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Mathematical Foundation of Information Processing: A Consciousness Based Study

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Abstract: In this article, our research focuses on the mathematical foundation of information processing. We assume that mind processes information in terms of the basic units named models. The concepts, information and knowledge are compared and differentiated. Then we take into account one of the mental processes, "comparison" to explain, how certain higher mathematical concepts are effectively accommodated in consciousness. We also discuss how such concepts are processed by the consciousness of a mathematician and the consciousness of a common man.

Keywords: Consciousness, Mathematical Models, Models, Comparison, Similarity, Equality.

1. INTRODUCTION

Consciousness is the end result of all processes taking place in our brain. We use the word mind in a general sense that it includes all sense organs, the brain, the network of neurons, all related processes and consciousness. A major share of the mental processes involve the manipulations of the various kinds of information received from the outer world as well as those generated in our mind by itself. Information includes all kinds of data received by our mind both from the outer world and from the inner self. But knowledge is the form of information, which is purified undergoing some mental processes. There are various kinds of mental processes that generate different forms of knowledge. For example, if mind uses exact definitions, examples, axioms and mathematical logic, then the generated output is a mathematical knowledge. If definitions, examples, hypothesis and verification are used, then a scientific knowledge is generated. On the other hand if the principle of belief is applied, then other forms of knowledge are obtained. Is it possible to describe these mental activities mathematically? An affirmative answer would facilitate further developments in the study of consciousness. Such a study is possible due to the following reasons. First, realty can be modeled more or less effectively, with the use of some mathematical techniques or methods. Consciousness is a part of the reality; so it can be studied mathematically. Second, mathematical ideas are generated by higher order mental activities, which are the consequence of human consciousness. One could argue that mathematical principles are inseparable even from the early stage of the development of consciousness. In the sequel, we will discuss enough evidences supporting this argument. As such, it is quite natural to incorporate the mathematical ideas, which are relevant to the study of the subject.

Every theory is built on a set of axioms and definitions. An axiom is a self-evident or universally recognized truth. It is a self-evident principle that is accepted as true, without proof, as the basis of arguments. In nonmathematical contexts, a statement is accepted as an axiom, only if there is no counterexample against that statement. But in mathematics, an axiom is a mathematical statement that serves as the starting point from which other statements are logically derived.

In [5], a mathematical theory of consciousness is presented. This theory is built on the main assumption that, "models are the building blocks of consciousness". Webster's new world dictionary defines the word model in three different ways. First, "a small copy or imitation of an existing object, as a ship, building, etc., made to scale". Second is "a preliminary representation of something, serving as the plan, from which

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the final, usually larger, object is to be constructed”. Third definition is, “the original pattern, or model, from which all other things of the same kind are made. In other words, model is a prototype”. The models used in this study have given a meaning taking together all the above definitions. So a model is a preliminary representation of something which carries some character or property of the object which is represented by it. Some times the models are expected to have just a correspondence with the object.

Model based approach to the study of consciousness has history dating back to the middle of twentieth century. In 1943, the computational scheme of metal model was set forth by Craik [1], which was then extensively elaborated by Johnson-Laird [4]. In their views thought processes construct mental models, which are imitative, small scale computational representations of the external world that retain the eternal worlds relation structure. Later this idea was adopted by Ito [2, 3] to describe the processes in cerebellum related to movement and thought. Following their path, Vandervert tried to explain how working memory and cognitive modeling functions of the cerebellum contribute to discoveries in mathematics [9].

Models used in the current study is so comprehensive that the representation of any sort of information received by the mind is treated as a model. It is either a representation of an external reality or a representation of the inner self. If we closely analyze various representations, it is realized that there are different kinds of models. Classification of models based on their variations in the character is the subject matter of a previous study done by Reji Kumar [6, 7, 8]. There are representations, which simply relate a sound or symbol with the object it represents. Such models are the primary forms of information. It is accepted through the sense organs and processed and stored in the mind as models. For example, sound, light signals, colors, taste etc. Such models are α models. In the initial stages of the development of consciousness, mind handles only α models, but its creation does not stop in the subsequent stages. Man might have faced the problem of sharing the representations that he has already in mind, with other members of his group. They might have created their own representations through correspondence. In our study such representations are called β models. The β models are the building blocks of languages. Words in a language are examples of β models. It should be noted that α models are created in the mind unconsciously. These models create firsthand experience in our mind. It exists without any conscious interference of man. So these models can be called pre-linguistic models. The third kind of models, the γ models are very complex, which are created by human being to explain their feelings and experiences. Stories and poems in literature, theories in mathematics or physics are examples.

