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Testing Goodwin's Growth Cycle Disaggregated Models: Evidence from the Input-Output Tables of the Greek Economy for the years 1988-1997

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This paper tests two of Goodwin's growth cycle disaggregated models empirically, using data from the symmetric input-output tables of the Greek economy for the years 1988-1997. It is found from a qualitative as well as a quantitative point of view that both models are not adequate to describe the long-run workers' share-employment rate trajectories of the Greek economy. However, in the medium-run analysis, the evidence presented here is more encouraging: at a qualitative level, one of the models considered is found to be adequate to describe the cyclical behaviour of the workers' share and employment rate.

INTRODUCTION

As is well known, Goodwin's growth cycle (Goodwin, 1967) model captures the interdependence of income distribution, capital accumulation and (un)employment. It is analogous to the famous biological Lotka-Volterra predator–prey system and has as a solution a family of closed, elliptical cycles in the phase variables workers' share of national income and employment rate. In the five decades since its publication, Goodwin's model has not only been further developed theoretically but also there have been many attempts to 'fit the model to real-world data' and,[1] therefore, to test the cyclical relationship between these phase variables. The evidences of these attempts show that the model is capable of describing the dynamics of a real economy, i.e. as Solow (1990, p. 38) has stressed, 'we must take seriously the story that the model tells'.[2]

However, Goodwin's original growth cycle model neglects the implications of capital heterogeneity, and this has been recognized by

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Goodwin and his followers as a fundamental weakness. More specifically, Goodwin (1984, p. 67) stressed that 'aggregated models, including my own, are less than totally satisfactory; they are useful in helping to conceptualize and as preliminary skirmishes prior to elaboration in disaggregative form'. The most notable attempts to overcome this weakness are the following models: (i) the model which was originally developed by Goodwin in *Use of Normalised General Co-ordinates in Linear Value and Distribution Theory* (1976), and re-printed in Goodwin (1983, ch. 7) (hereafter, GDM1 = Goodwin's Disaggregated Model 1); and (ii) the model which was developed by Goodwin in *Disaggregating Models of Fluctuating Growth* (1984) and re-proposed again in Goodwin and Landesmann (1996, pp. 180-81) (hereafter, GDM2). These models are both generalizations of Goodwin's original model into an n sector model. Contrary to the original model, these ones: (i) capture the interrelationships between the sectors; (ii) do not describe the dynamics of the aggregate economy, but the dynamics of each sector of the economy; and (iii) generate dynamic behaviours which depend on the matrix of input-output coefficients, i.e. the dynamics of the system are *a priori* unknown (see Rodousakis, 2012).[3]

The purpose of this paper is to test these two Goodwin's models empirically using data from the symmetric input-output tables (SIOT) of the Greek economy for the years 1988-1997.[4] For this purpose, we also use data for the Organisation of Economic Co-operation and Development (OECD), Department of Economics and Statistics' publications National Accounts and Labour Force Statistics for various years.

The remainder of the paper is organized as follows. Section 2 presents the analytic framework. Section 3 provides the results of the empirical analysis. Section 4 presents the conclusions.

GOODWIN'S MODELS

Consider a closed, linear system, involving only single products, circulating capital and homogeneous labour. Furthermore, assume that (1) the input–output coefficients are fixed; (2) the system is 'viable', i.e. the Perron-Frobenius eigenvalue of the $n \times n$ matrix of input-output coefficients, $\mathbf{A}$, is less than 1 (for more details, see Kurz and Salvadori 1995, chs 3–4), and 'diagonalizable', i.e. $\mathbf{A}$ has a complete set of n linearly independent eigenvectors; (3) wages are paid at the beginning of the common production period; and (4) the profit (growth) rate, r (g), is uniform.[5]

On the basis of these assumptions, we can write

$$\mathbf{p}^{\mathrm{T}} = (1+r)(\mathbf{p}^{\mathrm{T}}\mathbf{A} + m\mathbf{b}^{\mathrm{T}}), m \equiv \mathbf{p}^{\mathrm{T}}\mathbf{w} \quad (1)$$

$$\mathbf{x} = (1+g)(\mathbf{A}\mathbf{x} + \mathbf{c}), \mathbf{c} \equiv \mathbf{w}\mathbf{b}^{\mathrm{T}}\mathbf{x} \quad (2)$$

where **p** denotes a vector of production prices, m the money wage rate, **b** the vector of direct labour coefficients, **w** the vector of the real wage rate, **x** the vector of gross outputs, and **c** the consumption vector.[6]

If $\mathbf{Q} \equiv [q_{ij}]$ and the diagonal matrix $<\mathbf{\Lambda}>$ are matrices formed from the right eigenvectors and the eigenvalues of **A**, respectively, then $\mathbf{A} = \mathbf{Q} < \mathbf{\Lambda} > \mathbf{Q}^{-1}$ and, therefore, equations (1) and (2) can be written as

$$\boldsymbol{\pi}^{\mathrm{T}} = (1+r)(\boldsymbol{\pi}^{\mathrm{T}} < \mathbf{\Lambda} > + m\mathbf{a}^{\mathrm{T}}) \quad (3)$$

