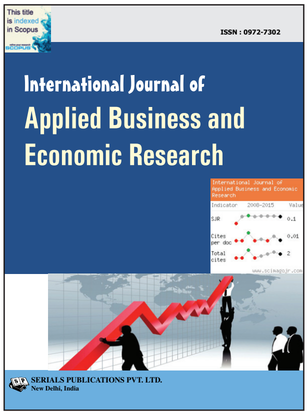

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System Approach in Modeling of Insurance Systems Management

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ABSTRACT

The article describes theoretical, methodological principles and practical basics of system analysis of processes occurring in insurance systems, the basic principles of general model construction are formulated. The activity of insurance systems is defined as a set-theoretic relationship, determined through various systems of equations. Each one of these equations is composed in relation to the variables of the system. The general theory of systems is given a concrete expression to construct the inflow and loss of clients functions, these functions being considered with two interpretations: indicator function and summing function. These functions serve as the basis for constructing specific mathematical queuing models for different cases, on the basis of general model. Realization of such models allows calculation of the values of the main performance indicators of insurance systems. The analysis of the simulation results allows to optimize the activity of the system under study.

Keywords: System, system analysis, insurance system, model, function, set, customer flow, customer loss, differential equations.

1. INTRODUCTION

Analysis of the of the Russian insurance business status shows that insurance companies are mostly unstable. To increase the stability of insurance systems, it is necessary to provide a fundamental theoretical basis to study the processes occurring in insurance companies, using the probability theory, mathematical statistics, mathematical and simulation modeling and system analysis.

The systematic approach was used including the standard scheme of systems theory and system analysis. The general systems theory is adopted for the analysis of the customers inflow and loss functions, based on which it is possible to build mathematical and simulation models of the insurance company activities. Generalized models of systems theory are used to obtain specific queuing models that allow determining the basic operational characteristics of queuing systems in some cases.

2. CONSTRUCTION OF THE INSURANCE SYSTEM GENERAL MODEL

An insurance company can be considered as a complex hierarchical system in which the input effects are transformed into output quantities. When describing the system, to apply the standard the systems theory framework (Buslenko 1978; Denissov 2005;Prokhorov 2006; Fetisov 2014;Okhrimenko 2000).

We will build a generalized model of the insurance system based on the followingprinciples:

1. All the basic concepts are formalized, i.e. the verbal description of a certain concept can be represented in mathematical form, using a minimal mathematical structure with particular properties.
2. We'll study various properties of the system with the help of mathematical theory and taking into account the applicability of obtained results to the solution of specific practical problems.

Let us build a formal system based on theory of setsand the relationships in given sets.

Let there be given a family of sets.

Given a collection of sets: $\mathrm{M} = \{m_i : i \in \mathrm{I}\}$,

where m_i – object insured,

I – set of indices.

Let us consider the model as a system of insurance companies C, represented by its own subset of the Cartesian product of insurance objects $m_i : i \in \mathrm{I}_x$. All subsets of this system are objects of the system C.

Thus, the activity of insurance systems is defined as a set-theoretics relationship, which is determined through various systems of equations. Each one of the equations is composed in relation to the system variables.

Let's assume that:

1. A system is a relationship of the following kind
2. $\mathrm{C} \subset m_1 \times m_2 \times \ldots \times m_k, i = 1, 2, \ldots, k \in I,$

   where

   $\times$ – symbol of a Cartesian product operation,

   II – set of indices,

   m_i – system object.

3. Let $\mathrm{I}_x \in \mathrm{I}$ and $\mathrm{I}_y \in \mathrm{I}$ form the set partition I, *i.e.* $\mathrm{I}_x \cap \mathrm{I}_y = \varnothing$ and $\mathrm{I}_x \cup \mathrm{I}_y = \mathrm{I}$. The set X – Cartesian product of system objects $m_i : i \in \mathrm{I}_x$ is called system inlet. The set – Cartesian product of system objects $m_i : i \in \mathrm{I}_y$ is called system output. Then system C can take a form of $\mathrm{C} = \mathrm{X} \times \mathrm{Y}$. The system like this is called "input-output" or "black box"(Buslenko 1978).

   When studying the insurance systems, C is the event-space of the system C. The process of insuring can be represented as a process of linking the number of concluded contracts x_i (system inlets) ($i = 1,2,\ldots,$N) with the number of completed contracts y_j (system outputs) ($j = 1,2,\ldots,$R).