2. THE MODEL AXIOMS

In the axiomatic approach to the study of consciousness we accept a set of axioms, named the model axioms of consciousness. These axioms are given below.

- Models are the fundamental units of consciousness.
- Models can represent realty.
- Some model can be expressed in terms of sub-models.
- A collection of models can make a new model.
- Relationship among models is a new model.

The null model (ϕ), which can represent any reality and the universal model (U) are particularly important in our study. These models can make the theory complete. It is impossible to verify or prove the existence of the null model and the universal model. We denote the collection of all models that constitute the consciousness of an individual, say x by C_x and any model in the consciousness by M. The collection of all models, which generates a model M is denoted by M_c. If $M_1, M_2, \ldots, M_n$ are the models that make a model M, then we will write $M_c = \{M_1, M_2, \ldots, M_n\}$. Here, M_i is a sub-model of the model M and it is denoted by

$M_i \leq M$. Also, if M_i is a member of the class of models, say M_c, then it is denoted by $M_i \in M_c$. Note that every model is a sub-model of the same model. If there exists another collection of models say, $N_1, N_2, \ldots, N_m$ such that, $M_c = \{N_1, N_2, \ldots, N_m)$ then both collections are equivalent. If M is either an α model or a β model then $M_c = M$.

If a model M represents a part of the reality, say r, then we denote it by M_y. Relations in mathematics are idealization of mental states, which connects two models together. We can generalize this notion for the class of all models. From the fourth axiom it follows that, if there is a relation R and two models N_1 and N_2 such that N_1 is related to N_2 (we denote it by N_1RN_2) then there exists a collection of models say R_c such that $N_1 \leq R_c$ and $N_2 \leq R_c$. More precisely, R_c is the collection of all models having the form 'x is related to y'. The following relations are particularly important in mathematics. If a relation R defined on a collection of elements relates an element to the same element, then the relation is *reflexive*. If xRy always ensures yRx, then the relation is *symmetric*. If a relation is such that xRy and yRz together imply xRz then the relation is *transitive*. A relation, which is reflexive symmetric and transitive simultaneously, that relation is an equivalence relation. We can define these notions on the classes of models.

There are two relations which are particularly important in our study. First relation is the *sub-modal relation* (R_M). We denote $M_1 R_M M_2$ if and only if there exists a model M such that $M \leq M_1$ and $M \leq M_2$. The second relation is $M_1 R^M M_2$ if and only if there exists a model M such that $M_1 \leq M$ and $M_2 \leq M$. The relation R^M is the *super-model relation*. The following result can be proved.

Theorem 1: The sub-modal relation and the super-model relation are equivalence relations.

Proof: Let $M \leq M_1$. Then trivially $M_1 R_M M_1$. Thus the sub-model relation is reflexive. Next let two models satisfy $M_1 R_M M_2$. Then $M \leq M_1$ and $M \leq M_2$. Reversing the order we get, $M \leq M_2$ and $M \leq M_1$. Thus $M_2 R_M M_1$. Hence the relation is symmetric. To prove the transitivity, let us assume $M_1 R_M M_2$ and $M_2 R_M M_3$. Then $M \leq M_1$ and $M \leq M_2$. Also, $M \leq M_3$. Thus we have $M \leq M_1$ and $M \leq M_2$. Hence we get $M_1 R_M M_3$. These three relations imply that R_M is an equivalence relation. The case of super-model relation is similar to the above.

Let R_{m_c} denote the collection of all models such that M is its sub-model. We can prove the following.

Theorem 2: If M_1 is a sub-model of M_2, then $R_{M_{2_c}} \subseteq R_{M_{1_c}}$.

Proof. If $M' \in R_{M_{2_c}}$, then M_2 is a sub-model of M'. So M_1 is also a sub-model of M'. Consequently $M' \in R_{M_{1_c}}$. Hence $R_{M_{2_c}} \subseteq R_{M_{1_c}}$.

If there exists a sequence of models $M_1, M_2, \ldots, M_n$ such that $M_1 \leq M_2 \leq \ldots \leq M_n$, Then $R_{M_{nC}} \subseteq R_{M_{n-11C}} \subseteq \cdots \subseteq R_{M_{1C}}$. Let S_1 and S_2 be any two collections of models. If for all $M \in S_1$ we have $M \in S_2$, then $S_1 \subseteq S_2$. Two collections are *equal* ($S_1 = S_2$) if and only if $S_1 \subseteq S_2$ and $S_2 \subseteq S_1$.. Two models M_1 and M_2 are *similar* (denoted by $M_1 \cong M_2$) if there exists a model M such that $M \leq M_1$ and $M \leq M_2$. Two models M_1 and M_2 are *equal* (denoted by $M_1 = M_2$) if and only if for any $M \leq M_1$ we have $M \leq M_2$ and vice versa.

