

## Manifestations of Langmuir Solitons in Satellites of Dipole-forbidden Spectral Lines of Helium, Lithium, and of the Corresponding Ions

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**ABSTRACT:** We show that in the case of Langmuir solitons in plasmas, the peak intensity of the satellites of dipole-forbidden spectral lines of helium, lithium, and of the corresponding ions can be significantly enhanced – by orders of magnitude – compared to the case of non-solitonic Langmuir waves. This distinctive feature of satellites under Langmuir solitons allows distinguishing them from non-solitonic Langmuir waves and seems to be the best way to diagnose Langmuir solitons in plasmas.

In paper [1] there was calculated analytically the shape of satellites of dipole-forbidden lines in a spectrum *spatially-integrated* through a Langmuir soliton (or through a sequence of Langmuir solitons separated by a distance  $L$ ). The Langmuir solitons have the following form in space [2]:

$$F(x, t) = E(x) \cos \omega_p t, \quad E(x) = E_0 / \text{ch}(x/\lambda), \quad \lambda \ll L. \quad (1)$$

Here

$$\omega_p = \omega_{pe} - 3T_e / (2m_e \omega_{pe} a^2), \quad (2)$$

where  $\omega_{pe}$  is the plasma electron frequency. For diagnosing solitons it is necessary not only to find experimentally an electric field oscillating at the frequency  $\sim \omega_{pe}$ , but also to make sure that the spatial distribution of the amplitude corresponds to the formfactor  $E(x)$  from (1).

Under any quasimonochromatic electric field, dipole-forbidden spectral lines of helium, lithium, and of the corresponding ions can exhibit satellites – see Appendix J. In cases of a relatively large separation between the forbidden and allowed lines, or a relatively weak amplitudes, the intensities of the far (+) and near (–) satellites are  $S_{\pm} = a_{\pm} E^2(x)$ , where  $a_{\pm}$  does not depend on  $x$ .

The spatially-integrated profile of the satellites, calculated with the allowance for the quadratic shift of their frequencies  $bE^2(x)$ , where  $b$  does not depend on  $x$ , has the form:

$$S + (\Delta\omega) = (1/L) \int_{L/2}^{L/2} dx a_{\pm} E^2(x) \delta[f(x)], \quad f(x) = \Delta\omega + \omega_p - bE^2(x). \quad (3)$$

After calculating the integral in Eq. (7.3), in paper [1] it was obtained:

$$S_{\pm}(\Delta\omega) = (\lambda/L) [a_{\pm} / (2|b|)] / [1 - (\Delta\omega \pm \omega_p) / (bE_0^2)]^{1/2}. \quad (4)$$

The profiles  $S_{\pm}(\Delta\omega)$ , formally calculated by Eq. (4), have singularities at  $\Delta\omega \pm \omega_p = bE_0^2$ . From the physical point of view, for obtaining a finite result at  $\Delta\omega \pm \omega_p = bE_0^2$ , it is necessary to replace the  $\delta$ -function in Eq. (3) by a real profile, e.g., by the Lorentz profile representing the dynamical Stark broadening by electrons and by some ions:

$$\delta[f(x_0)] \rightarrow (1/\pi)\gamma/\{\gamma^2 + [f(x_0)]^2\}, \quad (5)$$

where  $x_0$  is the root of the equation  $f(x) = 0$ . Using the Taylor expansion of  $f(x)$  at  $x = x_0$  and taking into account the first derivative  $df/dx$  vanishes at  $x = x_0$ , the right side of Eq. (5) can be approximated as follows:

$$(1/\pi)\gamma/\{\gamma^2 + [d^2f(x_0)/dx^2]^2(x - x_0)^4\}. \quad (6)$$

Then using the integral

$$(1/\pi) \int_{-\infty}^{\infty} dz \gamma/(\gamma^2 + z^4) = 1/(2\gamma)^{1/2}, \quad (7)$$

we find

$$S_{\text{soliton}}^{\text{max}} = S_{\pm}(\Delta\omega = \pm \omega_p - bE_0^2) = [\lambda a_{\pm}/(2^{1/2}\pi L b^{1/2})]E_0^2/\gamma^{1/2}, \quad (8)$$

where  $S_{\text{soliton}}^{\text{max}}$  is the peak intensity of the satellites in the case of solitons.

In the case of non-solitonic Langmuir waves, the peak intensity of the satellites is (see, e.g., book [3]):

$$S_{\text{nonsoliton}}^{\text{max}} = a_{\pm}E_0^2/(\pi\gamma). \quad (9)$$

So, for the ratio of the peak intensity of the satellite under solitons to the peak intensity of the same satellite under nonsolitons we get

$$S_{\text{soliton}}^{\text{max}}/S_{\text{nonsoliton}}^{\text{max}} = [\lambda/(2^{1/2}L)][\gamma/(bE_0^2)]^{1/2} \gg 1 \quad (10)$$

for the typical situation where  $\gamma \gg bE_0^2$ , i.e., where the dynamical Stark width  $\gamma$  is much greater than the quadratic shift  $bE_0^2$  of the satellites.

Thus, in the case of Langmuir solitons, the peak intensity of the satellites can be significantly enhanced – by orders of magnitude – compared to the case of non-solitonic Langmuir waves. This distinctive feature of satellites under Langmuir solitons allows distinguishing them from non-solitonic Langmuir waves and seems to be the best way to diagnose Langmuir solitons in plasmas.

Finally we note that for a more general case, where both the dynamical Stark broadening and the Doppler broadening are taken into account, the  $\delta$ -function in Eq. (3) should be substituted by the Voigt profile resulting in

$$S_{\text{soliton}}^{\text{max}}/S_{\text{nonsoliton}}^{\text{max}} \sim \Delta\omega_{\text{Voigt}}/(bE_0^2)^{1/2} \gg 1, \quad (11)$$

where  $\Delta\omega_{\text{Voigt}}$  is the halfwidth of the Voigt profile. Again, in the typical situation where

$\Delta\omega_{\text{Voigt}} \gg bE_0^2$ , the peak intensity of the satellites is significantly enhanced – by orders of magnitude – compared to the case of non-solitonic Langmuir waves.

Finally we mention that Hannachi *et al.* [4] performed simulations for finding the effect of Langmuir solitons on the hydrogen Ly $_{\alpha}$  line. The effect was an additional broadening. However, even at the low electron density  $N_e = 10^{14} \text{ cm}^{-3}$ , the effect was very small compared to the Stark broadening by plasma microfields. Moreover, the additional broadening rapidly diminished with the increase of  $N_e$ , so that there would be practically no additional broadening at  $N_e > 10^{15} \text{ cm}^{-3}$ . Therefore, it seems that the results by Hannachi *et al.* [4] could not be useful for the experimental diagnostics of Langmuir solitons.

## REFERENCES

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