Insurance contracts may be terminated prematurely, when the insured item is lost, for example. Let $x_i = 0$, if the contract is not completed at time *t*, and xi =1, if at time tthe contract is completed.

The random variable $(t) = x_1 + x_2 + ... + x_N$, shows the number of completed contracts at time *t*.

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Insurance contracts may be terminated prematurely, when the insured item is lost, for example. Let $x_i = 0$, if the contract is not completed at time *t,* and $x_i = 1$*,* if at time *t* the contract is completed. The random variable $(t) = x_1 + x_2 + ... + x_N$, shows the number of completed contracts at time *t*.

3. BUILDING THE FUNCTIONS OF INFLUX AND LOSS OF INSURANCE COMPANY CLIENTS

Let us introduce two functions $n(t)$ and $m(t)$ (Okhrimenko 2000). The function $n(t)$ is the client loss function, and the function $m(t)$ is the function of clients influx, which can be interpreted as follows:

1. If the number of clients loss is counted for the time span [0, T], then the function sums the number of losses of the insurance company clients (summing function). The function $n(t)$ is random, nondecreasing, and changes abruptly.
2. The function $n(t)$ indicates the number of lost clients at time *t* (indicator function).

The function $m(t)$ is interpreted in a similar way.

Let us identify the set of system inlets m_i with the system status

$$m(t) \equiv E_{n(t)}^{m(t)},$$

where

$$i \equiv t,$$

$$i - \text{index},$$

$$[0, T] \equiv I_x - \text{time span}.$$

$E_{n(t)}^{m(t)}$ depends on the interpretation of the clients influx and loss function:

1. If $n(t)$ is a summing function, $m(t)$ is a summing function, then $E_{n(t)}^{m(t)}$ means that in the system C$n(t)$clients were lost and $m(t)$contracts were concluded during the time span [0, T].
2. If $n(t)$ is a summing function, $m(t)$ is an indicator, then $E_{n(t)}^{m(t)}$ means that over the time span [0, T]$n(t)$ clients were lost and $m(t)$contracts were concluded at time *t*.
3. If $n(t)$ – is an indicator, $m(t)$ is a summing function, then $E_{n(t)}^{m(t)}$ means that over the time span [0, T] $m(t)$contracts were concluded and $n(t)$ contracts were terminated at time *t* in the system C.
4. If $n(t)$ is an indicator, $m(t)$ is an indicator, then $E_{n(t)}^{m(t)}$ means that $n(t)$ clients were lost and $m(t)$ contracts were concluded at time *t*.

Thus, the inlet X of the system C is the Cartesian product of events $E_{n(t)}^{m(t)}$: $t \in [0, T]$. The system status at its output depends on its status at the inlet.

To build mathematical models and to analyze the activities of insurance companies, it is necessary to build systems of ordinary differential equations that describe the processes occuring in insurance companies. These systems allow to determine all the main characteristics of the processes under study(Okhrimenko 2000).

Let a random process be characterized by probabilities $P_{n(t),\, m(t)}$, which evaluate the system status E^{m}_{n}.

We obtain the differential equations in unknown probabilities $P_{n(t),\, m(t)}$, having two properties:

Property 1. The system C is in status E^{0}_{0}. at time $t = 0$. Not more than one change can occur in the system C over the time span $(t, t + h)$(h is sufficiently small, $h > 0$). The system C changes only in going from a state to the nearest neighboring state: from E^{m}_{n} to E^{m}_{n+1}, from E^{m}_{n} to E_{n}^{m+1} , from E^{m}_{n} to E^{m}_{n}. If the system C is in the state E^{m}_{n} at some instant, the transition probabilities are determined from the formula (Feller 1984):

$$P(E^{m}_{m} \to E^{m}_{n+1}) = \lambda_{n,\, m}\, h + O(h),$$

$$P(E^{m}_{m} \to E^{m}_{n+1}) = \mu_{n,\, m}\, h + O(h),$$

where $h > 0$, h is sufficiently small, $\lambda_{n\,,\, m}$ is the intensity of clients influx, $\mu_{n\,,\, m}$ is the intensity of output, $0(h)$ is infinitesimal of higher order than h at $h \to 0$.

