of the gain A_i which ensures the stability of the controller is $10 \times I_m \leq A_i \leq 70 \times I_m$

In our case, the chosen matrix A_i is equal to $A_i = 50 \times I_3$, to ensure the stability of the system $G(z)$ and a sampling period $T_e = 0.05s$.

3.1. Case of Absence of Disturbance

For a unit step reference applied at $t = 0s$ and a sampling period $T_e = 0.05s$, let's consider the case characterized by the absence of disturbance.

We obtain the following simulation results:

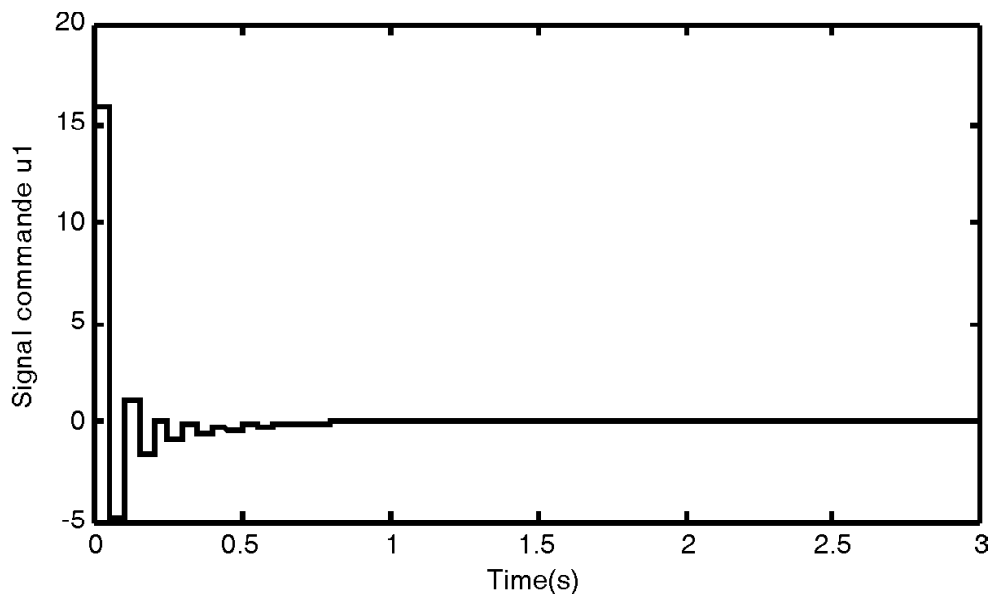


Figure 4: IMC control input u_1

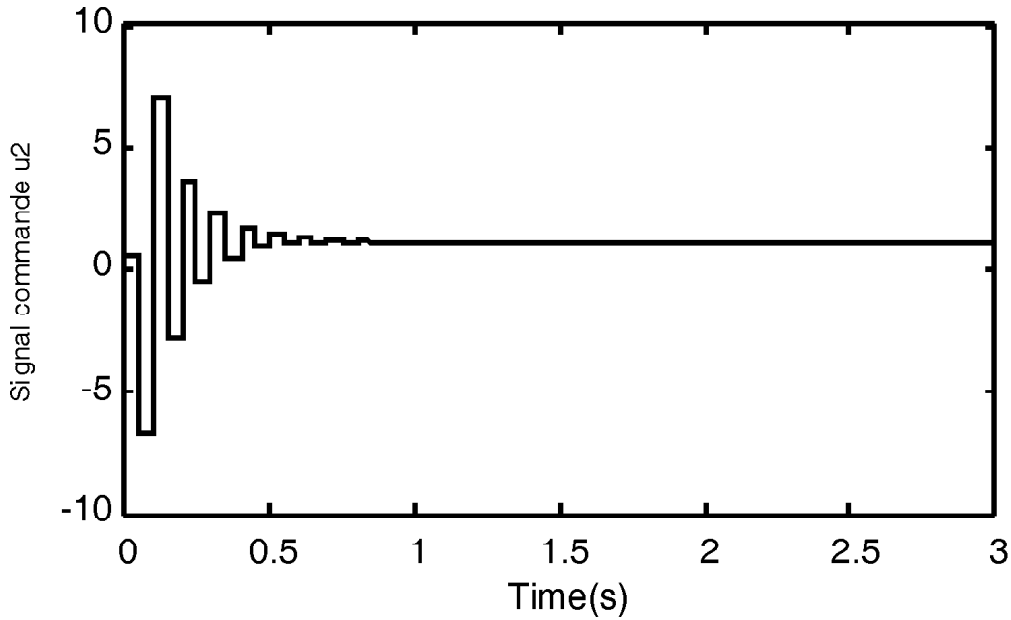


Figure 5: IMC control input u_2

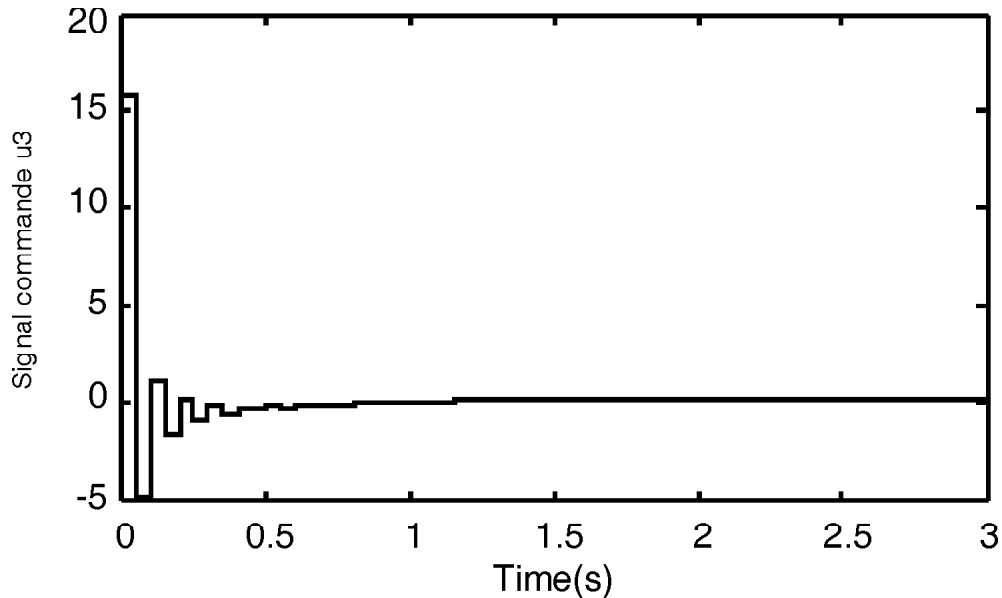


Figure 6: IMC control input u_3

With the simulation results we can clearly see a significant peak of the input U_1 (Figure 4) which appears at the initial instant, this peak is due to the multiplication of the input signal by the matrix gain A_1 , then it is eliminated by the feedback effect.

The evolution of output signal are presented in Figure 7 and Figure 8.

The simulation results show that the system output reach perfectly the reference signal. The IMC approach is applied maintaining the stability of the chosen sampled non-square systems. In order to show the accuracy and the disturbance rejection of our proposed IMC method, let's consider the case of a disturbed system.