$$\mathbf{q} = (1+g)(< \mathbf{\Lambda} > \mathbf{q} + \mathbf{k}) \quad (4)$$

where $\pi^{\mathrm{T}} \equiv \mathbf{p}^{\mathrm{T}}\mathbf{Q}$, $\mathbf{a}^{\mathrm{T}} \equiv \mathbf{b}^{\mathrm{T}}\mathbf{Q}$, $\mathbf{q} \equiv \mathbf{Q}^{-1}\mathbf{x}$ and $\mathbf{k} \equiv \mathbf{Q}^{-1}\mathbf{c}$. As observed by Goodwin, this transformation defines n independent, one-commodity, simpler systems (known as eigensectors), i.e. the system (1)-(2) is transformed from its own 'original coordinates' to another set of normalized 'principal coordinates' (see Goodwin and Punzo, 1987, ch. 2, section 5). There are of course the difficulties that (i) these systems are fictitious; and (ii) in general, they have no economic interpretation, in the sense that the eigenvalues of **A** can be negative or complex (with only exception the P-F eigenvalue which is always real and positive, i.e. the Sraffa's, 1960, ch. 4, 'Standard system'). Regarding this, Goodwin observes that we can always go back to our original coordinates where these difficulties disappear. Thus, the main advantage of the transformation under discussion is that '[b]y separating variables, the complications of inter dependence have been removed, without being ignored since when transforming back the solutions they are taken account of' (Goodwin 1984, p. 68).

Therefore, the GDM1 may be described by the following relations

$$y_j \equiv (1-\lambda_j)x_j, \lambda_j < 1, j = 1,2,\dots,n \quad (5)$$

$$L_j / x_j = a_j, \hat{a}_j = -b \quad (6)$$

$$\hat{N}_j = n \quad (7)$$

$$\hat{m}_j = \varepsilon(L_j / N_j) - \zeta,\ \zeta < \varepsilon \tag{8}$$

$$g = r \equiv r_n - \hat{\pi} \tag{9}$$

$$\hat{\pi}_j = \hat{\pi} = (\lambda_j + \theta_j)(1 + r_n) - 1 \tag{10}$$

where b, n, ε, ζ are positive constants. As usual, a 'dot' ('hat') above a variable denotes the time derivative (logarithmic derivative with respect to time). Furthermore, y_j, L_j, N_j, and θ_j ($\equiv a_j(m/\pi_j)$) denote the net output, employment, labour force, growth rate, and the unit labour cost of the j th eigensector, respectively. Finally, λ_j denotes the j eigenvalue of **A**, and r_n the nominal profit rate.

Equation (5) captures the assumption that capital lasts for one period of production. Equations (6) and (7) capture the assumption of steady ('disembodied') technical progress and steady growth of the labour force, respectively. Equation (8) captures the assumption that the money wage rate rises in the neighbourhood of full employment. Equation (9) describes the relationship between the real and the nominal profit rate. Finally, the nominal profit rate is assumed to be fixed and *uniform* across eigensectors, whilst the cost in one period determines price in the next, i.e. $\pi_j(t+1) = (\pi_j(t)\lambda_j + m(t)a_j(t))(1+r_n)$. Hence, from the last equation, 'if we ignore the difference between differentials and differences' (Goodwin, 1983, p. 147), we get equation (10).[7]

Equations (5)-(10) reduce to the linearized system[8]

$$\ddot{\mathbf{u}} = -\mathbf{K}\dot{\mathbf{u}} - \mathbf{N}\mathbf{u}' \tag{11}$$

$$\dot{\mathbf{v}} = -\mathbf{\Omega}\mathbf{u}' \tag{12}$$

the vector $\mathbf{u} \equiv [u_j]$ $(\mathbf{v} \equiv [v_j])$ denotes the *sectoral* workers' shares (the *sectoral* employment rates) of the original system, while $\ddot{\mathbf{u}} \equiv [\ddot{u}_j]$, $\dot{\mathbf{u}} \equiv [\dot{u}_j]$, $\mathbf{u}' \equiv \mathbf{u} - \mathbf{u}^*$, $\mathbf{u}^* \equiv [u_j^*]$ and $\dot{\mathbf{v}} \equiv [\dot{v}_j]$. Finally **K**, **N** and $\mathbf{\Omega}$ denote the matrices formed from the matrices **Q** and $\mathbf{Q}^{-1}$ and the parameters α_j, $\alpha_j c_j$ and $e_j(c_j / d_j)$, respectively.

The system (11) is easily recognizable as a 'free vibration of damped multiple degree of freedom system'. Hence, from system (11) we obtain the solutions u_j', in terms of t. Then substituting $\mathbf{u}'$ in (12), we obtain $\mathbf{v}'$. Therefore, as mentioned by Goodwin (1983, p. 148), the above system exhibits the following dynamic: 'A given initial condition chooses one of the curves, and each sector spiral on to its stable equilibrium point'.

We next describe the GDM2. *Ceteris paribus*, we assume that 'operating profit equals investment and growth' (Goodwin, 1984), i.e.

$$\hat{x}_j = \Pi_j \equiv 1 - \lambda_j - w_j a_j \tag{13}$$

where w_j ($\equiv m/\pi_j$) and Π_j denote the real wage rate ('in units of eigengood j'; Goodwin 1983, p. 156) and profits per unit activity level of the j th eigensector, respectively. Furthermore, we assume that 'all prices are constant' and, therefore, equation (8) is substituted by the following equation

$$\hat{w}_j = \rho(L_j / N_j) - \gamma, \gamma < \rho \tag{8a}$$

where ρ, γ are positive constants.[9]

Equations (5)-(7), (8a) and (13), reduce to the linearized system[10]

$$\ddot{\mathbf{u}} = -\mathbf{M}\mathbf{u}' \tag{14}$$

$$\dot{\mathbf{v}} = -\mathbf{\Phi}\mathbf{u}' \tag{15}$$

where **M** and **Φ** denote the matrices formed from the matrices **Q** and $\mathbf{Q}^{-1}$ and the parameters $\phi_j s_j$ and $\chi_j(s_j / l_j)$, respectively. The system (14) is easily recognizable as a 'free vibration of undamped multiple degree of freedom system'. Whereas, solving the system (14) we obtain $\mathbf{u}'$, and, then substituting $\mathbf{u}'$ in (15), we obtain $\mathbf{v}'$. Finally, we conclude the analysis of the GDM2 by noting that the theoretical investigation of the system (14) has shown that its dynamic behaviour depends on the eigenvalues of the matrix **A** (see Rodousakis, 2012). More specifically, there are the following cases:

(i) If all the eigenvalues are real, then the result is a '2n dimensional system, with n Lotka Volterra oscillating pairs', where the motion of each sector is a linear combination of these pairs.