Property 2. At the time $t = 0$ the system is in the state E^{0}_{i} ($i \geq 1$). Not more than one change can occur in the system C over the time span $(t, t + h)$. (The system changes only in going from an intermediate state to the nearest neighboring state). The transition probabilities are determined by such rules (Feller 1984):

$$P\left(E_{i+n}^{m} \to E_{i+n+1}^{m}\right) = \lambda_{n,\, m}\, h + O\ (h),$$

$$P\left(E_{i+n}^{m} \to E_{i+n}^{m+1}\right) = \mu_{n,\, m}\, h + O\ (h),$$

$$P\left(E_{i+n}^{m} \to E_{i+n+x}^{m+y}\right) = O(h),\ x \geq 1, y \geq 1,$$

$$P\left(E_{i}^{0} \to E_{i}^{1}\right) = \mu_{0,\, 0}\, h + O\ (h),$$

$$P\left(E_{i}^{0} \to E_{i+1}^{0}\right) = \lambda_{0,\, 0}\, h + O\ (h),$$

$$P\left(E_{i}^{0} \to E_{i+x}^{y}\right) = O(h),\ x \geq 1, y \geq 1,$$

Let us denote the probability of the state E^{m}_{i+n} by $P_{i+n,\, m}(t)$. Thus, the random process under consideration is given by three matrices: the matrix of transition probabilities P, the matrix of customer loss intensity L, the matrix of clients inflow intensity M.

The matrices L and M are defined by the most random process(Okhrimenko 2000). The properties 1 and 2 are formulated based on the study of the insurance companies activities.

4. MATHEMATICAL MODELS OF THE PROCESSES OCCURRING IN INSURANCE COMPANIES

To derive the basic differential equations of the model, let us consider the time intervals $(0, t)$ and $[t, t+h]$. In (Gnedenko 1987; Feller 1984)the probability $P_{n,m}(t)$ is found, which is the probability of the system status E^{m}_{n} at the end of time span $t = h$

The following cases are possible.

Case 1. During the time span $(0, t)$ the system passed to the state E^{m}_{n} and, in the interval of time h there were no changes in the system. Events corresponding to this process form a complete group, so, taking into account that the sum of a finite number of infinitesimal functions is an infinitesimal function of a higher order of smallness, we obtain

$$P\left(E^{m}_{n} \to E^{m}_{n}\right) = 1-\lambda_{n,m}h-\mu_{n,m}+O(h)$$

Assume that the events in the time spans $(0, t)$ and $(t, t+h)$ are independent, then the probability of case 1 is equal to:

$$P_{n,m}(t) - \left(\lambda_{n,m}+\mu_{n,m}\right)hP_{n,m}(t)+O(h)$$

Case 2. In the time span $(0, t)$ the system passed in the state E^{m}_{n-1}, and one client was lost in time h, *i.e.* the transition $E^{m}_{n-1} \to E^{m}_{n}$ was completed. In virtue of property 1:

$$P\left(E^{m}_{n-1} \to E^{m}_{n}\right) = \lambda_{n-1,m}h+O(h)$$

and then, the probability of case 2 is equal to :

$$\lambda_{n-1,m}P_{n-1,m}(t)h + O(h)$$

Case 3. In the time span $(0, t)$ the system passed in the state E^{m-1}_{n} and, one contract was concluded in time h, *i.e.* the transition $E^{m-y}_{n-x} \to E^{m}_{n}$ was completed.

Based on property 1:

$$P\left(E^{m-1}_{n} \to E^{m}_{n}\right) = \mu_{n,m-1}h+O(h)$$

and the probability of case 3 is equal to

$$\mu_{n,m-1}P_{n,m-1}(t)h + O(h)$$

Case 4. The system passed in the state $E^{m-y}_{n-x}\left(x, y \geq 1\right)$ in the time span $(0, t)$, x clients were lost and y contracts were concluded in time h, *i.e.* the transition $E^{m-y}_{n-x} \to E^{m}_{n}$ was completed. The probability of this transition is equal to $O(h)$, and the probability of this case is equal to:

$$P_{n-x,\, m-y}(t) = O(h)$$

We define the sought probability $P_{n,m}(t)$ using the addition law of probabilities of incompatible events (Feller 1984):

$$P_{n,m}(t+h) = P_{n,m}(t)-\left(\lambda_{n,m}+\mu_{n,m}\right)P_{n,m}(t)h+\lambda_{n-1,m}P_{n-1,m}(t)h+\mu_{n,m-1}P_{n,m-1}(t)h+O(h)$$

