Classical Mechanics Tutor: A graphical kit

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ABSTRACT

Classical Mechanics Tutor is an e-learning tool that simulates popularly taught systems in classical mechanics such as inclined planes, springs and pendulums. It provides a means to experiment and learn properties of the above mentioned systems. User can manipulate input values from a predefined set of parameters and initial conditions, to view the corresponding animation of the user-selected system, for a better understanding. The main aim of this application is to act as a guide to students i.e. as an educational tool to help school students visualize and build strong foundational hold of important classical mechanics concepts.

Keywords: Opengl, computer graphics, mechanical simulation, inclined plane, pendulum, spring, mechanics.

1. INTRODUCTION

Classical Mechanics Tutor is an effective tool consisting of a user friendly Graphical User Interface (GUI), which facilitates understanding the concepts of classical mechanics. Classical mechanics is defined as the study of motion of bodies (including at rest) in accordance with the principles first put forward by Sir Isaac Newton[1]. The proposed tool includes three classical mechanics systems namely inclined planes, simple pendulums and springs.

An object when placed on an inclined plane, tends to slip in a downward direction [3]. This motion occurs due to existence of an unbalanced forces namely gravitational force and normal force [2]. In a pendulum, a periodic back and forth motion about the equilibrium position is observed, which is caused by the restoring force acting upon the bob at all times. A spring system comes under simple harmonic motion systems, where a restoring force acts upon the mass, increasing when the mass moves towards the equilibrium position and vice versa, as explained by Hooke's law [5].

Rest of the paper is organized as follows. Section II gives motivation for the proposed work. Section III gives a brief review about the different kinds of works done in the same field. The mathematical model of implementation is given in Section IV. Section V gives an overview of the level of accuracy and precision being taken care of. The paper concludes with the results observed and the features of the interactive animation described in Section VI.

2. MOTIVATION

Getting a strong fundamental hold of classical mechanics concepts is extremely crucial for students from all the streams. Improving the user interface and allowing the user to play with values stimulates the mind and opens it up to all possible combinations of problems. This enables the user to face any variation given in a specific kind of problem and enhances the ability to understand a concept. Simulations of concepts with actual scientific parameters is lagging in the field of tutor simulations. This is a motivation to choose this topic.

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3. LITERATURE SURVEY

Scholarly articles related to graphical simulations in the domain of mechanics has revealed that most of the existing papers are based on the study of the motion of the bodies and the depiction of the nature of the motion in graphics.

The simulation of pulleys and cable, which can be used in Real Time Computer Graphics [6] is proposed. This model involves coupling of a damped spring and a variable coefficient wave equation integrated with cranes, elevators etc. However this does not involve showing the motion of the pulleys or changing any parameters such as the number of pulleys, mass of bodies etc. Using physically based deformable models in real-time applications such as motion pictures and video games are simulated using physics laws of motion [7]. Though it involves a lot of physics concepts, it only covers the theoretical aspects. An efficient algorithm for detecting self-collisions in complex collisions resulting from highly deformable surfaces, as well as some techniques for evaluating collision inside-outside orientation in a robust way is presented [9]. A physics-based controller for animating 3-D biped characters that can react to dynamic environments addresses the usage of inverted pendulum motion to adjust the desired motion trajectory in a 3D Character Motion (human motion) [10]. The main focus is on the application in real-time human motion simulation and does not aim to give an insight into the fundamentals of the pendulum motion dynamics.

The literature survey indicates that most of the earlier works integrate the physics involved in mechanics to real-time applications such as those involving wave motions, collision detection, pulley simulation, etc. However, it is observed that there is no such product which offers many categories of mechanics problems as one complete package and simulation of motions in applied mechanics with minute details. This makes the proposed tool unique and useful as it does not have any system specific requirements and is fairly simple to use.

4. METHODOLOGY

Each component, namely inclined plane, pendulum and spring, has its own set of figures to be drawn on the screen, set of input parameters to be manipulated by the user and set of formulae governing their motion. A library to render 2D vector graphics is used to display simulated objects on the screen. Coordinate geometrical equations and laws of motion formulae are applied to manipulate animations on the screen.

4.1. Inclined Plane

The user can change the parameters, mass and the angle of inclination of the plane, after which clicking on the set button starts the simulation.

For a rigid body to move on a rough plane, the net horizontal force should always be greater than the static frictional force. [11]. Once it overcomes the static frictional force, kinetic friction comes into play. Also coefficient of static friction has to be greater than the coefficient of kinetic friction and both the coefficients of friction are bound by 1 as the upper limit. Algorithm 1 describes how the animation is drawn on the screen, where these concepts are used in calculating the results.

Algorithm 1 Inclined plane

```
Clear screen
Draw plane on screen
if (set) then . set indicates that the user has pressed on the set button
calculate results including acceleration
move = velocity acceleration.: moves the block on inclined plane at the rate of the calculated acceleration
print result on screen
end if
```

where mass of block is m and g is gravity of Earth or Moon. Net Vertical Force refers to the vector sum of forces acting in the upward direction and Normal Reaction is the perpendicular force exerted by the surface on the body. The coefficient of kinetic friction is μ_k .

4.2. Pendulum

The user can set the parameters - length of the pendulum, mass of the bob and the angle of oscillation. Once the parameters are set, the pendulum begins to oscillate. The potential energy is maximum and the kinetic energy is minimum when the pendulum is at the highest position and vice versa for the mean position [2]. The variation in velocity caused due to this, has been simulated in the tool. Algorithm 2 describes the way animation is drawn on the screen, where these concepts are used for calculating the results.

