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Sliding Mode Controller for Parallel Rotary Double Inverted Pendulum: An Eigen Structure Assignment Approach

Bipin Krishna* Deepak chandran* and V.I. George*

Abstract : This paper proposes a special approach of sliding mode controller so called Eigen structure assignment approach for parallel rotary double inverted pendulum (PRDIP) system. This offers a simple method to control a class of under-actuated systems by reducing the higher order dynamics to simpler one. Here, a particular canonical form so called the regular form was used to produce an appropriate understanding of the reduced order sliding mode dynamics. In addition to this we adopt a robust Eigen structure assignment approach by using 'place' command in mat lab. In this the modeling of the system has been carried out through classical mechanics, considering the inertia tensors of all the principle axis.

Keywords : Rotary Double Inverted Pendulum (RDIP), Classical mechanics, Sliding mode control.

1. INTRODUCTION

Parallel Rotary Double Inverted Pendulum (PRDIP), an expanded version of inverted pendulums are one of the commonly studied material in control area. Variations of pendulum often represent the dynamics of robotic arms which make them quite popular. Here the Rotary Double Inverted Pendulum (RDIP) dynamics by means of rotational geometry are derived [2] which is described in section II. Instead of considering the inertia of the single principal axis, the dynamics with complete inertia tensor using Lagrangian formulation are presented. [11].

During the formation of any control problem there will be normally inconsistencies between the model developed for the controller design and the real plant. This disparity may due to unmodelled dynamics [4], difference in the system parameters or the estimation of complex plant behavior by a straight forward model. We must confirm that the ensuing controller should produce the required performances regardless of mismatches. This has directed to a powerful interest in the development of so called robust control method to solve this. One specific methodology to robust control design is the so called sliding mode control methodology.

Sliding-mode control (SMC) method is considered as one of the most commanding design procedures for many practical systems (e.g. robotics.) due to its simple design procedure and robustness against system uncertainties and external instabilities [4].

Section III describes the concept of sliding mode controller. The strategy and implementation of Sliding mode controller (SMC) has been carried out here. The result is a variable structure system, considered as the mixture of subsystem [9] whose control structure is stable and valid for specified region of system behavior.

* Instrumentation & control Engineering, Manipal Institute of Technology, Manipal University, Manipal India *E-Mail: bipin.monayi@gmail.com*

2. MATHEMATICAL MODELING OF PARALLEL ROTARY DOUBLE INVERTED PENDULUM

Figure 1. Indicates the structure of RDIP and the parameters of the system are taken according to the Quanser standards. A torque τ_1 is applied to the rotating arm through DC motor. The rotating arm and Pendulum arms are free to rotate [15].