Theorem 3: For any two models M_1 and M_2, $M_1 = M_2$ if and only if $M_1 \leq M_2$ and $M_2 \leq M_1$.

Proof. The result is a direct consequence of the definition.

Theorem 4: For any two collections of models $R_{M_{1C}}$ and $R_{M_{2C}}$, $R_{M_{1C}} = R_{M_{2C}}$ if and only if $M_1 = M_2$.

Proof. First suppose that for the collections of models $R_{M_{1C}}$ and $R_{M_{2C}}$, $R_{M_{1C}} = R_{M_{2C}}$. Then $M_1 \in R_{M_{2C}}$. So $M_2 \leq M_1$. Similarly we get $M_1 \leq M_2$. Hence by the previous theorem, $M_1 = M_2$. To prove the converse, suppose that $M_1 = M_2$. Let $M \in R_{M_{1C}}$. Then $M_1 \leq M$ and this implies $M_2 \leq M$ and $M \in R_{M_{2C}}$. So $R_{M_{1C}} \subseteq R_{M_{2C}}$. Similarly we can show $R_{M_{2C}} \subseteq R_{M_{1C}}$.

Let M be a model and R_{M_C} be the class of all models associated with it. We say that the model M *completely defines* the class R_{M_C}. Human consciousness identifies characters in objects, processes, phenomena etc. The identified characters are represented as models in our mind. Models give us an easy way to describe the mysteries of human mind. We can explain how a mathematical concept or a mathematical theory is developed based on models. We have in our mind the representations of objects. Consciousness identifies some characters present in the objects. Based on the identified characters mind compares and discriminates things. It makes new models, which are the classes of objects having the characters. Our consciousness identifies some common characters possessed by all objects in a class. It examines whether the common characters completely determine the class. If it determine, then the set of characters completely define the class. If not, it is possible to find some objects having the same characters, outside the class. It means, the existing class is extendable to a bigger class. It is reasonable to argue that a simplified version of this procedure is taking place throughout the various stages of development of consciousness. Next we proceed to present a modeling version of this theory. In addition to the model axioms we need an axiom of consciousness.

"*Two parts of reality are different if and only if some characters of one part is not found in the other*".

For the purpose of comparison, consciousness normally considers difference in space and time, in addition to all other differences. If there were no difference in characters, the consciousness would experience everything alike. A consciousness, which does not see any difference between two parts of reality is an undeveloped consciousness. The following theorem is very important this study of consciousness.

Theorem 5: The models M_1 and M_2 are different if and only if there exists a model N such that $N \leq M_1$ and N is not a sub-model of M_2 or $N \leq M_2$ and N is not a sub-model of M_1.

Proof. The models M_1 and M_2 are equal if and only if for any model N such that $N \leq M_1$ must imply $N \leq M_2$ and vice versa. The theorem follows directly from this.

A group of models, such that the model is a sub-model of each of its members is denoted by S_M. We say that, the model M *completely defines* S_M, if $S_M = R_{M_C}$. Let M_1 and M_2 be any two models. We define the *union* of the two models $M_1 \bigcup M_2$ as a new model which contains all sub-models of both M_1 and M_2. The *intersection* of the models contains all models which are common to the given models and it is denoted by $M_1 \bigcap M_2$. The model $M_1 * M_2$ is the new model M_1 and M_2. The model $M_1 + M_2$ is the new model, either M_1 or M_2.

Theorem 6: Let M_1 and M_2 be any two models. Then,

1. $R_{M_1 * M_{2C}} = R_{M_{1\,C}} \bigcap R_{M_{2\,C}}$

2. $R_{M_1+M_{2C}} = R_{M_1\,C} \bigcup R_{M_2\,C}$

3. $R_{M_{1C}} \bigcup R_{M_{2C}} \subseteq R_{M_1 \cap M_{2C}}$

Proof.

1. If $M \in R_{M_1 * M_{2C}}$, then $M_1 * M_2 \leq M$. So $M \in R_{M_{1C}}$ and $M \in R_{M_{2C}}$. Conversely, if $M \in R_{M_{1C}}$ and $M \in R_{M_{2C}}$, then $M_1 \leq M$ and $M_2 \leq M$. So $M_1 * M_2 \leq M$.

2. A model $M \in R_{M_1+M_{2C}}$ if and only if $M_1 \leq M$ or $M_2 \leq M$ or $M_1 * M_2 \leq M$. Consequently we get $R_{M_1+M_{2C}} = R_{M_1\,C} \bigcup R_{M_2\,C}$.

