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Modeling, Identification and Control of Nonlinear System Based on Adaptive Particle Swarm Optimization

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ABSTRACT

This paper addresses the problems of identification and controller for nonlinear systems identified by Takagi and Seguno (T-S) approach. A T-S modeling method using clustering algorithms is introduced at first. The fuzzy c-means models algorithm is sensitive to initialization which leads to the convergence to a local minimum of the objective function. In order to overcome this problem, an adaptive particle swarm optimization is employed to achieve global optimization of FCM algorithm. The second level is devoted to the synthesis of an optimal control law in order to ensure the global stability of the closed loop system. Indeed, this synthetic approach is based on the minimization of a quadratic criterion which leads to calculate the optimal control matrices. Thus, the gradient technic is applied to the Lagrange function in order to obtain necessary conditions for minimizing the quadratic criterion. Finally, the developed approach is applied an inverted pendulum system states.

Keywords: Modeling, Identification, Takagi-Sugeno, FCM algorithm, particle swarm optimization.

1. INTRODUCTION

The diversity of problems in automatic, notably in control theory, has evolved considerably during the last decades. A substantial amount of research has focused on automatic control problems for discrete nonlinear systems. This is motivated by the fact that the control theory applied to complex systems is the most important issue in the field of automation. However, before addressing the control problem, a large interest is devoted to modeling and identification, which reflects the dynamics of studied systems.

For this reason, several researches have focused on the modeling and the control of nonlinear systems and have lead to the consideration of some particular classes of nonlinear models [13], [14], [16]. Other attempts were geared towards large systems [6]. Indeed, the difficulty of stability analysis and controller synthesis is related to the complexity of the considered model [11]. Hence, it looked necessary to think of simpler models. In this context, several works that aim to represent a nonlinear system by some number of linear models have been developed, in the last few years. Indeed, the difficulty of stability analysis and controller synthesis is related to the complexity of the considered model. Hence, it looked necessary to think of simpler models. In this context, several works that aim to represent a nonlinear system by some number of linear models have been developed.

In the last few years, fuzzy modeling, especially, the T-S fuzzy model draw the attention of several researchers in recent decades, because of it excellent ability of describing nonlinear systems [20]. In this context, several clustering algorithms based on Takagi-Sugeno (T-S) fuzzy model has been proposed in the literature. Fuzzy c-means (FCM) is one of the most used clustering algorithms because it is efficient, straightforward, and easy to implement. However, the FCM algorithm suffers from premature convergence, and it is trapped easily into local minimum of the objective function, which will significantly affect the

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model accuracy. The important issue is how to avoid getting a bad local minimum value to improve the cluster accuracy. To overcome these drawbacks, Many optimization algorithms has been proposed in the literature such as genetic algorithm, GA [5], [7] and particle swarm optimization (PSO) [23], [10], [3], [18]. There is also interest in various methods for control systems [24-27].

Because of the feasibility of the PSO scheme, its efficacy has been demonstrated by many studies. The most important advantages of the PSO are that PSO is easy to implement and there are few parameters to adjust. The inertia weight is one of PSO's parameters to bring about a balance between the exploration and exploitation characteristics of PSO.

In this paper, we used an adaptive inertia weight in PSO algorithm (APSO) [4]. In order to overcome the shortcomings of the fuzzy c-means we integrate it with Adaptive particle swarm algorithm. However, the hybrid algorithm (FCM-APSO) can avoid sinking into local solution and diminish the sensitivity to the isolated points and the initial parameters. Once the modeling part is completed. The second level in this paper is devoted to the synthesis of an optimal control law in order to ensure the global stability of the closed loop system. The basic idea of existing control design methodology is to design a quadratic-optimal state feedback controller for each local model and then to construct a global controller from these local gains so that the global stability of the overall fuzzy system is guaranteed. Such a control design approach easily leads to linear system problems [21], which can be solved through various linear system techniques, such as linear matrix inequalities (LMI). However, it is easy to see that when the number of rules become large, the problem may become difficult to solve.Furthermore, the stabilization by quadratic optimal state feedback is not a convex problem, and thus techniques based on solving linear matrix inequalities (LMIs) are not directly applicable to solve this problem. Some recent control methods are discussed in [24-28].

