International Journal of Mathematical Sciences
Vol. 17, Nos. 1-2, January-June 2018, pp. 135-144

C-BOCHNER CURVATURE TENSOR ON $(LCS)_n$ – MANIFOLDS

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Abstract: The aim of this paper is to study *C*-Bochner curvature tensor of $(LCS)_n$ -manifold. Here first we describe C-Bochner flat $(LCS)_n$-manifold. Next we study *C*-Bochner pseudo-symmetric $(LCS)_n$-manifold. Moreover, we also consider the conditions $B(\xi, x).R = 0$, $B(\xi, x).\ B = 0$ and $B(\xi, X).S = 0$ on a$(LCS)_n$-manifold.

AMS Subject Classification: 53C15, 53C25.

Key Words: Lorentzian metric, $(LCS)_n$-manifold C-Bochner curvature tensor, scalar curvature and η-Einstein manifold.

1. INTRODUCTION

In 1949, Bochner developed a Kahler analogue of the Weyl conformal curvature tensor called Bochner curvature tensor [2] and the geometric interpretation of the Bochner curvature tensor is given by Blair [1]. By considering the Boothby-Wang's fibration [3]. Matsumoto and chuman defined C-Bochner curvature tensor[11] from the Bochner curvature tensor. The *C*-Bochner curvature tensor is given by

$$
\begin{aligned}
(1,1)\quad B(X, Y)Z = R(X, Y)Z + \frac{1}{n+3}&[S(X, Z)Y - S(Y, Z)X + g(X, Z)QY \\
&- g(Y, Z)QX + S(\varphi X, Z)\Phi Y - S(\phi Y, Z)\varphi X + g(\phi X, Z)Q\phi Y \\
&- g(\phi Y, Z)Q\phi X + 2S(\phi X, Y)\phi Z + 2g(\phi X, \phi Y)Q\phi Z \\
&- S(X, Z)\eta(Y)\xi + S(Y, Z)\eta(X)\xi - \eta(X)\eta(Z)QY + \eta(Y)\eta(Z)QX] \\
&- \frac{p+n-1}{n+3}[g(\phi X, Y)Z - g(\phi Y, Z)\phi X + 2g(\phi X, Y)\phi Z] \\
&- \frac{p-4}{n+3}[g(x, 2)y - g(y, 2)x] + p/(n+3)[g(X, Z)\eta(Y)\xi \\
&+ \eta(X)\eta(Z)Y - g(Y, Z)\eta(X)\xi - \eta(Y)\eta(Z)X],
\end{aligned}
$$

where S is the Ricci tensor of type(0,2), Q is the Ricci operator defined by $g(QX, Y) = S(X, Y)$, $p = \frac{r+n-1}{n+1}$, and r being the scalar curvature of the manifold. The geometry of C-Bochner curvature tensor in a Riemannian manifold with different structures have been studied extensively by many geometers such as [5, 6, 8, 9, 21] and many others.

The present paper is organized as follows: In Section 2, we give brief information about $(LCS)_n$-manifold and C-Bochner curvature tensor needed throughout the paper. In Section 3, we consider C – Bocher flat$(LCS)_n$-manifold. We show that scalar curvature r is always constant. The next section is devoted to the study of C-Bochner pseudo-symmetric $(LCS)_n$–manifold. Thus, we obtained that either $(\alpha^2 - \rho) = LB$ or the manifold reduces to η-Einstein. In Section 5, we describe$(LCS)_n$-manifold satisfying the condition $B(\xi, U).R=0$. In this case we find an expression for Riemannian Curvature tensor. Next in Section 6, weconsider $(LCS)_n$-manifold satisfying the condition $B(\xi, U).\ B = 0$ and proved that manifold becomes η-Einstein. Finally, our discussion ends with the condition $B(\xi, U).\ S = 0$ on a $(LCS)_n$-manifold and we find that the manifold reduces to Einstein manifold.

2. PRELIMINARIES

A Lorentzian manifold M is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type (0,2) such that for each point $p \in M$, the tensor g: $T_p M \times T_p M \to R$ is a non-degenerate inner product of signature (–, +,…,+), where T_pM denotes the tangent vector space of M at p and R is the real number space.