(ii) If some eigenvalues are complex, mathematical theorems to be applied for an appropriate analysis of the properties of the considered system do not exist. Therefore it has to be studied by means of *ad hoc* numerical simulation methods, which give explosive oscillations. It should be remembered that the analysis was restricted to the non-zero equilibrium point and its local properties and, therefore, it would be completely unwarranted to extend these results to the non-linear system (see, e.g. Medio, 1992, p. 51). This further indicates that although the linear system exhibits explosive oscillations, the system in its non-linear form may generate complex dynamic behaviours (including chaos).

RESULTS AND THEIR EVALUATION

The application of the previous analysis to the data of the Greek economy, for the years 1988-1997, gives the results summarized in Tables 1-2 and Figures 1-3.[11] Table 1 shows the parameters of the models. The parameters n, b, ρ, γ, which have been estimated econometrically using ordinary least squares (OLS) regression, are taken from Harvie (2000). The parameter r_n, which is estimated on the basis of the available input-output data, is taken from Tsoulfidis and Mariolis (2007). Finally, following Harvie (2000, p. 356), we estimate the parameters ε and ζ (see Appendix I for details). It should be noted that (i) the parameters n, b, ρ, γ, ε, ζ cannot be estimated from the given input-output data; and (ii) we have to assume that these parameters are uniform across the eigensectors. Thus, it is reasonable, for the estimation of n, b, ρ, γ, ε, ζ, to deal with aggregate data.

Table 1
The parameters of GDM1 and GDM2

Parameters	
n	0.003568
b	0.0401
ρ	53.48
γ	46.02
ε	1.44764
ζ	1.19995
r_n	0.272

Table 2 shows the eigenvalues of the matrices of input-output coefficients.

For reasons of clarity of presentation and economy of space, the following set of figures is associated only with the sector 1 and year 1988: Figures 1 and 2 display the solution paths of u_1' (v_1') which is associated with GDM1 and GDM2, respectively.

Table 2
The eigenvalues of the matrix of input-output coefficients; 1988-1997

1988	1989	1990	1991	1992
0.550	0.540	0.533	0.521	0.481
0.440	0.445	0.435	0.417	0.367
0.337	0.338	0.323	0.302	0.291+0.010i
0.333	0.306+0.014i	0.300+0.006i	0.294+0.008i	0.291-0.010i
0.306	0.306-0.014i	0.300-0.006i	0.294-0.008i	0.273
0.240	0.265	0.262	0.262	0.222
0.181+0.047i	0.191+0.045i	0.195+0.051i	0.193+0.046i	0.178+0.047i
0.181-0.047i	0.191-0.045i	0.195-0.051i	0.193-0.046i	0.178-0.047i
-0.065+0.079i	0.108	0.111	-0.0703+0.08i	-0.069+0.078i
-0.065-0.079i	-0.058+0.081i	-0.064+0.088i	-0.070-0.085i	-0.069-0.078i
0.077+0.053i	-0.058-0.081i	-0.064-0.088i	0.099	0.091
0.077-0.053i	0.075+0.047i	0.072+0.044i	0.066+0.0441i	0.060+0.036i
0.079+0.013i	0.075-0.047i	0.072-0.044i	0.066-0.0441i	0.060-0.036i
0.079-0.013i	0.032+0.014i	0.042	0.031+0.015i	0.046
-0.020+0.027i	0.032-0.014i	0.031	0.031-0.015i	0.026
-0.020-0.027i	-0.017+0.022i	-0.015+0.020i	-0.014+0.020i	-0.016+0.019i
0.023	-0.017-0.022i	-0.015-0.020i	-0.014-0.020i	-0.016-0.019i
-0.011	-0.008	0.009	0.008	-0.005
0.006	0.007	-0.007	-0.005	0.002
1993	**1994**	**1995**	**1996**	**1997**
0.493	0.498	0.525	0.505	0.531
0.393	0.402	0.420	0.404	0.420
0.296	0.294	0.297+0.004i	0.281+0.004i	0.284
0.255	0.261	0.297-0.004i	0.281-0.004i	0.257
0.237	0.245	0.237	0.242	0.239
0.207	0.205	0.214	0.192	0.215+0.050i
0.181+0.045i	0.167+0.039i	0.179+0.037i	0.175+0.043i	0.215-0.050i
0.181-0.045i	0.167-0.039i	0.179-0.037i	0.175-0.043i	0.192
-0.072+0.082i	-0.070+0.075i	-0.072+0.082i	-0.057+0.073i	-0.072+0.094i
-0.072-0.082i	-0.070-0.075i	-0.072-0.082i	-0.057-0.073i	-0.072-0.094i
0.096	0.097	0.097	0.091	0.104
0.074	0.066+0.035i	0.079+0.033i	0.078+0.025i	0.089
0.054+0.032i	0.066-0.035i	0.079-0.033i	0.078-0.025i	0.045
0.054-0.032i	0.055	0.021+0.013i	0.068	0.030+0.023i
-0.014+0.021i	-0.013+0.022i	0.021-0.013i	-0.016+0.024i	0.030-0.023i
-0.014-0.021i	-0.013-0.022i	-0.011+0.012i	-0.016-0.024i	-0.009+0.016i
0.016	0.021	-0.011-0.012i	0.018	-0.009-0.016i
-0.005	-0.006	0.005	0.002	0.014
0.001	0.005	-0.004	-0.001	0.001