The important issue is how to avoid these limitations. To avoid this problem, we will use a approach based on iterative algorithms. Thus, this work focuses on the synthesis of an optimal state feedback controller based on minimize a quadratic criterion to satisfy the desired dynamic performance and to reduce the used energy [2]. The application of the gradient technique to the Lagrange function in order to obtain necessary conditions for minimizing criterion. The developed approach is applied to ensure an optimal convergence for inverted pendulum system states.

2. TAKAGI-SUGENO FUZZY MODEL

The Takagi-Sugeno (T-S) fuzzy model is a system described by fuzzy IF-THEN rules which can generate a local linear representation of the nonlinear system by dividing the whole input space into many partial fuzzy spaces and representing each output space with a linear equation. Such a model is capable of approximating a wide class of nonlinear systems. The T-S fuzzy model of this system can be described by the following IF-THEN fuzzy rules [20]:

$$\begin{aligned} &if\ x_{k1}\ is\ F_{i1}\ \ and \ldots and\ x_{kn}\ is\ F_{in} \\ &then\ y_i = a_i^T\ x_k + b_i \\ &\qquad\qquad i = 1,\ 2 \ldots c \end{aligned} \tag{1}$$

where $i = 1,\ldots,c$ (c is the number of fuzzy rules) $a_i \in R^n$, $b_i \in R^{n+1}$ are the polynomial coefficients that forms the consequent parameter of the i^{th} rules, $x_k = [x_{k1},..,x_{kn}]^T \in R^n$ is the input vector of the fuzzy model and $F_{i1}, F_{i2}, \cdots, F_{in}$ are multidimensional antecedents of fuzzy sets, y_i is the output of i^{th} fuzzy rule.

The global estimated output is calculated by a weighting of the others output of local models according to the expression:

$$\hat{y}(k) = \frac{\sum_{i=1}^{c} \mu_{ik} . y_i(k)}{\sum_{i=1}^{c} \mu_{ik}} \tag{2}$$

μ_i is the weight of the i^{th} rules can be calculated as follows:

$$\mu_i(k) = \frac{\prod_{j=1}^{n} w_{F_{ij}}(x_j)}{\sum_{i=1}^{c} \sum_{j=1}^{n} w_{ij}(x_j)} \tag{3}$$

where $w_{A_{ij}}(x_j)$ is the membership function of the fuzzy set, A_{ij} in the antecedent of R_i and $\mu_i(k)$ are weighting functions that ensure the transition between sub-models and have the following properties:

$$\begin{aligned} &\sum_{i=1}^{c} \mu_i(k) = 1 \ \ \forall k \\ &0 \le \mu_i(k) \le 1 \ \ \forall i = 1 \ldots c \ \ \forall k \end{aligned} \tag{4}$$

3. FUZZY CLUSTERING ALGORITHMS

3.1. Fuzzy C-Means algorithm

The fuzzy C-mean (FCM) algorithm is proposed by [1], it is a powerful clustering technique with a large number of applications in various fields including image processing, classification and system identification. This algorithm is based on the minimization of the following criterion:

$$J = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^m \cdot d_{ik}{}^2 - \lambda(\sum_{i=1}^{c} \mu_{ik} - 1) \tag{5}$$

$$D_{ik}{}^2 = \|x_k - v_i\|^2 = (x_k - v_i)^T (x_k - v_i) \tag{6}$$

where

D_{ik} is the square euclidean distance between data object x_k to center v_i, m is a weighting exponent chosen between 1.5 and 2.5c, is the cluster number and$N$is the number of observation. After the minimization the criterion (5) by canceling the derivative of$J$, with respect to λ, $\mu_{ik}$and v_i