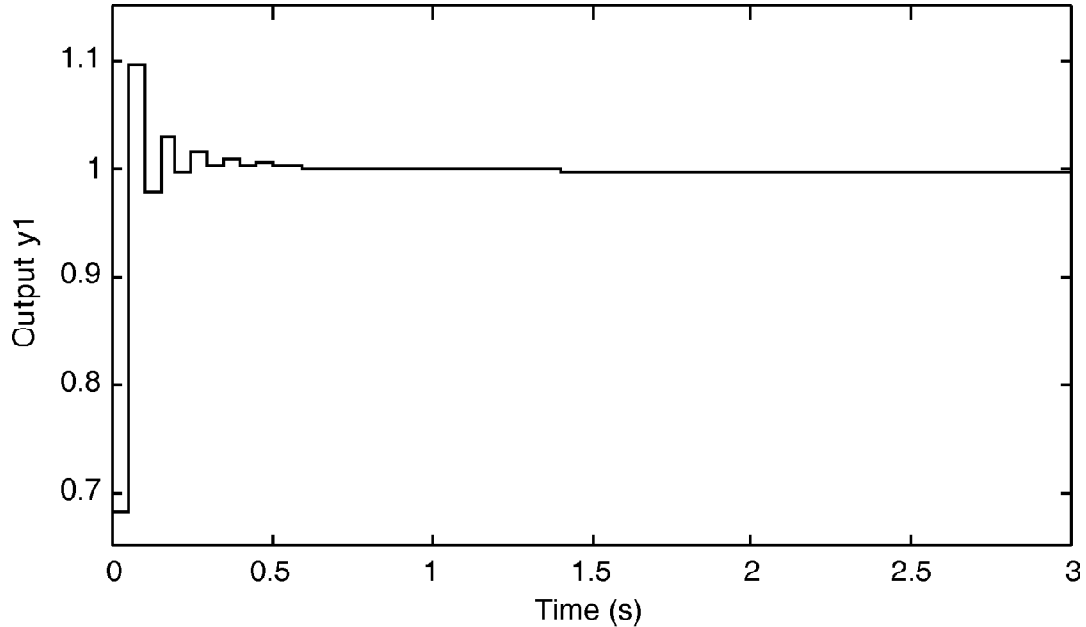


Figure 7: The step response output y_1

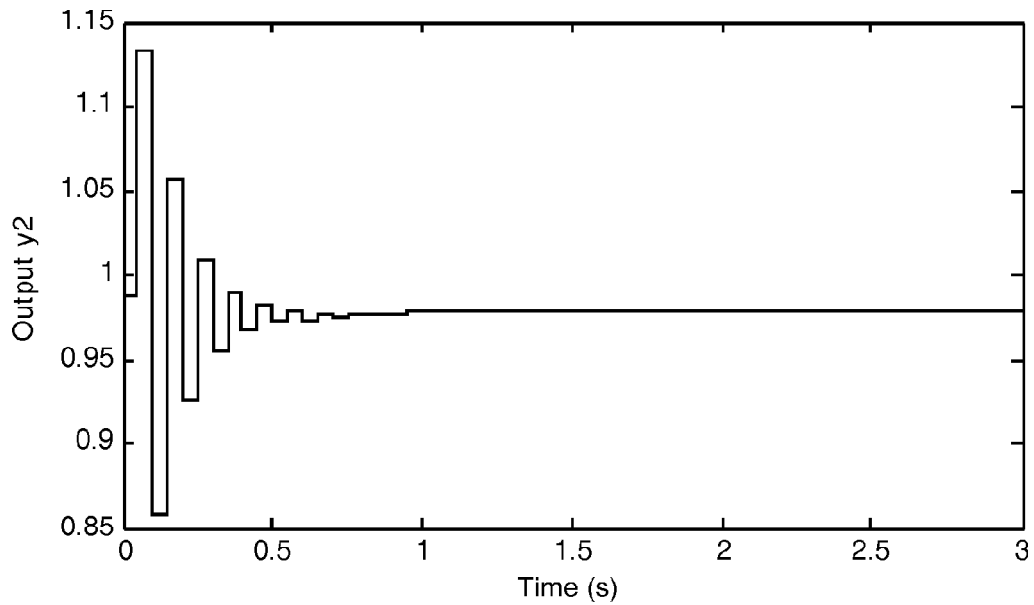


Figure 8: The step response output y_2

3.2. Case of Disturbed System

For the same reference and the sampling period $T_e = 0.05s$, the simulation results in the case of the presence of an external disturbance in the form at a unit step with amplitude 1 and which appears at $t = 1.5s$ are as follows:

From these results, we can notice that the control structure reject the external disturbance and the IMC approach is capable to ensure stability. Our proposed IMC structure gives a perfect reference tracking.