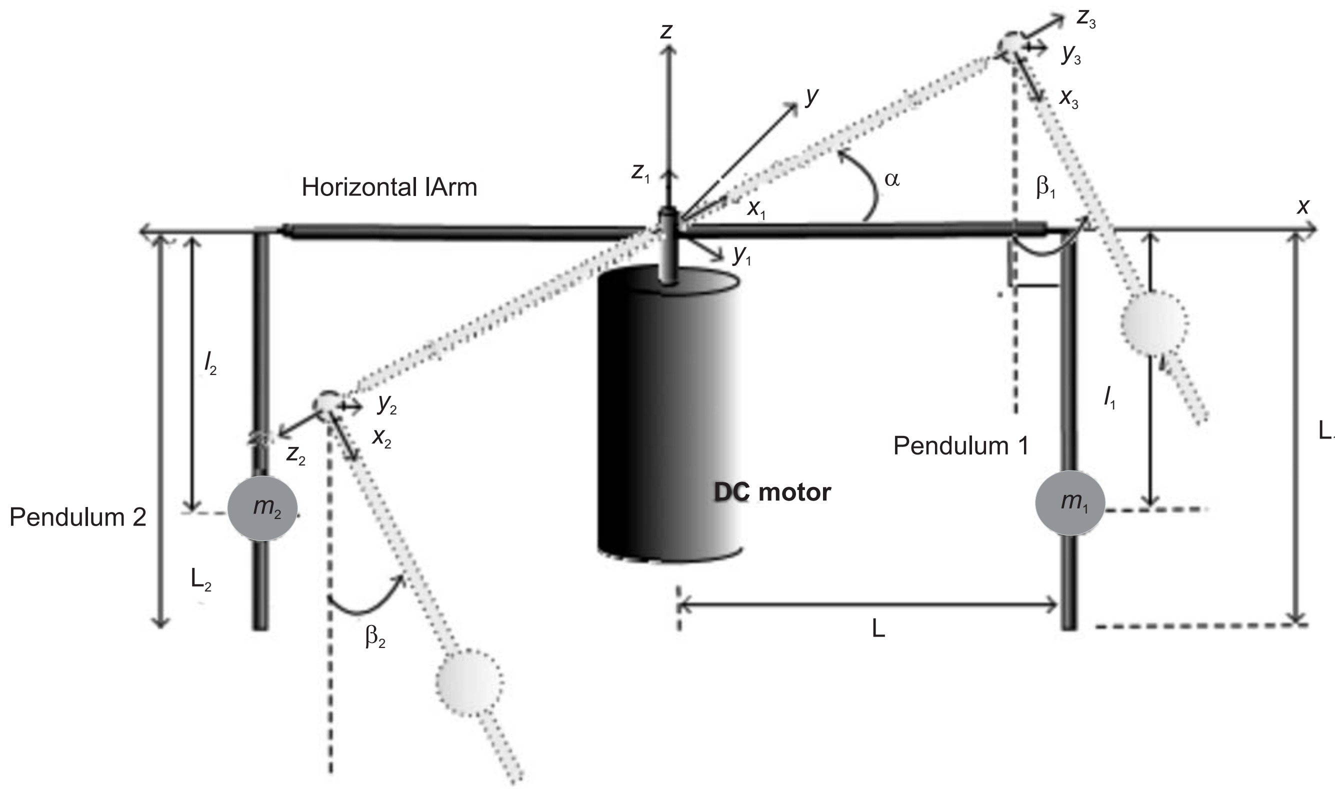


Figure 1: Representation of Rotary double inverted pendulum system

The rotations are defined by right hand coordinate system [2]. The inertia tensors are taken in diagonal form as in (1), so that the coordinate axes of arms are the principal axes.

Energies [4] of the arms are defined as:

For rotating arm,

Potential energy is $$p_0 = 0$$

Kinetic energy is, $$k_0 = \frac{1}{2}(V_1^T m V_1 + \omega_1^T J_1 \omega_1) = \frac{1}{2}\dot{\alpha}^2 (mL^2 + Jz_1 z) \quad (1)$$

Potential energy of pendulum1 is given by

$$P_1 = m_1 g l_1 \cos(\beta_1) \quad (2)$$

And Kinetic energy is, $$k_1 = \frac{1}{2}(V_2^T m_1 V_2 + \omega_2^T J_2 \omega_2)$$

$$= \frac{1}{2} m_1 (\dot{\alpha}^2 L^2 \sin^2(\beta_1) + (\dot{\alpha} L \cos(\beta_1) + l_1 \dot{\beta}_1)^2$$

$$+ \dot{\alpha}^2 l_1^2 \sin^2(\beta_1)) + \frac{1}{2}(\dot{\alpha}^2 Jx_2 x \cos^2(\beta_1)$$

$$+ \dot{\alpha}^2 \sin^2(\beta_1) Jy_2 y + \dot{\beta}_1^2 Jz_2 z) \quad (3)$$

Potential energy of pendulum 2 is given by

$$P_2 = m_2 g l_2 \cos(\beta_2) \tag{4}$$

Kinetic energy is given by $K_2 = \frac{1}{2}(V_3{}^T m_2 V_3 + \omega_3{}^T J_3 \omega_3)$

$$= \frac{1}{2} m_2 (\dot{\alpha}^2 L^2 \sin^2(\beta_2) + (\dot{\alpha} L \cos(\beta_2) + l_2 \dot{\beta}_2)^2 + \dot{\alpha}^2 l_2{}^2 \sin^2(\beta_2))$$

$$+ \frac{1}{2}(\dot{\alpha}^2 Jx_3 x \cos^2(\beta_2) + \dot{\alpha}^2 \sin^2(\beta_2) Jy_3 y + \dot{\beta}_2^2 Jz_3 z) \tag{5}$$