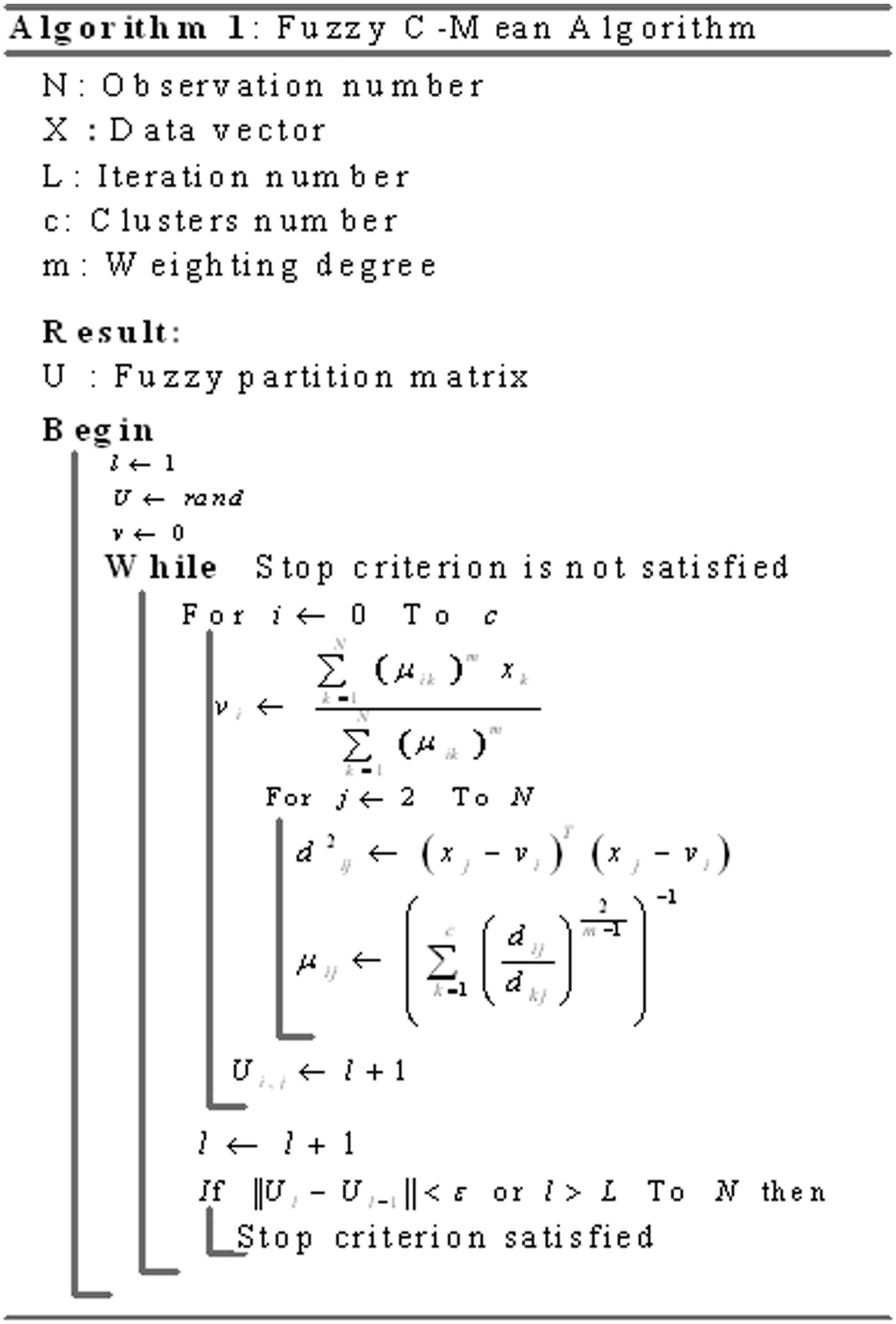
$$\begin{cases} \dfrac{\partial J}{\partial \lambda} = 0 \\ \dfrac{\partial J}{\partial \mu_{ik}} = 0 \\ \dfrac{\partial J}{\partial v_i} = 0 \end{cases} \tag{7}$$

we obtain the following expressions:

$$v_i = \frac{\sum_{k=1}^{N} (\mu_{ik})^m . x_k}{\sum_{k=1}^{N} (\mu_{ik})^m} \tag{8}$$

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c} (\frac{d_{ik}}{d_{jk}})^{\frac{2}{m-1}}} \tag{9}$$

The FCM clustering algorithm is summarized by (Algorithm 1):



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Algorithm 1: Fuzzy C-Mean Algorithm
N: Observation number
X : Data vector
L: Iteration number
c: Clusters number
m: Weighting degree