Finally, the *actual* trajectories of the workers' share of national income, u^a, and the employment rate, v^a, over the period 1959-2007 are shown in Figure 3. The evidence presented in Figure 3 suggests the existence of a large cycle over the period 1959-2007 (Figures 3a), and a slightly shorter one starting in the 1990s (Figure 3b).

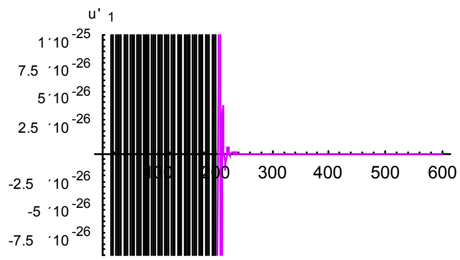

Figure 1a: The path of u_1' ; t : 0-600

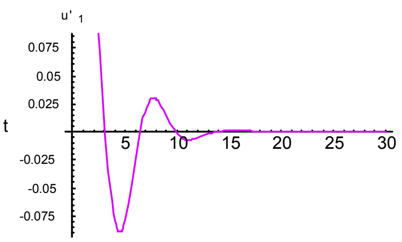

Figure 1b: The path of u_1' ; : 0-30

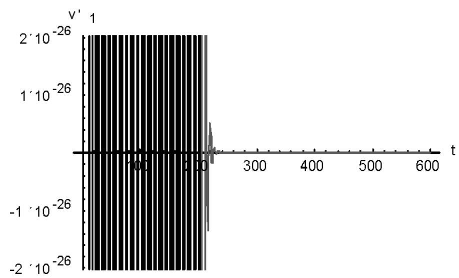

Figure 1c: The path of v_1'; t : 0-600

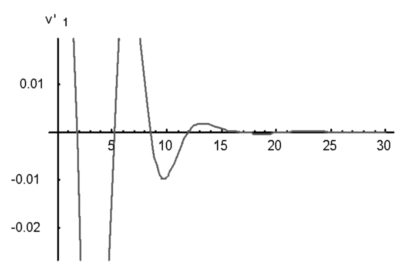

Figure 1d: The path of v_1'; t : 0-30

Figure 1: The solution paths of u_1' and v_1'; GDM1; t_0 = 1988

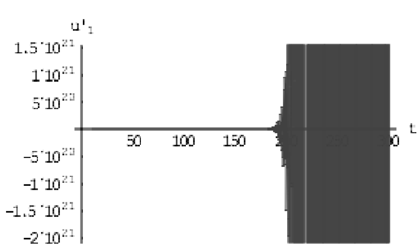

Figure 2a: The path of u_1'; t : 0-600

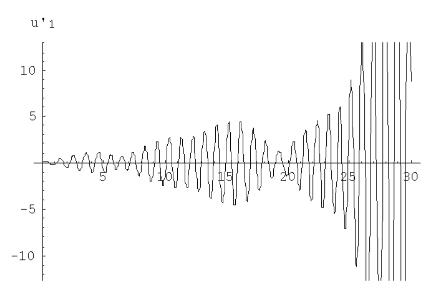

Figure 2b: The path of u_1'; t : 0-30

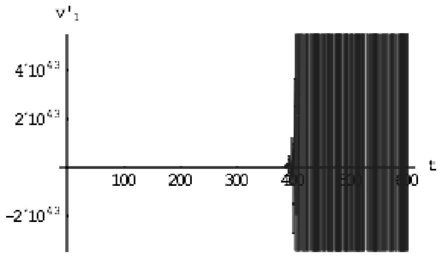

Figure 2c: The path of v'_1; t : 0-600

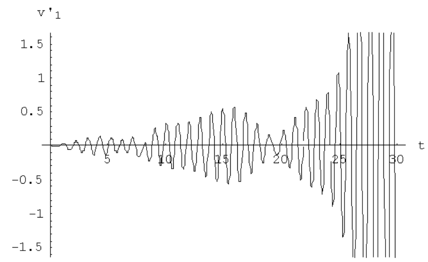

Figure 2d: The path of v'_1; t : 0-30

Figure 2: The solution paths of u'_1 and v'_1; GDM2; $t_0 = 1988$

On the basis of these estimates, we may remark the following:

(i) From the system (12)-(13), we derive that each sector exhibits damped oscillations, i.e. both u'_j and v'_j becomes smaller and smaller over time, tending to zero in the limit (see e.g. Figure 1). Since all u'_j and v'_j tend to zero, i.e. each sector tends to its equilibrium point, it can be expected that the whole system will tend to equilibrium. This evidence can be compared with the actual aggregative workers' share-employment rate trajectories for the Greek economy (see Figure 3). The motion of the actual sectors of the Greek economy cannot be estimated from the given data, therefore, it is reasonable to compare the result of our investigation with