A unit time like concircular vector field ξ on a LorentzianManifold M is called characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1 \tag{2.1}$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that

$$g(X, \xi) = \eta(X). \tag{2.2}$$

The equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha\, [g(X, Y) + \eta(X)\, \eta(Y)],\ (\alpha \neq 0) \tag{2.3}$$

For all vector fields X, Y, where ∇ denotes the covariant differentiation operator with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla x\, \alpha = (X\alpha) = d\, \alpha(X) = \rho\eta(X) \tag{2.4}$$

Where ρ being a certain scalar function given by $\rho = -(\xi\, \alpha)$. If we put

$$\phi X = \frac{1}{\alpha} \nabla x \, \xi, \tag{2.5}$$

Then from (2.3) and (2.4), we have

$$\phi X = X + \eta(X)\xi, \tag{2.6}$$

From which it follows that ϕ is a symmetric (1, 1) tensor. Thus the Lorentzian manifold M together with the unit time like concircular vector field ξ, its associated1-form η and (1,1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n$-manifold) [13]. In a $(LCS)_n$-manifold, the following relations hold:

$$\eta(\xi) = -1, \ \phi \, \xi = 0, \ \ \eta(\phi \, X) = 0, \tag{2.7}$$

$$g\ (\phi X, \phi Y) = g(X, Y) + \eta(X)\, \eta(Y), \tag{2.8}$$

$$R(X, Y)Z = (\alpha^2 - \rho)\ [g(Y, Z)X - g(X, Z)Y], \tag{2.9}$$

$$(\nabla x\ \phi)\ (Y) = \alpha\ [g(X, Y)\ \xi + 2\ \eta(X)\ \eta\ (Y)\ \xi + \eta\ (Y)X], \tag{2.10}$$

$$S(X, \xi) = (n - 1)\ (\alpha^2 - \rho)\ \eta(X), \tag{2.11}$$

$$S(\phi X, \phi Y) = S(X, Y) + (n - 1)\ (\alpha^2 - \rho)\ \eta(X)\ \eta\ (Y), \tag{2.12}$$

For any vector fields X, Y, Z, where R and S denotes respectively the curvature tensor and the Ricci tensor of the manifold.

In a $(LCS)_n$-manifold M, the C-Bochner curvature tensor satisfies the following relations:

$$B(X,Y)\xi = \frac{2(r-n-3) - 2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}[\eta(Y)X - \eta(X)Y], \tag{2.13}$$

$$\begin{aligned} B(\xi,X)Y &= \frac{2(r-n-3) - 2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}[g(X,Y)\xi - \eta(Y)X], \\ &= - B(X,\xi)Y, \end{aligned} \tag{2.14}$$

$$\begin{aligned} B(\xi,X)\xi &= \frac{2(r-n-3) - 2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}[\eta(X)\xi + X], \\ &= - B(X,\xi)\xi, \end{aligned} \tag{2.15}$$

$$B^e\ (\xi, \xi)\ X = 0. \tag{2.16}$$

On contracting (3.1), we get

$$\Sigma_{i=1}^{n} g(B(ei, X)W, ei) = lS(X,W) + mg(X,W) + n\eta(X)\eta(W), \tag{2.17}$$

Where $l = \left(\frac{11-n}{n+3}\right), m = \frac{n[n^2 - 7(n+1)] + r(n-5) + 13}{(n+3)}$ and

$$n = \frac{n(n^2 + 5n - 3) + r(n-1) + 7 - (n-5)(n-1)(n+1)(\alpha^2 - \rho)}{n+3}$$

Definition 2.1: An n-dimensional $(LCS)_n$-manifold M, is said to be η-Einstein if its Ricci tensor S if of the form

$$S(X, Y) = ag(X, Y) + b\,\eta(X)\,\eta(Y),$$

for any vector fields X and Y, wherea and b are constants. If $b = 0$ then the manifold is Einstein and if $a = 0$, the manifold is a special type of η-Einstein.