Lagrange function [14] can be written as

$$L = KE - PE,$$

$$L = \frac{1}{2}\dot{\alpha}^2 (mL^2 + Jz_1 z) + \frac{1}{2} m_1 (\dot{\alpha}^2 L^2 \sin^2(\beta_1) + (\dot{\alpha} L \cos(\beta_1) + l_1 \dot{\beta}_1)^2$$

$$+ \dot{\alpha}^2 l_1{}^2 \sin^2(\beta_1)) + \frac{1}{2}(\dot{\alpha}^2 Jx_2 x \cos^2(\beta_1) + \dot{\alpha}^2 \sin^2(\beta_1) Jy_2 y + \dot{\beta}_1^2 Jz_2 z)$$

$$+ \frac{1}{2} m_2 (\dot{\alpha}^2 L^2 \sin^2(\beta_2) + (\dot{\alpha} L \cos(\beta_2) + l_2 \dot{\beta}_2)^2 + \dot{\alpha}^2 l_2{}^2 \sin^2(\beta_2))$$

$$+ \frac{1}{2}(\dot{\alpha}^2 Jx_3 x \cos^2(\beta_2) + \dot{\alpha}^2 \sin^2(\beta_2) Jy_3 y + \dot{\beta}_2^2 Jz_3 z)$$

$$- m_2 g l_2 \cos(\beta_2) - m_2 g l_2 \cos(\beta_2) \tag{6}$$

The inertia tensor can be approximated as follows [2],

$$J_1 = \begin{pmatrix} J_{x1x} & 0 & 0 \\ 0 & J_{y1y} & 0 \\ 0 & 0 & J_{z1z} \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_1 \end{pmatrix}$$

$$J_2 = \begin{pmatrix} J_{x2x} & 0 & 0 \\ 0 & J_{y2y} & 0 \\ 0 & 0 & J_{z2z} \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_2 \end{pmatrix} \tag{7}$$

$$J_3 = \begin{pmatrix} J_{x3x} & 0 & 0 \\ 0 & J_{y3y} & 0 \\ 0 & 0 & J_{z3z} \end{pmatrix} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & J_3 & 0 \\ 0 & 0 & J_3 \end{pmatrix}$$

Total moment of inertia of rotating arm, pendulum1 and pendulum2 are given by,

$$\hat{J}_1 = J_1 + mL^2 \tag{8}$$

$$\hat{J}_2 = J_2 + m_1 l_1{}^2 \tag{9}$$

$$\hat{J}_3 = J_3 + m_1 l_2{}^2 \tag{10}$$

Total moment of inertia is given by

$$\hat{J}_0 = \hat{J}_1 + m_1 L^2 + m_2 L^2$$

$$= J_1 + mL^2 + m_1 L^2 + m_2 L^2 \tag{11}$$

The torque [6] can be related to the voltage by,

$$\tau = \frac{K_m V}{R} - \frac{K_m K_b \dot{\alpha}}{R} \tag{12}$$

Where K_m, K_b, R are the motor parameters.

A. Linearized state equation

Linearized equation for the upright position is derived below.

$$\begin{aligned}
\hat{J}_0(\ddot{\alpha}) - m_1 l_1 L \ddot{\beta}_1 - m_2 l_2 L \ddot{\beta}_2 + \left(b_0 + \frac{K_m K_b}{R}\right)\dot{\alpha} &= \frac{K_m V}{R} \\
-m_1 l_1 L(\ddot{\alpha}) + \hat{J}_2 \ddot{\beta}_1 + m_1 l_1 g(\beta_1 - \pi) + b_1 \dot{\beta}_1 &= \tau_2 \\
-m_2 l_2 L(\ddot{\alpha}) + \hat{J}_3 \ddot{\beta}_2 + m_2 l_2 g(\beta_2 - \pi) + b_2 \dot{\beta}_2 &= \tau_3
\end{aligned} \tag{13}$$

Where,

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{J}_0 & -m_1 l_1 L & -m_2 l_2 L \\ 0 & 0 & 0 & -m_1 l_1 L & \hat{J}_2 & 0 \\ 0 & 0 & 0 & -m_2 l_2 L & 0 & \hat{J}_3 \end{pmatrix}$$

$$Q = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\left(b_0 + \frac{K_m K_b}{R}\right) & 0 & 0 \\ 0 & -m_1 g l_1 & 0 & 0 & -b_1 & 0 \\ 0 & 0 & -m_2 g l_2 & 0 & 0 & -b_2 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{K_m}{R} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{14}$$

After neglecting the Disturbance torque,

$$R = \begin{bmatrix} 0 & 0 & 0 & \frac{K_m}{R} & 0 & 0 \end{bmatrix}^T \tag{15}$$

3. SLIDING MODE CONTROL FOR PARALLEL ROTARY DOUBLE INVERTED PENDULUM

The strategy and implementation of Sliding mode controller (SMC) has been carried out here. In spite of the mismatches between the real plant and derived model, the designed controller should possess the capacity to produce the required performance. This has headed to the growth of so called robust control method [4]. One specific method to robust control design is so called Sliding mode control methodology

A. Theory Of Sliding Mode Control

Here the dynamics are controlled by sliding surface parameters not the system parameters. Sliding mode controller designing involves two steps. Designing a sliding surface and defining a control law. The two main objectives are to drive the nonlinear system trajectory to the defined surface and maintain it on the surface [7].