Therefore

$$P_{n,m}(t+h) - P_{n,m}(t) = -\left(\lambda_{n,m} + \mu_{n,m}\right)P_{n,m}(t)h + \lambda_{n-1,m}P_{n-1,m}(t)h + \mu_{n,m-1}P_{n,m-1}(t)h + O(h)$$

or
$$\frac{P_{n,m}(t+h) - P_{n,m}(t)}{h} = -\left(\mu_{n,m} + \nu_{n,m}\right)P_{n,m}(t) + \lambda_{n-1,m}P_{n-1,m}(t) + \mu_{n,m-1}P_{n,m-1}(t) + \frac{O(h)}{h},$$

passing to the limit at $h \to 0$, we obtain

$$\lim_{h\to 0}\frac{P_{n,m}(t+h) - P_{n,m}(t)}{h} = -\left(\mu_{n,m} + \nu_{n,m}\right)P_{n,m}(t) + \lambda_{n-1,m}P_{n-1,m}(t) + \mu_{n,m-1}P_{n,m-1}(t) + \lim_{h\to 0}\frac{O(h)}{h}$$

So we have one differential equation:

$$P'_{n,m}(t) = -\left(\lambda_{n,m} + \mu_{n,m}\right)P_{n,m}(t) + \lambda_{n-1,m}P_{n-1,m}(t) + \mu_{n,m-1}P_{n,m-1}(t).$$

In (Gnedenko 1987; Feller 1984) a differential equation associated with initial state of the system E_0^0 is established for case 5.

Case 5. The system is in the state E_0^0 and no changes occur in time h, *i.e.* the transition $E_0^0 \to E_0^0$ was completed in time span $(t, t + h)$.

Consider the transition $E_0^0 \to E_0^1$ The state E_0^0 means that there is no clients loss in the system C. The events $E_0^0 \to E_0^0$, $E_0^0 \to E_0^1$, $E_0^0 \to E_1^0$, $E_0^0 \to E_n^m$ form a complete group of events, their probabilities are, respectively, equal to (Feller 1984):

$$\begin{aligned} P(E_0^0 &\to E_0^0), \\ (E_0^0 &\to E_1^0), &= \lambda_{0,0}\, h + O(h), \\ P(E_0^0 &\to E_0^1), &= \mu_{0,0}\, h + O(h), \\ P(E_0^0 &\to E_n^m), &= O(h). \end{aligned}$$

This implies:

or $$P\left(E_0^0 \to E_0^0\right) + P\left(E_0^0 \to E_1^0\right) + P\left(E_0^0 \to E_0^1\right) + P\left(E_0^0 \to E_n^m\right) = 1$$

$$P\left(E_0^0 \to E_0^0\right) = 1 - (\lambda_{0,0} + \mu_{0,0})h + O(h)$$

and, by the addition theorem of probabilities of independent events, the probability of an event for case 5 is equal to:

$$P_{0,0}(t) = \left(\lambda_{0,0} + \mu_{0,0}\right)h + O(h),$$

where
$$P_{0,0}(t) = P\left(E_0^0 \to E_0^0\right)$$

Then, by the addition theorem of probabilities of independent events (Feller 1984):

$$P_{0,0}(t+h) = P_{0,0}(t) - \left(\lambda_{0,0} + \mu_{0,0}\right)hP_{0,0}(t) + O(h)$$

or $$\frac{P_{0,0}(t)-P_{0,0}(t)}{h} = -(\lambda_{0,0}+\mu_{0,0})P_{0,0}(t)+\frac{O(h)}{h}.$$

Passing to the limit at $h \to 0$ we get

$$\lim_{h\to 0}\frac{P_{0,0}(t+h)-P_{0,0}(t)}{h} = -(\lambda_{0,0}+\mu_{0,0})P_{0,0}(t)+\frac{O(h)}{h},$$

and the differential equation of initial states of the system will be written as:

$$P'_{0,0}(h) = -(\lambda_{0,0}+\mu_{0,0})\,P_{0,0}(t).$$

A system of infinitely many equations is finally obtained for this case

$$\begin{cases} P'_{n,m}(t) = -(\lambda_{n,m}+\mu_{n,m})P_{n,m}(t)+\lambda_{n-1,m}P_{n-1,m}(t)+\mu_{n,m-1}P_{n,m-1}(t) \\ P_{0,0}(t) = -(\lambda_{0,0}+\mu_{0,0})P_{0,0}(t), \end{cases}$$

where n, $m \in$ N.

This system of equations represents the mathematical model of a random process of concluding contracts, under condition of feasibility of property 1.

A similar system of equations can be obtained for random processes possessing property 2, which differs from property 1 by initial states.