3. *C*-BOCHNER FLAT $(LCS)_n$ – MANIFOLD

Let us consider a C-Bochner flat $(LCS)_n$-manifold. Then C-Bochner curvature tensor vanishes i.e.,

$$B(X, Y)Z = 0$$

By using above condition in (3.1), we get

$$\begin{aligned}
(3.1)\quad 0 = R(X, Y)Z + \frac{1}{n+3}&[S(X, Z)Y - S(Y, Z)X + g(X, Z)QY \\
&- g(Y, Z)QX + S(\phi X, Z)\,\phi Y - S(\phi Y, Z)\,\phi X + g(\phi X, Z)\,Q\phi Y \\
&- g(\phi Y, Z)\,Q\phi X + 2S(\phi X, Y)\,\phi Z + 2g(\phi X, \phi Y)Q\phi Z \\
&- S(X, Z)\,\eta(Y)\xi + S(Y, Z)\,\eta(X)\xi - \eta(X)\,\eta(Z)QY + \eta(Y)\,\eta(Z)QX] \\
&- \frac{p+n-1}{n+3}[g(\phi X, Y)Z - g(\phi Y, Z)\phi X] + 2g(\phi X, Y)\phi Z] \\
&- \frac{p-4}{n+3}[g(X, Z)Y - g(Y, Z)X] + p/(n+3)[g(X, Z)\eta(Y)\xi \\
&+ \eta(X)\,\eta(Z)Y - g(Y, Z)\eta(X)\xi - \eta(Y)\eta(Z)X],
\end{aligned}$$

Putting $Z = \xi$ in (3.1) and by virtue of (2.7), (2.9) and (2.11), we obtain

$$\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)[\eta(Y)X-\eta(X)Y]=0. \tag{3.2}$$

Again putting $Y = \xi$ in (3.2), yields

$$\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)[X+\eta(X)\xi]=0 \tag{3.3}$$

Which implies either $\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}=0$ or $X + \eta(X)\,\xi = 0$, also for a $(LCS)_n$-manifold $X + \eta(X)\,\xi \neq 0$, we get

$$r = (n + 3) + (n - 3)\,(n + 1)\,(\alpha^2 - \rho) \tag{3.4}$$

This leads us to the following theorem:

Theorem 3.1. In a *C*-Bochner flat $(LCS)n$-manifold, the scalar curvature is constant and is given by (3.4).

4. *C*-BOCHNER PSEUDO-SYMMETRIC $(LCS)_n$-MANIFOLD

An n-dimensional $((LCS)_n$-manifold M is said to be *C*-Bochner pseudo-symmetric if

$$(R(X, Y)\,.B)\,(U,V)W = L_B\,[((X \wedge Y).B(U,\ V)W], \tag{4.1}$$

holds on the set $U_B = \{x \in M : B \neq 0\}$ at x, where L_B is some function on U_B and B is the C - Bochner curvature tensor.

Let M be an n- dimensional *C*-Bochner pseudo-symmetric$(LCS)_n$-manifold, then it follows from (4.1) that

$$\begin{aligned}(R\,(X, \xi)\,.\,B)\,(U,V)\,W = L_B\,[((X \wedge \xi).B\,(U,V)\,W] - B((X \wedge \xi)U,V)\,W \\ - B\,(U, (X \wedge \xi))\,W - B(U,V)\,(X \wedge \xi)W].\end{aligned} \tag{4.2}$$

Now the left hand side of (4.2) gives

$$\begin{aligned}R\,(X, \xi)\,B\,(U,V)\,W - B\,(R\,(X, \xi)\,U,\,V)\,W - B\,(U,\,R\,(X, \xi)\,V)\,W \\ - B\,(U,\,V)\,R\,(X, \xi)\,W\end{aligned} \tag{4.3}$$

In view of (2.9) the above equation reduces to

$$(\alpha^2 - \rho)\,[\eta\,(B\,(U,V)W)X - g\,(X, B(U,\,V)W)\xi - \eta\,(U)\,B(X\,,\,V)W \qquad (4.4)$$
$$+\, g(X,\,U)B(\xi,\,V)\,W - \eta(W)\,B\,(U,\,V)\,X + g\,(X,\,W)\,B\,(U,\,V)\,\xi$$
$$-\,\eta(V)B\,(U,\,X)\,W + g\,(X,\,V)\,B\,(U,\,\xi)\,W].$$

Similarly, right hand side of (4.2) gives

$$L_B\,[\eta\,(B\,(U,V)\,W)\,X - g\,(X,\,B(U,V)\,W)\,\xi - \eta(U)B\,(X,V)\,W \qquad (4.5)$$
$$+\, g\,(X,\,U)\,B\,(\xi,\,V)\,W - \eta\,(W)\,B\,(U,\,V)\,X + g\,(X,\,W)\,B\,(U,\,V)\,\xi$$
$$-\,\eta(V)B\,(U,\,X)\,W + g\,(X,\,V)\,B\,(U,\,\xi)\,W].$$