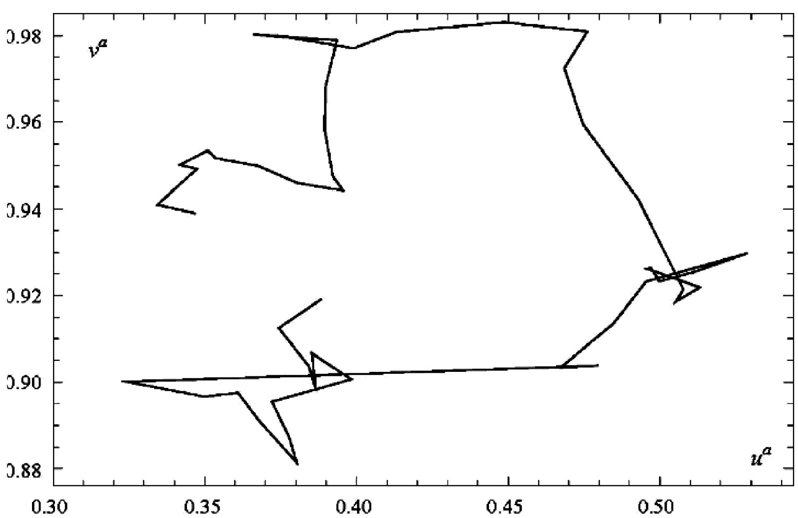

Figure 3a.i. The $u^a v^a$-trajectories for the Greek economy; 1959-2007

Figure 3.a.ii. The path of u^a; 1959-2007

Figure 3.a.iii. The path of v^a; 1959-2007

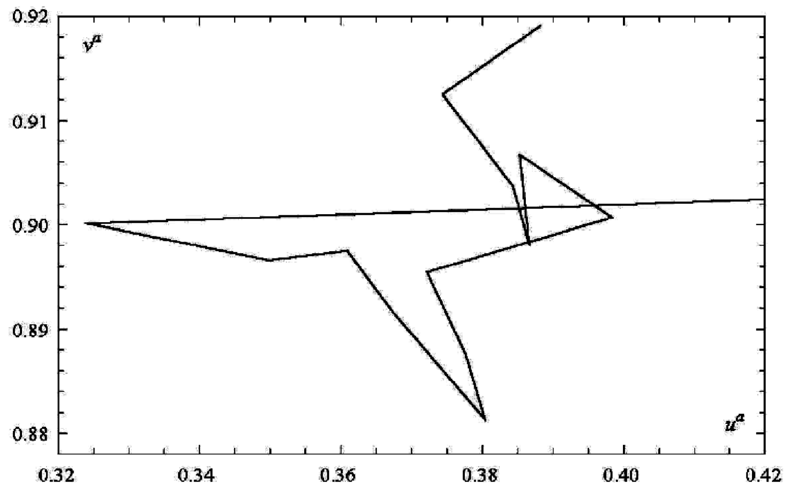

Figure 3b.i. The $u^a v^a$-trajectories for the Greek economy; 1988-2007

Figure 3.b.ii. The path of u^a; 1988-2007

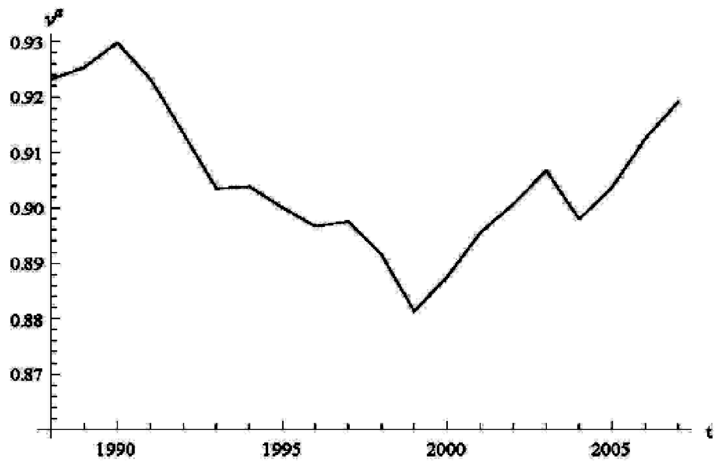

Figure 3b.iii. The path of v^a; 1988-2007

Figure 3: The trajectories of the actual workers' share and the employment rate

the aggregate data of the Greek economy. The evaluation of the results shows that from a qualitative as well as a quantitative point of view, GDM1 is found to be inadequate: it predicts damped oscillations and, therefore, cannot exhibit the cyclical behaviour of the actual system.

(ii) The result obtained with GDM2 can be summarized as follows. All u'_j and v'_j oscillate with increasing amplitude (see e.g. Figures 2). At first they exhibit cyclical oscillations (see e.g. Figure 2b), then a critical value of t is reached for which the oscillations become explosive (see e.g. Figure 2a). As noted above, one can say that the dynamic behaviour (the motion) of the whole system is 'equivalent' to the motions of the 19 sectors of Greek economy. Comparing now this motion with the motion of the actual system over the period 1988-2007 (see Figurer 3b), from a qualitative point of view the model is found to be adequate to exhibit the cyclical movements of the workers' share and the employment rate. However, as mentioned above, there is a critical value of t for which the oscillations become explosive. So, the model cannot describe long-run cycles. Finally, from a quantitative point of view, the model is found to be inadequate: both the workers' share and the employment rate exceed unity and if the 'investigated period' is long then the values of and become 'exotic' (see e.g. Figure 2a). As is well known, Goodwin's original model suffers from the same basic defect. Recent works show that the original model can be reformulated to ensure that the workers' share and the employment rate do not exceed unity (see Desai *et al.*, 2006).