During this process the defined control law will constrain the states within the locality of the sliding surface. The two major advantage is that the closed loop response will be unaffected to the uncertainty and disturbance. Moreover the dynamics of the system will be directed by sliding surface parameters rtather than system parameters.

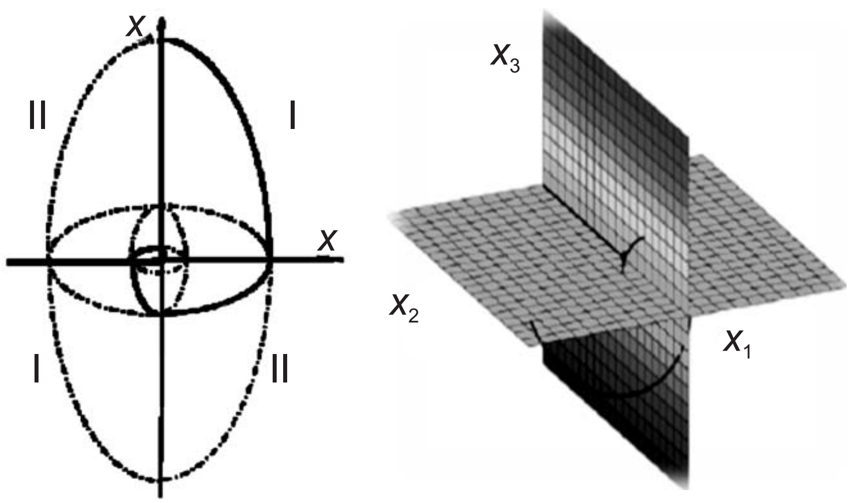


Figure 2: Phase Portrait of The System Under VSCS

The phase portarit is attained by merging together the suitable region from the two assymptotically unstable phase portrait.

B. Sliding Mode Design Approach

This work has been carried out by emphasising more on numerical tractability and writing MATLAB mfiles for implementing the design procedures. The procedure is grounded on robust Eigen structure assignment as this propose a real method of minimizing the effect of unmatched parameter variation.

A linear model of the system can be conveyed in the regular form as

$$\begin{aligned} \dot{z}_1(t) &= A_{11}z_1(t) + A_{12}z_2(t) \\ \dot{z}_2(t) &= A_{21}z_1(t) + A_{22}z_2(t) + B_2u(t) \end{aligned} \quad (16)$$

The switching function is as follows

$$s(t) = S_1z_1(t) + S_2z_2(t) \quad (17)$$

Where coordinate transformation is defined by T

T– Orthogonal matrix so that $z(t) = T_r x(t)$ (18)

4. SIMULATION RESULTS AND DISCUSSION

Simulation results explores in more details the properties of sliding motion and control action required to maintain such motion. The control actions related with the simulations are given in the following figures. The simulation results obtained here are based upon robust pole assignment approach. However, the minimization of the integral cost function is not considered here.