The solution of equations system (1) depends on the properties of matrices L and M, *i.e.* on the behavior of numerical sequences $\{\lambda_{n,m}\}$ and $\{\mu_{n,m}\}$.

Let $$n = m, \lambda_{n,m} = \lambda_n, \mu_{n,m} = \mu_n$$

$$\lambda_{n-1,m} = \lambda_{n-1}, \mu_{n-1,m} = \mu_{n-1}$$

then the system is written as

$$+\ P'_n(t) = -(\lambda_n\mu_n)P_n(t)+\lambda_{n-1}P_{n-1}(t)+\mu_{n+1}P_{n+1}(t)$$

$$P'_0(t) = -(\lambda_0+\mu_0)P_0(t),$$

where $$n = 1, 2, \ldots .$$

Let us form the matrix $$A = ((a_{ij})), \text{ where}$$

$$a_{i,i+1} = \lambda_i,\ a_{i,i-1}$$

$$= \mu_i,$$

$$a_{ii} = -(\lambda_i + \mu_i),$$

$$a_{ij} = 0,$$

if $$|i-j| > 1, \lambda_i > 0 \text{ at } ii \geq 0,$$

$$\mu_i > 0 \text{ at } i \geq 0$$

and $$\mu_0 = 0.$$

If $\sum_{n=1}^{\infty} a_{\text{in}} = 0$ at any *i,* then the matrix is conservative. If the system coefficients form a conservative matrix A and

$$\sum_{n=1}^{\infty}\left(\frac{1}{\lambda_n}+\frac{\mu_n}{\lambda_n\lambda_{n-1}}+\ldots+\frac{\mu_n\mu_{n-1}\ldots\mu_2}{\lambda_n\lambda_{n-1}\ldots\lambda_1}\right) = \infty,$$

then there exists a unique solution of system (1) and this solution satisfies the completeness condition

$$\sum_{n=1}^{\infty} \mathrm{P}_n(t) = 1 \text{ at any } t \text{ (Feller 1984).}$$

When determining solutions to the systems, initial conditions are given, for example,

$$\mathrm{P}_0(0) = 1,\ \mathrm{P}_n(0), \text{ if } n \neq 0.$$

To derive the differential equations, let us calculate $\mathrm{P}_n(t+h)$. A system can be in state E_n, at the point of time $t + h$, when and only when one of the following requirements is fulfilled:

1. At time t the system is in the state E_n and no changes occur in time h, that is, there is a transition $\mathrm{E}_n \to \mathrm{E}_{n'}$ the probability of this transition is equal to:
2. $\mathrm{P}_n(t)-(\lambda_n+\mu_n)\mathrm{P}_n(t)h+\mathrm{O}(h)$.
3. At time t the system is in the state E_{n-1} and in time h one client is lost, *i.e.* the transition $\mathrm{E}_{n-1} \to \mathrm{E}_{n'}$ is completed, its probability is equal to $\lambda_{n-1} h + \mathrm{O}(h)$.
4. At time t the system is in the state E_{n+1}, and in time h one contract is concluded, that is the transition $\mathrm{E}_{n+1} \to \mathrm{E}_n$ is completed with the probability equal to $\mu_{n-1} h + \mathrm{O}(h)$.
5. In time h to or more changes occur. The probability of this condition is $\mathrm{O}(h)$.

Based on these grounds, the following system of differential equations is obtained

$$\mathrm{P}'_n(t) = -(\lambda_n\mu_n)\mathrm{P}_n(t)+\lambda_{n-1}\mathrm{P}_{n-1}(t)+\mu_{n+1}\mathrm{P}_{n+1}(t),\ n\in \mathrm{N}$$

5. CONCLUSION

The insurance systems operation is described in terms of system analysis, which seems quite logical. This approach allows to use the system analysis to construct the clients inflow and loss functions which make it possible to develop specific mathematical queuing models for calculating the basic performance indicators of insurance companies.

Every insurance company's activities are designed to prevent their clients from financial losses or minimize them if the insured event occurs. Occurrence of the insured events is uncertain. Before the insurance contract is concluded, the client has the risk of an insured event, leading to the incidental loss X. If the client enters into an insurance contract, then, having paid a certain amount, he (or she) shifts the risks to the insurance company. Thus, the problem of ensuring the financial stability of the insurance company arises. This problem is complex and it can be solved with the help of a comprehensive analysis of the insurance company activities.

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