From the findings, for the whole period 1988-1997, become apparent that the dynamic behaviour of the two models is remarkably uniform over time. This is not unexpected, since: (i) GDM1 tends to equilibrium, independently of its parameters; and (ii) the dynamic behaviour of GDM2 strongly depends on the eigenvalues of the matrix of input-output coefficients (see Section 2). Also, it should be stressed that the eigenvalues presented in Table 2 have the following characteristics (Mariolis and Tsoulfidis, 2011):[12] (i) The moduli of the non-dominant eigenvalues fall quite rapidly in the 'beginning' and figuratively speaking their falling pattern can be described by an exponential curve that approaches asymptotically much lower values, where it is observed a concentration of moduli. (ii) The complex (as well as the negative) eigenvalues tend to appear in the lower ranks (i.e. their modulus is relatively small). However, even in the cases that they appear in the higher ranks (i.e. second or third rank) the real part has been found to be much larger than the imaginary part, which is equivalent to saying that the imaginary part may even be ignored. Moreover, in the fewer cases that the imaginary part of an eigenvalue exceeds the real one, not only their ratio is relatively small but also the modulus of the eigenvalue can be considered as a negligible quantity. Finally, it is observed that, in general, the imaginary part tends to fall. Furthermore, it goes without saying that, as Mariolis and Tsoulfidis (2011, 2016) have suggested, through the examination of the input-output data of many diverse economies, the above characteristics are remarkably uniform across countries and over time. Thus, since there is a tendency towards uniformity in the eigenvalue distribution across countries and over time, it can be expected that there is also uniformity in the dynamics of the model across countries and over time.

CONCLUDING REMARKS

This paper tested two of Goodwin's growth cycle disaggregated models empirically using data from the symmetric input-output tables of the Greek economy for the years 1988-1997. It was found that none of the models considered can describe a long-run cyclical behaviour of the phase variables, workers' share and employment rate. These results are likely to have been derived from limitations of the present models, such as the neglect of the role of effective demand. However, in the medium-run analysis, the evidence presented here is more encouraging: at a qualitative level, one of the models considered is found to be adequate to describe the cyclical behaviour of the phase variables. Furthermore, as Mariolis and Tsoulfidis (2011) have shown there is a tendency towards uniformity in the eigenvalue distribution across countries and over time, therefore it can be expected that there is also an uniformity in the dynamics of the two models across countries and over time. Future work should (i) test these models for various countries and years; and (ii) investigate the possibility of improving these models by taking effective demand considerations into account.

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Notes

1. For the US economy, see, Solow (1990), Flaschel and Groh (1995), Goldstein (1999), Harvie (2000), Nikiforos (2000), Flaschel *et al.* (2005), Barbosa-Filho and Taylor (2006), Mohun and Veneziani (2008), Stockhammer and Stehrer (2009), Tarassow (2010), *inter alia*. Furthermore, before the 1990's, two notable contributions were made by Atkinson (1969) and Desai (1984). Finally, for some current contributions in this field, see Massy *et al.* (2013), Jr. Moura and Ribeiro (2013).
2. For the model's contribution to economic policy issues, see e.g. Flaschel (2010, p. 466).
3. It should be noted that, as the original one, both these models neglect the role of capacity utilization (for this line of research, see Canry, 2005; Mariolis, 2006, pp. 202-214; 2013; Barbosa-Filho and Taylor, 2006; Flaschel and Luchtenberg, 2012, ch. 4).
4. It is important to note that we decided to use these input-output tables mainly because they are used in a number of empirical studies of the Greek economy, see Mariolis *et al.* (2006), Tsoulfidis and Mariolis (2007), Mariolis and Tsoulfidis (2011). For the available input output data, see Appendix I.
5. Matrices (and vectors) are denoted by boldface letters; and the transpose of an $n \times 1$ vector $\mathbf{z} \equiv [z_i]$ is denoted by $\mathbf{z}^T$.
6. The price system in (1) is essentially the same as Sraffa's (Sraffa, 1960, §11) price system, except that, following Goodwin, we hypothesize that wages are paid *ex ante*.
7. It should be mentioned that Goodwin considered: (i) n independent labour markets, each with its particular, given growth rates of productivity and labour force (or

b_j, n_j, ε_j, ζ_j); and (ii) a particular nominal profit rate in every eigensector. However, the present analysis is based on the assumption of a uniform labour market and nominal profit rate across eigensectors. It is not too difficult to show that if the eigenvalues of the matrix of input-output coefficients are complex, then Goodwin's assumptions lead to a system (of coupled equations) which is economically insignificant, see Appendix II for details. Finally, since r_n is uniform across eigensectors, $\hat{\pi}_j$ equals $\hat{\pi}$, i.e. it is uniform across eigensectors.

8. For details, see Appendix III.
9. Note that Goodwin considered a particular ρ and γ in every eigensector. However, it is not too difficult to show that if the eigenvalues of the matrix of input-output coefficients are complex, then this assumption of Goodwin leads to a system (of coupled equations) which is economically insignificant (see Appendix II for details).
10. See footnote 8.
11. The analytical results are available on request from the author; and *Mathematica 5.0* is used in the calculations.
12. For further empirical evidence by using 'spectral analysis', see Mariolis and Tsoulfidis (2016, ch. 5).