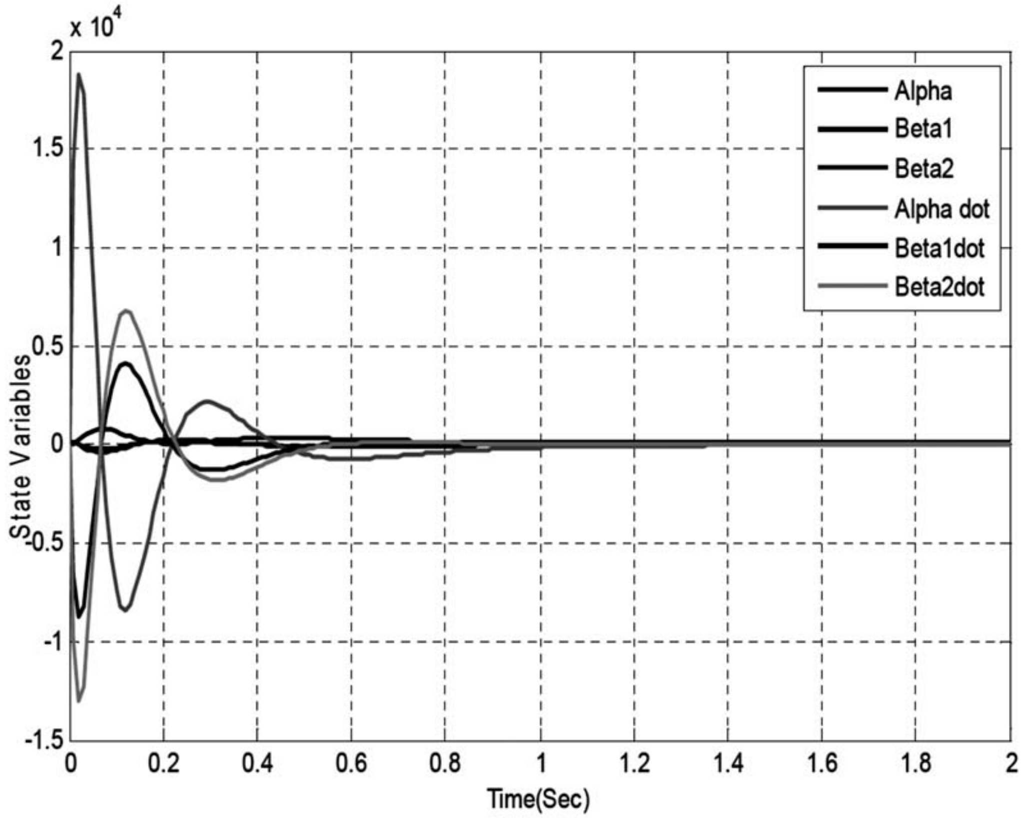
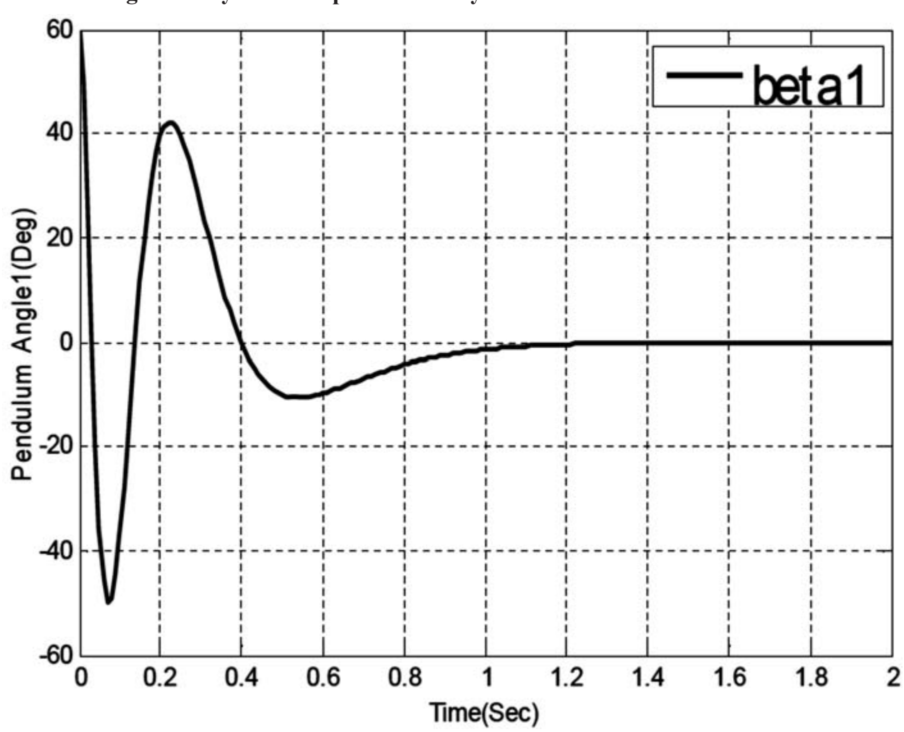
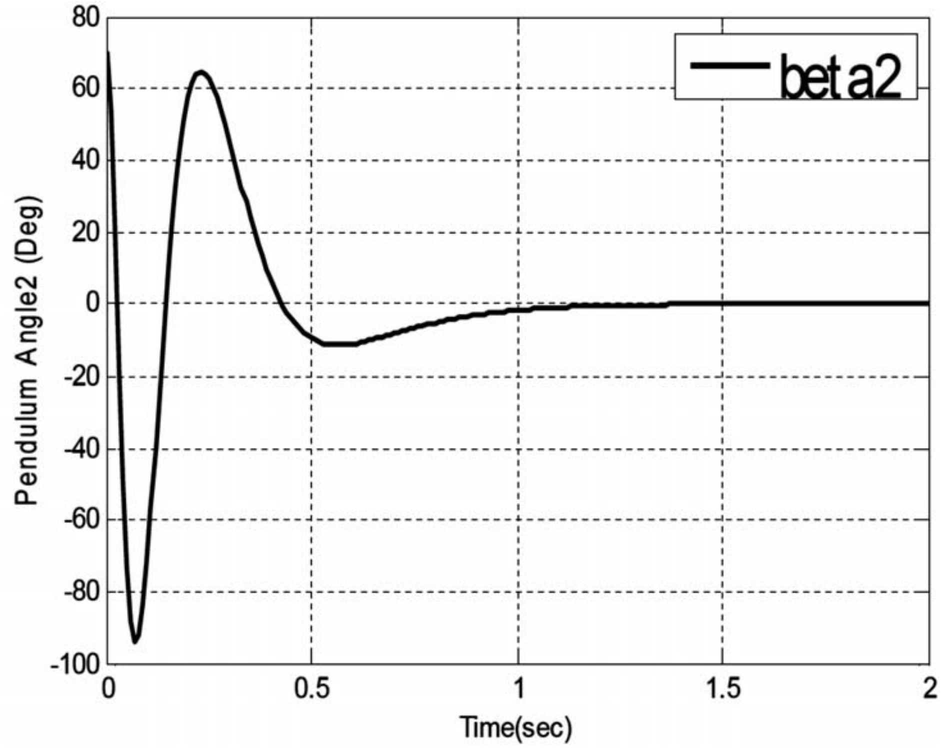