Result:
U : Fuzzy partition matrix
Begin
    $l \leftarrow 1$
    $U \leftarrow rand$
    $v \leftarrow 0$
    While Stop criterion is not satisfied
        For $i \leftarrow 0$ To $c$
            $v_i \leftarrow \frac{\sum_{k=1}^{N} (\mu_{ik})^m x_k}{\sum_{k=1}^{N} (\mu_{ik})^m}$
            For $j \leftarrow 2$ To $N$
                $d^2_{ij} \leftarrow (x_j - v_i)^T (x_j - v_i)$
                $\mu_{ij} \leftarrow \left( \sum_{k=1}^{c} \left( \frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{m-1}} \right)^{-1}$
            $U_{i,j} \leftarrow l + 1$
        $l \leftarrow l + 1$
        If $\|U_l - U_{l-1}\| < \varepsilon$ or $l > L$ To $N$ then
            Stop criterion satisfied
```

3.2. Adaptative Particle Swarm Optimisation Algorithm

These metaheuristic solutions became very popular as they are much better than mathematical solutions in terms of efficiency and complexity. The great benefit of the PSO among other optimization strategies is that it is easily implemented and there are not many parameters to adjust [8], [15]. This heuristic method is initialized with a population of random solutions called particles in the goal to get the optimal result. Each particle has a position represents the special parameter and a velocity to be used in the search space. At each iteration, the particle positions and velocities were updated. The velocity of each particle is updated using two best positions, personal best position and global best position. The personal best position, pbest, is the best position of the particle which has visited and gbest is the best position of the swarm which has visited from the first time step. For every generation, the velocity and position can be updated by the following equations [18].

$$\begin{cases} v_{kd}(t+1) = \omega v_{kd}(t) + \rho_1 (pbest_{kd} - X_{kd}(t)) + \rho_2 (gbest_{\mathrm{gd}} - X_{kd}(\mathrm{t})) \\ X_{kd}(t+1) = X_{kd}(t) + v_{kd}(t+1) \quad k = 1, 2, \cdots N_p \end{cases} \tag{10}$$

where X_{kd} and v_{kd} are position and velocity of particle respectively in the d^{th} dimension of the k^{th} particle, $pbest$ and $gbest$ are the memory of particle searched, N_p is the number of particles in the swarm and ρ_1 and ρ_2 represent two random variables defined by

$$\begin{cases} \rho_1 = c_1 \times r_1 \\ \rho_2 = c_2 \times r_2 \end{cases} \tag{11}$$

where the two variables r_1 and r_2 are randomly generated between $[0\ 1]$. Also, c_1 and c_2 are positive constants satisfy the following relationship:

$$c_1 + c_2 \leq 4 \tag{12}$$

The inertia weight ω in (10) was introduced by Shi and Eberhart [9]. They showed that ω is linearly decreasing with the iterative generations as

$$\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \frac{t}{t_{\max}} \tag{13}$$

The selected inertia weight range [$\omega_{\min}$, $\omega_{\max}$] is [0.4, 0.9], where $\omega_{s\max}$ and $\omega_{s\min}$ are minimum, maximum respectively values of ω, t is the current iteration and $t_{\max}$ is the maximum number of generations of the algorithm.

4. FUZZY C-MEANSALGORITHM BASED ON ADAPTIVE PARTICLE SWARM OPTIMIZATION

The FCM-APSO algorithm combines the advantages of FCM algorithm and APSO algorithm. To evaluate each particle, the fitness function is given by:

$$Fitness = \frac{G}{J_{FCM}(\mathrm{U}, \mathrm{V})} \tag{14}$$

The FCM-APSO clustering algorithm is summarized by the following steps:

```
Algorithm 2: FCM-APSO Algorithm
  N, X: Observation number, Data vector;
  L, c: Iteration number, Clusters number;
  $m, \varepsilon$ : Weighting degree, Stopping criterion;
  Np: Particles number;
  $x_j, v_j$ : Respectively the position and velocity of particle $P_j$;
  $p_{besti}$ : Bestfitnessobtained for particle $P_j$;
  $g_{besti}$ : Global best fitness obtained for particle $P_j$;
  $x_{pbesti}$: Particle position $P_j$ for better fitness;
  $x_{gbesti}$: Particle position $P_j$ for global better fitness
  Result:
  U : Fuzzy partition matrix
  Begin
    $l \leftarrow 1$
    $\{U, x_{pbest_i}, x_{gbest_i}, v_i\} \leftarrow rand$
    $v \leftarrow 0$
    While Stop criterion is not satisfied
      For $i \leftarrow 0$ To $c$
        $v_i \leftarrow \frac{\sum_{k=1}^{N} (\mu_{ik})^m x_k}{\sum_{k=1}^{N} (\mu_{ik})^m}$
        For $j \leftarrow 2$ To $N$ do
          $d^2_{ij} \leftarrow (x_j - v_i)^T (x_j - v_i)$
          $\mu_{ij} \leftarrow \left( \sum_{k=1}^{c} \left( \frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{m-1}} \right)^{-1}$
        For $j \leftarrow 1$ To $N$ do
          $F(x_j) \leftarrow \frac{G}{\sum_{k=1}^{c} \sum_{j=1}^{N} (\mu_{jk})^m d^2_{jk}}$
            If $F(x_i > p_{besti})$ then
              $p_{besti} \leftarrow F(x_i)$
              $x_{pbesti} \leftarrow x_i$
            If $F(x_i > g_{besti})$ then
              $g_{besti} \leftarrow F(x_i)$
              $x_{gbesti} \leftarrow x_i$
        For $j \leftarrow 1$ To $N$ do
          $v_i \leftarrow v_i + \rho_1 (x_{pbesti} - x_i) + \rho_2 (x_{gbesti} - x_i)$
          $x_i \leftarrow x_i + v_i$
      Stop criterion satisfied
```

5. QUADRATIC OPTIMAL CONTROL BY STATE FEEDBACK MULTI-MODEL DISCRETE-TIME SYSTEMS

The local model from the sub-models of the nonlinear system is given by

$$(M_i)\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) \\ y_i(k) = C_i x_i(k) \\ i = 1,\dots,c \end{cases} \tag{15}$$

where $x_i(k) \in R^n$ is the state vector, $y_i(k) \in R^p$ is the output vector, $u_i(k) \in R^m$ is the vector control and A_i, B_i, C_i are matrices of suitable dimensions.

We suppose that the system (15) is controllable. The problem is to determine an optimal state feedback control law of the sub-model which will be written as

$$u_i(k) = -F_i x_i(k) \tag{16}$$

where $F_i \in R^{m\times n}$ is the control gain matrix to be determined by minimizing a quadratic criterion.

The substitution of (16) in (15) leads to write the controlled system as

$$x_i(k+1) = (A_i - B_i F_i) x_i(k) \tag{17}$$

To determine the optimal control gain matrix, we consider the following local quadratic criterion

$$\begin{aligned} J_i &= \sum_{k=0}^{\infty}\left(x_i^T(k) Q x_i(k) + u_i^T(k) R u_i(k) \right) \\ &= \sum_{k=0}^{\infty} x_i^T(k)\left(Q + F_i^T R F_i\right) x_i(k) \end{aligned} \tag{18}$$