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Appendix I: A Note on the Data

The symmetric input-output tables of the Greek economy for the years 1988-1997 are provided at the 19×19 sector detail and are taken from Tsoulfidis and Mariolis (2007). Furthermore, in our estimation of parameters ε and ζ we follow Harvie (2000, p. 356), i.e. the long-run relationship between money wage growth, $\hat{m}_t \equiv (m_{t+1} - m_t)/m_t$, and employment rate, is estimated by the following relation

$$\hat{m}_t \equiv \sum_{j=0}^{k} \varepsilon_j v_{t-j} + \sum_{j=1}^{k} \sigma_j \hat{m}_{t-j} - \eta$$

where k denotes the number of lags necessary to ensure the model is dynamic well specified, and $\varepsilon = (1 - \varepsilon_0 - \varepsilon_1 - \varepsilon_2 - \ldots - \varepsilon_k)/(1 - \sigma_1 - \sigma_2 - \ldots - \sigma_k)$, $\zeta = \eta/(1 - \sigma_1 - \sigma_2 - \ldots - \sigma_k)$. The dada for this estimation are taken from OECD Department of Economics and Statistics' publications National Accounts and Labour Force Statistics for the period 1959-1994. Finally, once again, following Harvie (2000, p. 356), we estimate our state variables. We define the actual aggregative workers' share of national income and employment rate to be u^a = compensation of employs/ (compensation of employs + operating surplus) and v^a = total employment/ total labour force, respectively. The dada for this estimation are taken from OECD Department of Economics and Statistics' publications National Accounts and Labour Force Statistics for the period 1959-2007.

Appendix II: Proofs

Let us consider a 3×3 matrix **A** with one real eigenvalue (the P-F eigenvalue), λ_1 (<1), and a pair of complex conjugate eigenvalues λ_2 and λ_3. From (15) we get

$$\mathbf{K} \equiv \mathbf{Q} < \mathbf{K} > \mathbf{Q}^{-1}$$

where

$$< \mathbf{K} > = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}$$

From the above equation after rearrangement, we obtain

$$\mathbf{K} = \begin{pmatrix} \alpha_1 q_{11} q'_{11} + \alpha_2 q_{12} q'_{21} + \alpha_3 q_{13} q'_{31} & \alpha_1 q_{11} q'_{12} + \alpha_2 q_{12} q'_{22} + \alpha_3 q_{13} q'_{32} & \alpha_1 q_{11} q'_{13} + \alpha_2 q_{12} q'_{23} + \alpha_3 q_{13} q'_{33} \\ \alpha_1 q_{21} q'_{11} + \alpha_2 q_{22} q'_{21} + \alpha_3 q_{23} q'_{31} & \alpha_1 q_{21} q'_{12} + \alpha_2 q_{22} q'_{22} + \alpha_3 q_{23} q'_{32} & \alpha_1 q_{21} q'_{13} + \alpha_2 q_{22} q'_{23} + \alpha_3 q_{23} q'_{33} \\ \alpha_1 q_{31} q'_{11} + \alpha_2 q_{32} q'_{21} + \alpha_3 q_{33} q'_{31} & \alpha_1 q_{31} q'_{12} + \alpha_2 q_{32} q'_{22} + \alpha_3 q_{33} q'_{32} & \alpha_1 q_{31} q'_{13} + \alpha_2 q_{32} q'_{23} + \alpha_3 q_{33} q'_{33} \end{pmatrix}$$

Since λ_1 is the P-F eigenvalue, the first column (row) of $\mathbf{Q}$ (of $\mathbf{Q}^{-1}$) is real and positive. On the other hand, the corresponding eigenvectors to λ_2 and λ_3 will ordinary involve negative and complex numbers. Therefore, the element κ_{11} of $\mathbf{K}$ can be expressed as

$$\kappa_{11} = \alpha_1 q_{11} q'_{11} + (\pm\tau \pm \eta i)(\pm\sigma \pm \omega i)(\pm\xi \pm \delta i) + (\pm\tau \mp \eta i)(\pm\sigma \mp \omega i)(\pm\xi \mp \delta i)$$

where $\sigma, \omega, \xi, \delta \geq 0$ and $\tau, \eta, \alpha_1\ q_{11}\ q'_{11} > 0$. The above relation is a sum of real numbers, and, therefore the element κ_{11} is real. By contrast, if $n_2 \neq n_3$ or $b_2 \neq b_3$ or $r_{n2} \neq r_{n3}$, then $\alpha_2 \neq \alpha_3$ and κ_{11} is complex (and the same holds true for any κ_{ij} of a $n \times n$ matrix). Hence, if matrix $\mathbf{A}$ has complex eigenvalues and each eigensector has its particular n_j, b_j, r_{nj}, then the elements of matrix $\mathbf{K}$ will be complex.

In the same way, it can be proved that the elements of the matrices $\mathbf{N}$ and $\mathbf{\Omega}$ ($\mathbf{M}$ and $\mathbf{\Phi}$) are real *iff* the parameters b_j, n_j, ζ_j, r_{nj} (b_j, n_j, γ_j) are the same in all eigensectors.

Appendix III: Analytical Presentation of GDM1 and GDM2

Presentation of GDM1

Equations (5)-(10) reduce to *n 2D*-systems of differential equations in state variables l_j and β_j:

$$\hat{l}_j = \varepsilon\beta_j - (b + \zeta) - [\lambda_j + (1 - \lambda_j)l_j](1 + r_n) + 1 \qquad \text{(A1)}$$

$$\hat{\beta}_j = (1 + r_n)(1 - \lambda_j)(1 - l_j) - (b + n) \qquad \text{(A2)}$$

where $l_j (\equiv [(m / p_j)L_j]/ y_j)$ and $\beta_j (\equiv L_j / N_j)$ denote the workers' share and the employment rate of the j th eigensector, respectively.