Figure 3: Dynamic response of the system under the control of SMC



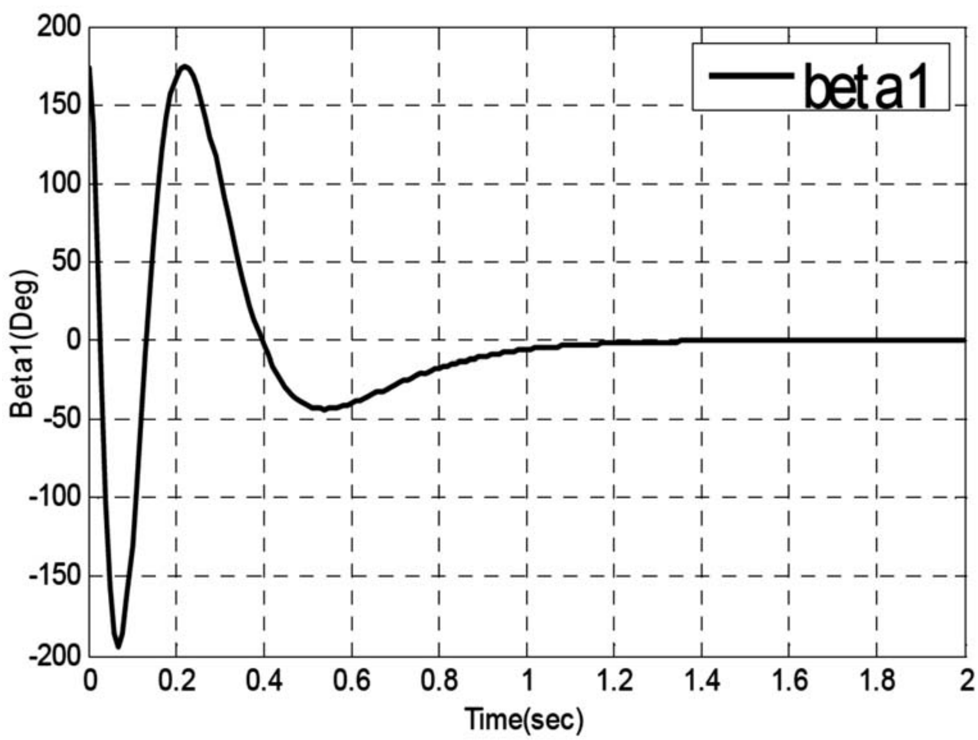
(*a*)



(*b*)

Figure 4: Asymptotic stabilization of Pendulum angles for (*a*) $\beta_1(0) = 60^0$ and (*b*) $\beta_2(0) = 70^0$

Sliding mode control was employed on RDIP system as a stabilizing controller for the pendulum. Stabilization response of RDIP system balanced with SMC for different switching are displayed in the above figure. The implementation of sliding mode control is often irritated by chattering [8], [13] which can be solved by the smooth control methodologies.Therefore we tried sliding mode-state feedback control, proposed for stabilization of RIP system which was indeed not successful.