Substituting (4.4)and (4.5) in (4.2), we get

$$((\alpha^2 - \rho) - LB)[\eta\,(B\,(U,\,V)\,W)\,X - g\,(X,\,B(U,\,V)W)\,\xi - \eta\,(U)\,B(X,\,V)W \qquad (4.6)$$
$$+\, g\,(X,\,U)\,B\,(\xi,\,V)\,W - \eta\,(W)\,B\,(U,\,V)\,X + g\,(X,\,W)\,B\,(U,\,V)\,\xi$$
$$-\,\eta(V)B\,(U,\,X)\,W + g\,(X,\,V)\,B\,(U,\,\xi)\,W] = 0.$$

On plugging $V = \xi$ in (4.6) and using (2.13) - (2.16), we obtain

$$0 = ((\alpha^2 - \rho) - L_B)\{B(U,X)W + \left(\frac{2(r-n-3) - 2(n+1)(n-3)(\alpha^2 - \rho)}{(n+1)(n+3)}\right)[g(X,W)U - g(U,W)X]\} \qquad (4.7)$$

Which implies either $(\alpha^2 - \rho) = L_B$ or

$$B(U,X)W = \left(\frac{2(r-n-3) - 2(n+1)(n-3)(\alpha^2 - \rho)}{(n+1)(n+3)}\right)[g(X,W)U - g(U,W)X]\}. \qquad (4.8)$$

On contracting above equation with respect to U and by virtueof (2.17), we have

$$S\,(X,\,W) = A'\,g\,(X,\,W) + B'\,\eta(X)\,\eta(W). \qquad (4.9)$$

Where $A' = \dfrac{2(n^2-1)(n-3)((\alpha 2 - \rho) + n(n^3 - 6n^2 - 10n + 10) + r(n^2 - 6n - 3) + 7}{(n-11)(n+1)}$

And $B' = \dfrac{n(n^2 + 5n - 3) + r(n-1) + 7 - (n^2-1)(n-5)(\alpha^2 - \rho)}{(n-11)}$

Hence we can state the following:

Theorem 4.2. Let M be a n-dimensional C-Bochner pseudo-symmetric $(LCS)_n$-manifold then either $(\alpha^2 - \rho) = L_B$ or the manifold becomes η-Einstein.

5. $(LCS)_n$ – MANIFOLD SATISFYING $B(\xi, U).\, R = 0$

Let us consider $(LCS)_n$-manifold satisfying the condition $B(\xi, U).\, R = 0$, then

$$0 = B(\xi, U)\, R(X, Y)Z - R(B(\xi, U)X, Y)Z \tag{5.1}$$

$$- R(X, B(\xi, U))Y)Z - R(X, Y)B(\xi, U)Z,$$

Which in view of (2.14) gives

$$0 = \left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)[g(U, R(X,Y)Z)\xi \tag{5.2}$$

$$-\eta(R(X, Y)Z)U - g(U, X)R(\xi, Y)Z + \eta(X)R(U, Y)Z - g(U, Y)R(X, \xi)Z$$

$$+ \eta(Y)R(X, U)Z - g(U, Z)R(X, Y)\xi + \eta(Z)R(X, Y)U].$$

Which implies that either $\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right) = 0$ i.e. scalar curvature $r = (n + 3) + (n - 3)(n + 1)(\alpha^2 - \rho)$ or

$$0 = g(U, R(X, Y)Z)\xi - \eta(R(X, Y)Z)U - g(U, X)R(\xi, Y)Z + \eta(X)R(U,Y)Z \tag{5.3}$$

$$- g(U, Y)R(X, \xi)Z + \eta(Y)R(X, U)Z - g(U, Z)R(X, Y)\xi + \eta(Z)R(X,Y)U.$$

On plugging $Z = \xi$ in (5.3) and then using (2.5) and (2.7), we get

$$R(X, Y)U = (\alpha^2 - \rho)[g(Y, U)X - g(X, U)Y]. \tag{5.4}$$

Hence we can state the following result:

Theorem 5.3. Let M be n-dimensional $(LCS)_n$-manifold satisfying the condition $B(\xi, U).\, R = 0$. Then either scalar curvature $r = (n + 3) + (n - 3)(n + 1)(\alpha^2 - \rho)$ or the curvature tensor R is given by (5.4).