It is immediately obvious that from the non-linear system of (A1) and (A2), we cannot derive any result for the dynamics of the 'original system', in the sense that its solutions cannot be expressed in terms of the original coordinates. The way out of this problem is to linearize the system around the non-zero equilibrium point. Hence, linearizing these systems around the non-zero equilibrium points $\left(l_j^*, \beta_j^*\right) = \left(\alpha_j / e_j, c_j / d_j\right)$, where $\alpha_j \equiv (1 + r_n)(1 - \lambda_j) - (b + n)$, $e_j \equiv (1 + r_n)(1 - \lambda_j)$, $c_j \equiv r_n + \zeta - n$, $d_j \equiv \varepsilon$, and using algebra, we obtain

$$\ddot{\mathbf{l}} = - < \mathbf{K} > \dot{\mathbf{l}} - < \mathbf{N} > \mathbf{l}' \qquad \text{(A3)}$$

$$\dot{\boldsymbol{\beta}} = - < \mathbf{\Omega} > \mathbf{l}' \qquad \text{(A4)}$$

where $\ddot{\mathbf{l}} \equiv [\ddot{l}_j]$, $\dot{\mathbf{l}} \equiv [\dot{l}_j]$, $\mathbf{l}' \equiv \mathbf{l} - \mathbf{l}^*$, $\mathbf{l} \equiv [l_j]$, $\mathbf{l}^* \equiv [l_j^*]$ and $\dot{\boldsymbol{\beta}} \equiv [\dot{\beta}_j]$, while $< \mathbf{K} >, < \mathbf{N} >$ and $< \boldsymbol{\Omega} >$ denote the diagonal matrices formed from the elements $< \alpha_j >, < \alpha_j c_j >$ and $< e_j (c_j / d_j) >$, respectively.

Going back to the original coordinates, i.e. pre-multiplying equations (A3) and (A4) by $\mathbf{Q}$ and taking into account that the vector $\mathbf{u} \equiv [u_j] \equiv \mathbf{Ql}$ $(\mathbf{v} \equiv [v_j] \equiv \mathbf{Q}\boldsymbol{\beta})$ denotes the *sectoral* workers' shares (the *sectoral* employment rates) of the original system, we obtain

$$\ddot{\mathbf{u}} = -\mathbf{K}\dot{\mathbf{u}} - \mathbf{N}\mathbf{u}' \tag{11}$$

$$\dot{\mathbf{v}} = -\boldsymbol{\Omega}\mathbf{u}' \tag{12}$$

where $\ddot{\mathbf{u}} \equiv [\ddot{u}_j]$, $\dot{\mathbf{u}} \equiv [\dot{u}_j]$, $\mathbf{u}' \equiv \mathbf{u} - \mathbf{u}^*$, $\mathbf{u}^* \equiv [u_j^*]$, $\dot{\mathbf{v}} \equiv [\dot{v}_j]$, $\mathbf{K} \equiv \mathbf{Q} < \mathbf{K} > \mathbf{Q}^{-1}$, $\mathbf{N} \equiv \mathbf{Q} < \mathbf{N} > \mathbf{Q}^{-1}$ and $\boldsymbol{\Omega} \equiv \mathbf{Q} < \boldsymbol{\Omega} > \mathbf{Q}^{-1}$.

Presentation of GDM2

From equations (5)-(7), (8a) and (13), we obtain

$$\hat{l}_j = \rho\beta_j - (b + \gamma) \tag{A5}$$

$$\hat{\beta}_j = (1 - \lambda_j)(1 - l_j) - (b + n) \tag{A6}$$

Consequently, the GDM2 can be described by the system of (A5) and (A6).

As above, the solutions of the non-linear system of (A5) and (A6), cannot be expressed in terms of the original coordinates. Therefore, we linearize these systems around the non-zero equilibrium point $\left(l_j^*, \beta_j^*\right) = \left(\phi_j / \chi_j,\ s_j / \iota_j\right)$, where $\phi_j \equiv (1 - \lambda_j) - (b + n)$, $\chi_j \equiv (1 - \lambda_j)$, $s_j \equiv b + \gamma$, $\iota_j \equiv \rho$. Hence, after some algebra, we obtain

$$\ddot{\mathbf{l}} = - < \mathbf{M} > \mathbf{l}' \tag{A7}$$

$$\dot{\boldsymbol{\beta}} = - < \boldsymbol{\Phi} > \boldsymbol{\beta}' \tag{A8}$$

where $\boldsymbol{\beta}' \equiv \boldsymbol{\beta} - \boldsymbol{\beta}^*$, $\boldsymbol{\beta} \equiv [\beta_j]$, $\boldsymbol{\beta}^* \equiv [\beta_j^*]$, $< \mathbf{M} >$ and $< \boldsymbol{\Phi} >$ are the diagonal matrices formed from the elements $< \phi_j s_j >$ and $< \chi_j (s_j / \iota_j) >$, respectively.

Therefore, going back to the original coordinates, we obtain

$$\ddot{\mathbf{u}} = -\mathbf{M}\mathbf{u}' \tag{14}$$

$$\dot{\mathbf{v}} = -\boldsymbol{\Phi}\mathbf{u}' \tag{15}$$

where $\mathbf{M} \equiv \mathbf{Q} < \mathbf{M} > \mathbf{Q}^{-1}$ and $\boldsymbol{\Phi} \equiv \mathbf{Q} < \boldsymbol{\Phi} > \mathbf{Q}^{-1}$.