3. We know that $M_1 \bigcap M_2 \leq M_i$ where $i = 1, 2$. So we get $R_{M_{iC}} \subseteq R_{M_1 \cap M_{2C}}$ where $i = 1, 2$. Hence $R_{M_{1C}} \bigcup R_{M_{2C}} \subseteq R_{M_1 \cap M_{2C}}$.

Here we initiate a study of the interrelationships that exist among the models, which represent a reality. First we focus on the few relations which are very clearly felt and leave the rest for another occasion. A collection of models say, *S* represents a reality if and only if there exists a one to one correspondence (or function) from the sub-models in *S* to the reality. This correspondence can be called the *representation function*. If the correspondence is such that more than one model represent the same reality or a part of the realty is represented by more than one sub-models, then the correspondence is a *representation relation*. Representation relation causes vagueness and subjectivity. The study of representation relations is more important than the study of the representation functions because the former is very closer to the consciousness of common men. The latter is suitable for explaining the consciousness of a mathematician or a scientist. In the current study we concentrate on representation functions, because proceeding from clear to vague or objective to subjective is a better approach.

In the following discussion by a reality we mean a physical reality or any part thereof. It does not mean any mental state or mental reality. It is possible to extend the study to the state of mind as well as the physical reality. If a part of reality occurs due to another part of reality, then the sub-model representing the latter should imply the sub-model representing the earlier. Both models together with the implication make a new model. If the second reality causes the first reality, then both parts of realities cannot occur at different time. So the models representing such realities cannot stand alone. For example “elephant” implies “black color”, but “black color” does not imply “elephant”. We can define the following relations among the models. Let M_1 and M_2 be two models. If the reality representing M_1 causes the reality representing M_2, then we say that M_1 *implies* M_2 and it is denoted by $M_1 \rightarrow M_2$. If $M_1 \rightarrow M_2$ and $M_2 \rightarrow M_1$, then we say M_1 and M_2 *occur together* (it is denoted by $M_1 \leftrightarrow M_2$).

Theorem 7: The relation $\rightarrow$ is reflexive and transitive.

Theorem 8: The relation $\leftrightarrow$ is an equivalence relation.

A reality and its negation does not exist simultaneously. If a model *M* represents a reality, then the negation of the model ($\bar{M}$) has only theoretical existence. Normally, if *M* matches with a reality, then $\bar{M}$ is expected to match with the negation of the reality, which is impossible due to the reason stated above. It does not match with any reality. The complement of a model *M* (denoted by M') represents the reality other than the reality represented by *M*.

3. HOW DOES CONSCIOUSNESS DEVELOP?

In the previous section we have seen a mathematical study of the processing of knowledge in human mind. It is presented using models as the basic units of consciousness. This describes the reality in a very objective, accurate and perfect way. On the other hand an ordinary man's consciousness is neither objective nor precise.

During the early stage of its development, only a very small number of models are created in or mind. It is very weak in differentiating two objects or phenomena because it can handle only a few models. As the number of models increases, its ability to differentiate objects too increases. The effectiveness of comparison is directly proportionate to the total number of models in the mind. Also, comparison is one of the fundamental operations taking place in our mind, which leads to differentiation. These activities together generate new models from the existing models in the mind.

Similarity and equality are two very general models created by comparison. In the light of the discussion given in the previous section, it is clear that two models M_1 and M_2 are similar if and only if there exists a model M such that $M \leq M_1$ and $M \leq M_2$. Equality, on the other hand, demands similarity with respect to each and every character (sub-model) of any two models. Similarity is quite simple and verifiable but equality is extremely difficult to verify. As there is no final representation for any realty, and the number of models in the representation increases indefinitely, when we continuously analyze the realty, we can realize that testing whether two representations are equal will remain incomplete for ever. In addition to this, there is the problem of combinatorial complexity of comparison. But consciousness overcomes these difficulties by simply limiting the number of comparison. The *comparison limit* (denoted by N_x), of a consciousness say x is the maximum number of comparisons that it is willing to make between any two models.

Theorem 9: The models $M_1, M_2 \in C$ are equal if there is no difference between the sub-models, identified with in the comparison limit. Otherwise they are different.

Proof. Quite easy.

4. CONCLUSION

In this short paper we have discussed a mathematical approach to the problem of explaining the functions of human consciousness. It is argued that the relations and functions in mathematics are idealizations of similar functions which take place in our mind. So a model based study of such functions is initiated. The importance of comparison and differentiation in the development of consciousness is discussed. This research is to be extended in many directions. For example, one can study the concepts of subjectivity, emotion and qualia in the background of modeling theory. In essence, the discussions given in this paper naturally motivates us to think that mathematical methods and techniques are inseparably intervened with all stages of the development of consciousness.

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