4.3. Spring

The parameters that can be varied by the user are k, which is the spring constant, and the mass of the block. The displacement of the spring can be set by the user by a simple drag motion. In Algorithm 3, equation 1 is used in calculating the result in Line 18 of the Algorithm 3.

Spring has a damping force which reduces the amount of displacement after each oscillation. The force of compres-sion/expansion is given by [5]:

$$\mathbf{F} = \mathbf{k}\mathbf{x} \tag{1}$$

where k is spring constant and x is displacement.

Algorithm 2 Pendulum
Clear screen
if !(set) then . set indicates that the angle of oscillation has been set and the user has pressed on the set button
if (dir) then . if user has dragged the pendulum right drawpendulum(xf; yf; 180 angss) . angss-
angle of oscillation when not set
else
drawpendulum(xf; yf; 180 + angss)
end if
else
if (angle < 180 then . pendulum moving towards equillibrium position
inc = $g \cos(180 \text{ angle}) 3:14 = 180)$
else
inc = $g \cos(180 \text{ angle}) 3:14 = 180)$
end if
vel + = inc
angle + = vel

draw Pendulum (1; 10; angle)

end if

Calculate result

Print result on screen

Algorithm 3 Draw spring

```
Clear screen
if (set) then
             . set indicates that the user has pressed on
the set button
   if displacement > dif f + equipos then
                                             . bob is
above equilibrium position
       displacement = eqipos + dif f
       velocity = 0
       inc = 0.05 \cos(equipos)
                                 displacement)
   else
       displacement = eqipos dif f
       velocity = 0
   end if
   if displacement < equipos then
                                         . bob moving
towards equilibrium position
       inc = 0.05
                      cos(equipos
                                             displacement)
          . *bob moving away from equilibrium position
   else
      inc = 0:05
                      cos(equipos
                                     displacement)
   end if
   velocity + = inc
   displacement + = velocity
   Calculate result
end if
Draw spring on screen
Print result on screen
```

5. EXPERIMENTS AND ANALYSIS

The possible scenarios and corresponding parameter values experimented, using the tool are shown below.

5.1. Inclined Plane

For an inclined plane, the parameters given as input are: mass, angle of inclination, coefficients of static and kinetic friction. The results of the simulations can be observed from the output parameters such as Net horizontal and vertical forces which are responsible for the motion of the body and its acceleration as shown in Table 1. Fig. 1 shows that the motion of block is in the middle of the plane when it is in kinetic motion. The block comes to rest when it reaches the end as shown in Fig. 2.

			Different possible combinations of parameter values for memory plane							
SI. No.	Mass (m)	Angle (\ominus)	Static Friction Coefficient (μ_s)	Kinetic Friction Coefficient (μ_k)	Net Vertical Force	Net Horizontal Force	Frictional Force	acceleration (a)		
1	2	30	0.8	0.3	19.60	0	13.58	0		
2	2	70	0.8	0.3	19.60	16.14	2.01	8.20		
3	6	30	0.5	0.5	58.80	14.12	15.28	2.35		
4	8	15	0.5	0.3	78.40	0	37.86	0		
5	8	25	0.8	0.6	78.40	22.17	33.26	2.77		

 Table 1

 Different possible combinations of parameter values for inclined plane



5.2. Pendulum

Angle of release, mass of the bob, length of the pendulum, selection of gravitational medium (earth or moon) are given as input to the simulation. It evaluates the potential energy and kinetic energy of the moving bob at different positions along with the restoring force and tension in the string which govern the motion of the pendulum as shown in Table 2. Fig. 3 and 4 depict the positions of the pendulum in motion.

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Table 2 Different possible combinations of parameter values for pendulum								
SI. No.	Mass (m)	Angle (\ominus)	Gravity (g)	Length (L)	Restoring Force	Tension (T)	Total Energy	Time Period t
1	4	30	9.8	20	19.58	33.96	78.4	8.98
2	4	60	9.8	20	33.94	19.61	352.8	8.98
3	6	30	9.8	16	29.33	50.96	117.6	8.03
4	6	30	1.67	16	5.02	8.67	20.04	19.4



Figure 3: Intermediate position of bob (right hand side of the position of equilibrium) when pendulum is in motion Figure 4: Intermediate position of bob (left hand side of the position of equilibrium) when pendulum is in motion

5.3. Spring

For a spring, the parameters given as input are displace-ment, spring constant and mass of the body which are used to compute the potential energy and kinetic energy values at different positions along with the spring force. Fig. 5 and 6 depict the positions of the spring when in motion. Table 3 shows a few combinations of input values and the corresponding results considered for the simulation in the experiments.

Table 3

Different possible combinations of parameter values for spring							
SI.	Mass	Spring	Distance	Spring	Total	Time Period	
No.	<i>(m)</i>	Constant (k)	(d)	Force	Energy	(t)	
1	2	4	0.6	44	242	4.44	
2	2	8	0.6	104	676	3.14	
3	2	4	1.5	124	1922	4.44	



6. SUMMARY AND CONCLUSION

The proposed tool simulates the concepts of classical mechanics namely the characteristic motion of incline plane, springs and pendulums. Mechanics, a challenging area in Physics, is now made much easier to understand by allowing visualization of the actual motion, rather than relying on mere imagination. The product is believed to be a good contribution to the society and can be improvised in the future by adding simulation of other systems such as pulleys and gears and also by including combination problems along with additional features such as drag and drop of the systems.

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