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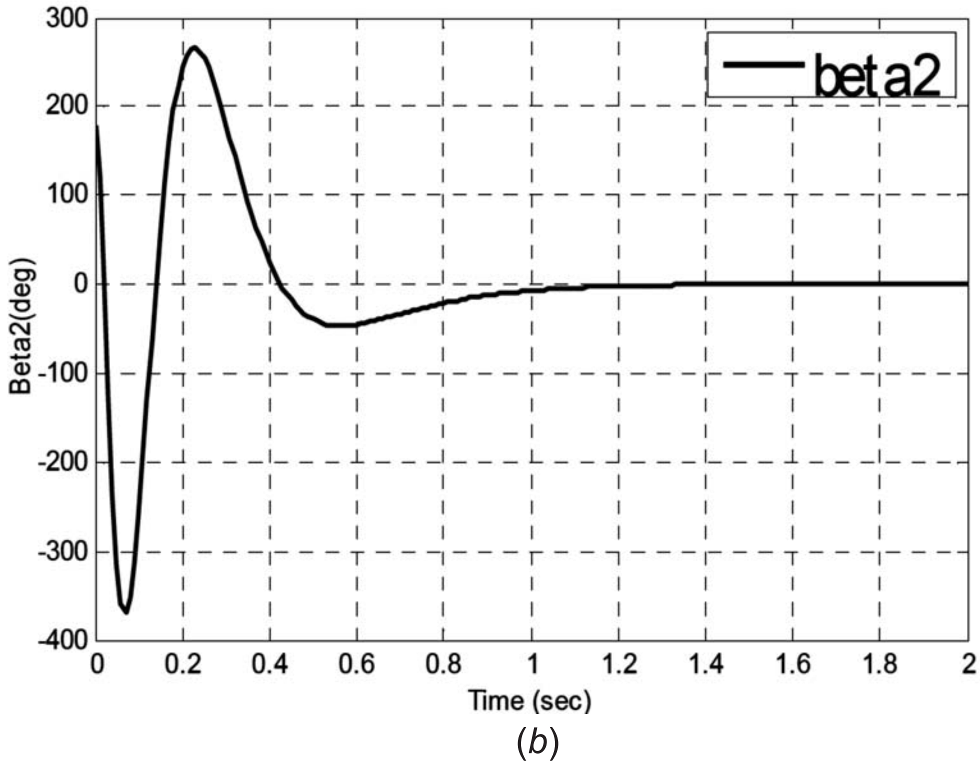

(*b*)

Figure 5: Asymptotic stabilization of Pendulum angles for (*a*) $\beta_1(0) = 175°$ and (*b*) $\beta_2(0) = 178°$

This figure indicate dynamic response of the pendulum angle1 balanced with sliding mode control. Here we can notice that the settling time is less. The simulation has been done for pendulum angle1 and pendulum angle2 for different initial values. Coming figures depicts the disturbance rejection of the system using Sliding mode controller.