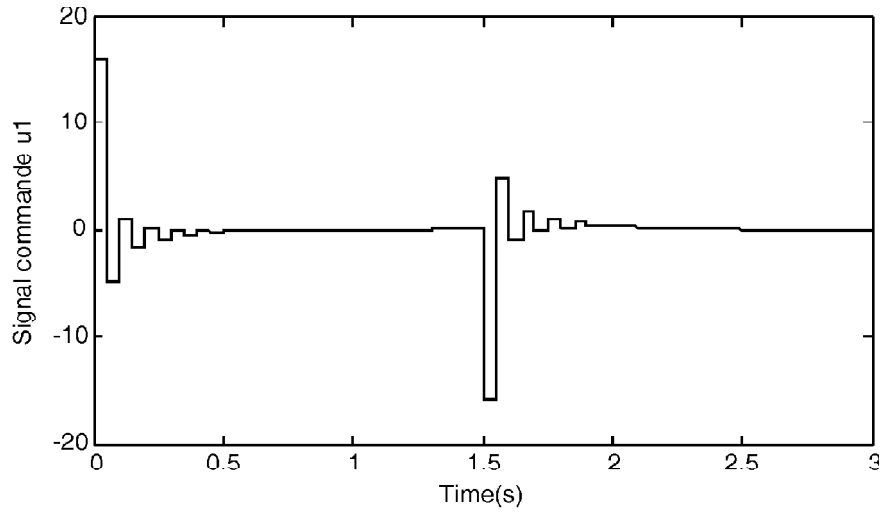


Figure 9: IMC control input U_1 with disturbance

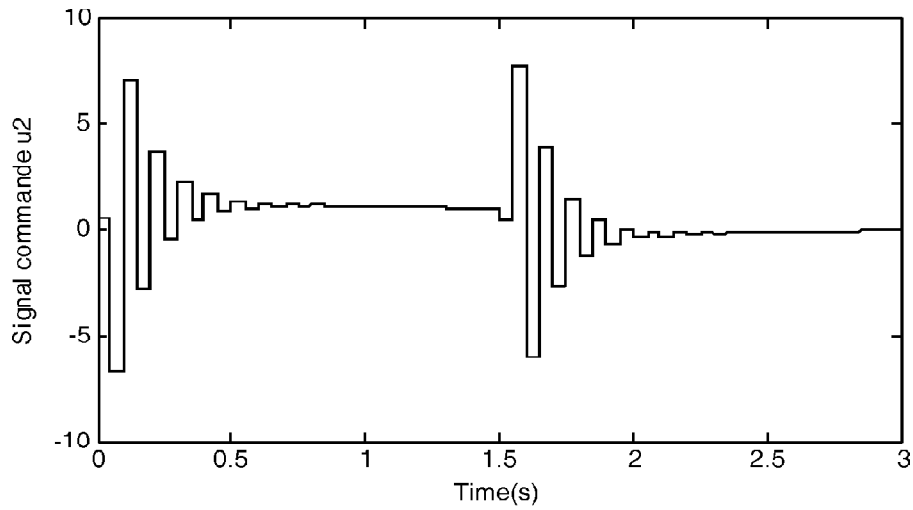


Figure 10: IMC control input U_2 with disturbance

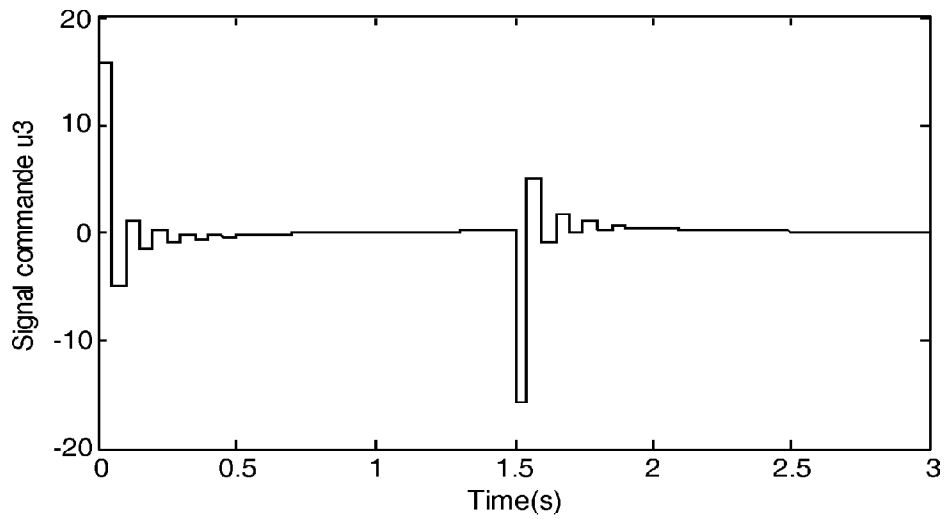


Figure 11: IMC control input U_3 with disturbance

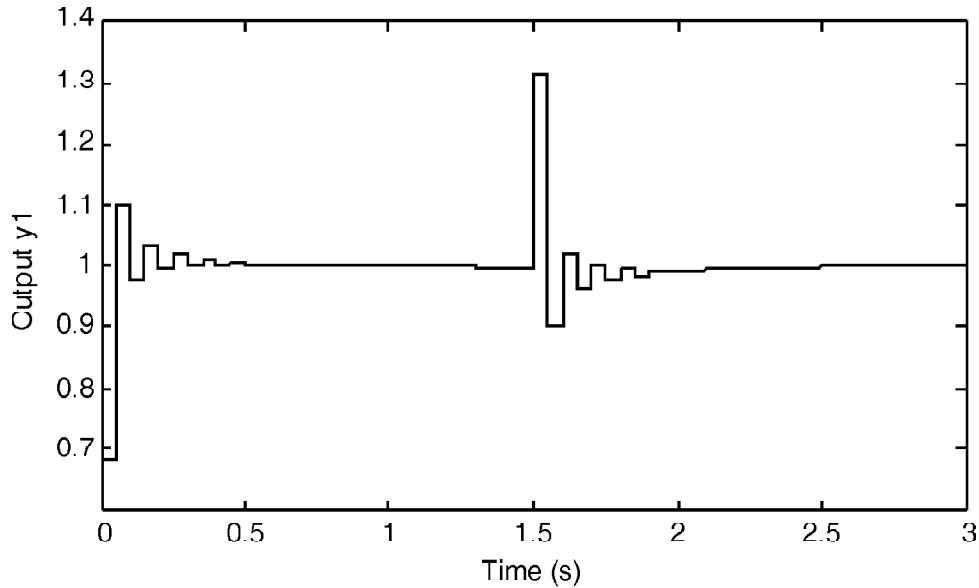


Figure 12: The step response output y_1 with disturbance

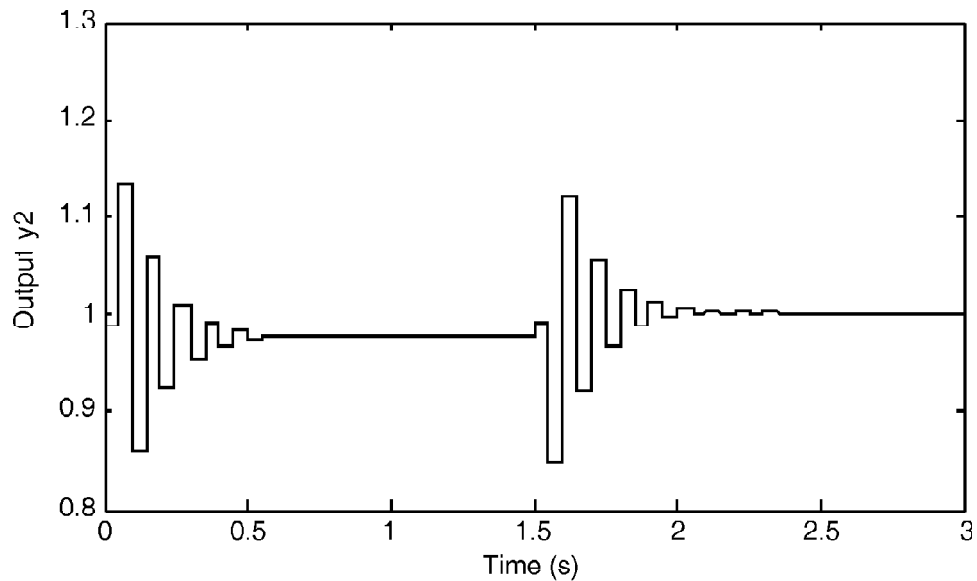


Figure 13: The step response output y_2 with disturbance

5. CONCLUSIONS AND FURTHER WORK

The paper presents a new technique for control of over-actuated multivariable process with outputs more than inputs. The proposed approach IMC ensures stability and preserves system performances despite the external disturbance by using matrix inequality approach.

The Internal Model Control (IMC) which due to its simple design, excellent robustness, and good control performance shows the strong vigor to solve the control problems of the multivariable systems. Satisfactory results have been obtained with the internal multivariable inputs -outputs.

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