6. $(LCS)_n$ – MANIFOLD SATISFYING B $(\xi, U).\, B = 0$

Theorem 6.4. If M is a n-dimensional $(LCS)_n$-manifold satisfying $B(\xi, U).\, B = 0$, then either scalar curvature $r = (n + 3) + (n - 3)(n + 1)(\alpha^2 - \rho)$ or the manifold is η-Einstein.

Proof. Let M be a n-dimensional $(LCS)_n$-manifold satisfying $B(\xi, U).B = 0$

Then above condition turns into

$$0 = B(\xi, U).B(X, Y)Z - B(B(\xi, U)X, Y)Z \tag{6.1}$$

$$- B(X, B^e(\xi, U)Y)Z - B(X, Y)B(\xi, U)Z.$$

Using (2.14) in (6.1), we have

$$0=\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)[g(U,B(X,Y)Z)\xi \tag{6.2}$$

$$-\eta(B(X,Y)Z)U-g(U,X)B(\xi,Y)Z+\eta(X)B(U,Y)Z-g(U,Y)B(X,\xi)Z$$

$$+\eta(Y)B(X,U)Z-g(U,Z)B(X,Y)\xi+\eta(Z)B(X,Y)U.$$

Which implies that either $\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)=0$ i.e., scalar curvature $r=(n+3)+(n-3)(n+1)(\alpha^2-\rho)$ or

$$0=g(U,B(X,Y)Z)\xi-\eta(B(X,Y)Z)U-g(U,X)B(\xi,Y)Z+\eta(X)B(U,Y)Z \tag{6.3}$$

$$-g(U,Y)B(X,\xi)Z+\eta(Y)B(X,U)Z-g(U,Z)B(X,Y)\xi+\eta(Z)B(X,Y)U.$$

Putting $Z=\xi$ in the (6.3) and by virtue of (2.13) - (2.16), we obtain

$$B(X,Y)U=\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)[g(U,Y)X-g(X,U)Y] \tag{6.4}$$

On contracting above equation with respect to X and then using (2.17), we get

$$S(Y,U)=A''g(Y,U)+B''\eta(Y)\eta(U), \tag{6.5}$$

Where $A''=\dfrac{2(n^2-1)(n-3)((\alpha 2-\rho)+n(n^3-6n^2-10n+10)+r(n^2-6n-3)+7}{(n-11)(n+1)}$

And $B''=\dfrac{n(n^2+5n-3)+r(n-1)+7-(n^2-1)(n-5)(\alpha^2-\rho)}{(n-11)}$

Hence M is an η-Einstein manifold:

7. $(LCS)_n$-MANIFOLD SATISFYING $B(\xi,X).S=0$

Theorem 7.5. Let M be a n-dimensional $(LCS)_n$-manifold satisfying $B(\xi,X).S=0$. Then either scalar curvature r is constant and is given by $r=(n+3)+(n+1)(n-3)(\alpha^2-\rho)$ or the manifold reduces to Einstein.

Proof. Let us consider $(LCS)_n$-manifold satisfying the condition $B(\xi,X).S=0$. Then it can be easily seen that

$$S\,(B\,(\xi, X)\,U, V) + S\,(U, B\,(\xi, X)\,V) = 0. \tag{7.1}$$

By virtue of (2.14) in (7.1), we get

$$= \left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)[g(X,U)S(\xi,V) \tag{7.2}$$

$$-\,\eta(U)\,S(X, V) + g\,(X, V)\,S\,(U, \xi) - \eta(V)S\,(X, U)]$$

Which implies that either $\left(\frac{2(r-n-3)-2(n+1)(n-3)(\alpha^2-\rho)}{(n+1)(n+3)}\right)=0$ i.e., scalar curvature $r = (n + 3) + (n - 3)\,(n + 1)\,(\alpha^2 - \rho)$ or

$$0 = g(X, U)S(\xi, V) - \eta(U)S(X,V) + g(X, V)S(U, \xi) - \eta(V)S(X, U). \tag{7.3}$$

Putting $U = \xi$ in (7.3) and by virtue of (2.11), we get

$$S(X, V) = (n - 1)\,(\alpha 2 - \rho)g(X, V). \tag{7.4}$$

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