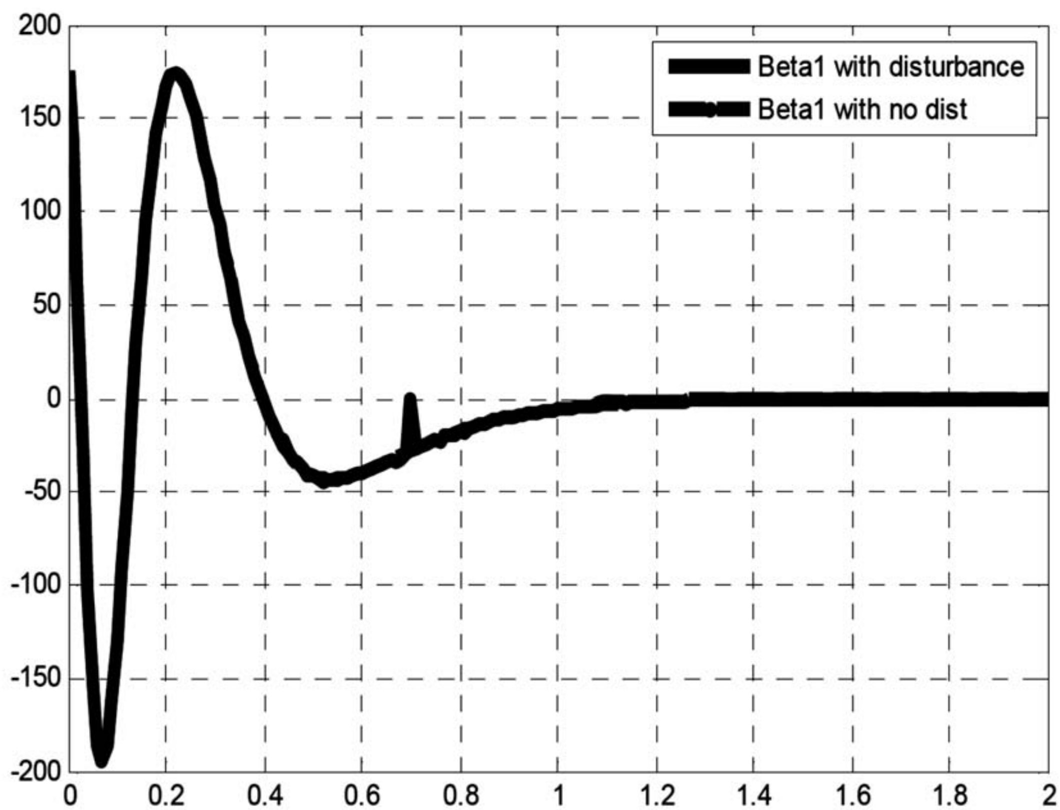

Figure 6: Variation of the pendulum angle1 with respect to disturbance

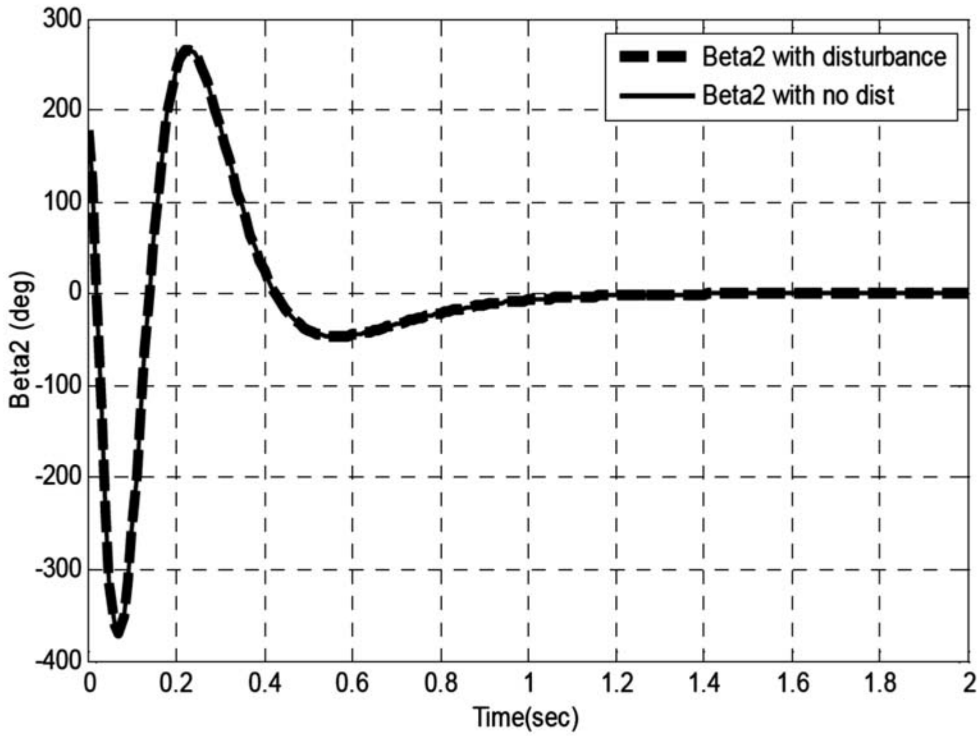

Figure 7: Variation of the pendulum angle1 with respect to disturbance

Fig (6) and Fig (7) depicts the insensitivity of sliding mode controller towards disturbance. Both figures describe the comparison of pendulum angle1 and pendulum angle2 responses with and without disturbances. It is clearly visible from the graph that the sliding mode controller is completely insensitive to the disturbances. Even though disturbances are added along with the input, sliding mode controller try to maintain the same response as earlier without any disturbances.

5. CONCLUSION

In this case, the problem of designing the sliding surface through regular method and reduction of higher order dynamics was considered. The simulation results illustrates the better performance of SMC. SMC exhibit excellent disturbance rejection capabilities while controlling the parallel RDIP. In the designing of SMC we used a canonical form [4] so called a regular form approach along with an Eigen value placement approach.

6. REFERENCES

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