where $Q \in R^{n\times n}$ is a symmetric positive semi-definite matrix and $R \in R^{m\times m}$ is a symmetric positive definite matrix. The minimization of the proposed criterion presents a compromise between the performances of the local model ($x_i^T(k) Q x_i(k)$) and the control energy ($u_i^T(k) R u_i(k)$). The quadratic global criterion is expressed by [17], [12].

$$\begin{aligned} J &= \sum_{i=1}^{c} \mu_i(x(k)) J_i = \sum_{i=1}^{c} \mu_i(x(k)) \sum_{k=0}^{\infty} x_i^T(k)\left(Q + F_i^T R F_i\right) x_i(k) \\ &= \sum_{i=1}^{c}\sum_{k=0}^{\infty} \mu_i(x(k)) x_i^T(k)\left(Q + F_i^T R F_i\right) x_i(k) \end{aligned} \tag{19}$$

where $\mu_i(x(k))$ values of membership functions, F_i $i = 1,\dots,c$ is the gain matrix.

We note here that the global quadratic criterion is obtained by melting the different partial criteria J_i. Thus, the relation (20) expresses the validity of each criterion relative to the overall quadratic criterion.

The solution of (18) is expressed by

$$x_i(k) = (A_i - B_i F_i)^K x_i(0) \tag{20}$$

where $x_i(0)=x_{i0}$ is the initial condition of the state vector M_i. Then, the quadratic global criterion is obtained using relation (20).

$$\begin{aligned} J &= \sum_{i=1}^{c}\sum_{k=0}^{\infty}\mu_i(x(k))x_i^T(0)\left((A_i-B_iF_i)^K\right)^T\left(Q+F_i^TRF_i\right)\left(A_i-B_iF_i\right)^K x_i(0) \\ &= \sum_{i=1}^{c}\mu_i(x(k))x_i^T(0)P_ix_i(0) \end{aligned} \tag{21}$$

where $P_i=\sum_{k=0}^{\infty}\left((A_i-B_iF_i)^K\right)^T\left(Q+F_i^TRF_i\right)\left(A_i-B_iF_i\right)^K$ is the symmetric positive definite matrix, solution of the following Lyapunov equation

$$(A_i-B_iF_i)^TP_i(A_i-B_iF_i)-P_i+Q+F_i^TRF_i=0 \tag{22}$$

By using the following property:

$$trace(a^Tb)=trace(ab^T) \tag{23}$$

Where a and b are matrices of suitable dimensions. The quadratic global criterion is expressed by

$$\begin{aligned} J &= \sum_{i=1}^{c}\mu_i(x(k))x_i^T(0)P_ix_i(0) \\ &= \sum_{i=1}^{c}\mu_i(x(k))trace\left\{P_ix_i(0)x_i^T(0)\right\} \end{aligned} \tag{24}$$

Indeed, it’s clear that the optimization problem (23) depends on the initial condition of the state vector $x_i(0)$. Using the following property:

$$E\left(x_i(0)x_i^T(0)\right)=I_n \tag{25}$$

The quadratic global criterion is given by

$$\tilde{J}=E(J)=\sum_{i=1}^{c}\mu_i(x(k))trace\{P_i\} \tag{26}$$

5.1. Necessary conditions for optimality gain control

To obtain the necessary conditions for minimizing the quadratic criterion with respect of the conditions (22) and (25), we can apply the gradient matrix operations to the following Lagrangian [12]:

$$\begin{aligned} \Im(F_i,P_i,S_i) &= \sum_{i=1}^{c}\mu_i(x(k))\,trace\{P_i\} \\ &\quad+\sum_{i=1}^{N}\mu_i(x(k))\,trace\left\{S_i^T[\Psi]\right\} \\ &\textit{where} \\ \Psi &= (A_i-B_iF_i)^TP_i(A_i-B_iF_i)-P_i+Q+F_i^TRF_i \end{aligned} \tag{27}$$

The necessary conditions for minimizing the quadratic criterionJ_i are obtained by canceling the gradient matrix of the Lagrange function (26). Thus, we obtain the following system [17]:

$$\begin{cases} \dfrac{\partial \Im(F_i,P_i,S_i)}{\partial F_i} = 2\sum_{i=1}^{c}\mu_i(x(k))\left[\Phi\right]=0 \\ \dfrac{\partial \Im(F_i,P_i,S_i)}{\partial P_i} = \sum_{i=1}^{c}\mu_i(x(k))\left[\Omega\right]=0 \\ \dfrac{\partial \Im(F_i,P_i,S_i)}{\partial S_i} = \sum_{i=1}^{c}\mu_i(x(k))\left[\Xi\right]=0 \end{cases}$$

$$\begin{aligned} & where \\ & \Phi = -B_i^T P_i A_i S_i + B_i^T P_i B_i F_i S_i + R F_i S_i \\ & \Omega = \left(A_i - B_i F_i\right) S_i \left(A_i - B_i F_i\right)^T - S_i + I_n \\ & \Xi = \left(A_i - B_i F_i\right)^T P_i \left(A_i - B_i F_i\right) - P_i + Q + F_i^T R F_i \end{aligned} \tag{28}$$

which gives

$$\begin{cases} F_i = (B_i^T P_i B_i + R)^{-1} B_i^T P_i A_i \\ (A_i - B_i F_i) S_i (A_i - B_i F_i)^T - S_i + I_n = 0 \\ (A_i - B_i F_i)^T P_i (A_i - B_i F_i) - P_i + Q + F_i^T R F_i = 0 \\ i = 1, \ldots, c \end{cases} \tag{29}$$

The first equation of the system (28) expresses the optimal control gain matrix of the local model M_i. Where P_i are the symmetric positive definite matrices which represent the solutions of the Lyapunov function from the third equation of the system (27).

6. EXAMPLE

6.1. System Description

To present the availability and the efficiency of the quadratic-optimal controller design for discrete-time nonlinear systems, we consider the system of an inverted pendulum with a cart. The motion equations for the proposed pendulum are given by

$$\begin{cases} \dot{x}_1(k+1) = x_1(k) + Te\, x_2(k) \\ \dot{x}_2(k+1) = x_2(k) + Te\left[\Theta\right] \end{cases} \tag{30}$$

where:

$$\Theta = \frac{g\sin(x_1(k)) - a m_1 l x_2^2(k)\sin(2x_1(k))/2 - a\cos(x_1(k))u(k)}{4l/3 - a m_1 l\cos^2(x_1(k))}$$

x_1 is the pendulum angle (in radians) from the vertical, x_2 is the angular velocity, $g = 9.8$ is the constant gravity, m_1 is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, u is the force applied to the cart (Newton) and $Te = 0.02$ is a fixed step of discretization. $a = 1/(m1 + M)$ m1 = 0.1 Kg, M = 1 Kg and .

6.2. Results of system identification

In this section, we analyze the identification results of the inverted pendulum system. The variables $y(k)$ and $u(k)$ are output and input data, respectively. We choose $y(k-1)$, $y(k-2)$, $u(k-1)$ as the variables of the fuzzy model (regression vector). The parameter settings are, $N_p = 50$, the value of weighting exponent m is set at 2.5 and $c_1 = c_2 = 1.5$. The sequences of input and output signal used for the identification process are shown in Figure 1. Figure 2 shows the real output superposed with the estimated output and the signal error generated by the difference between the real and estimated outputs. The obtained results show that the proposed FCM-APSO algorithm provides a good approximation modeling accuracy.

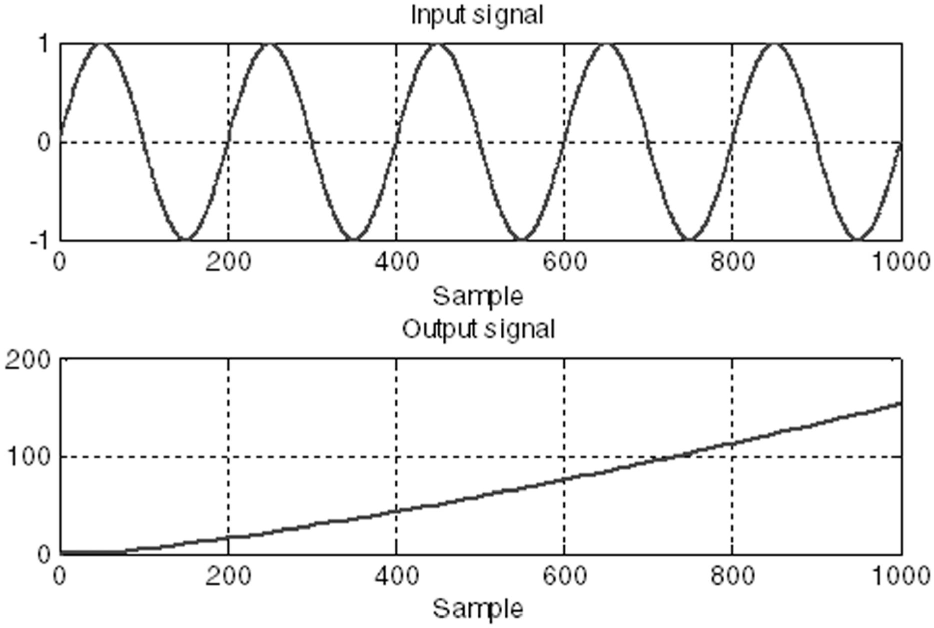

Figure 1: Sequences of input-output

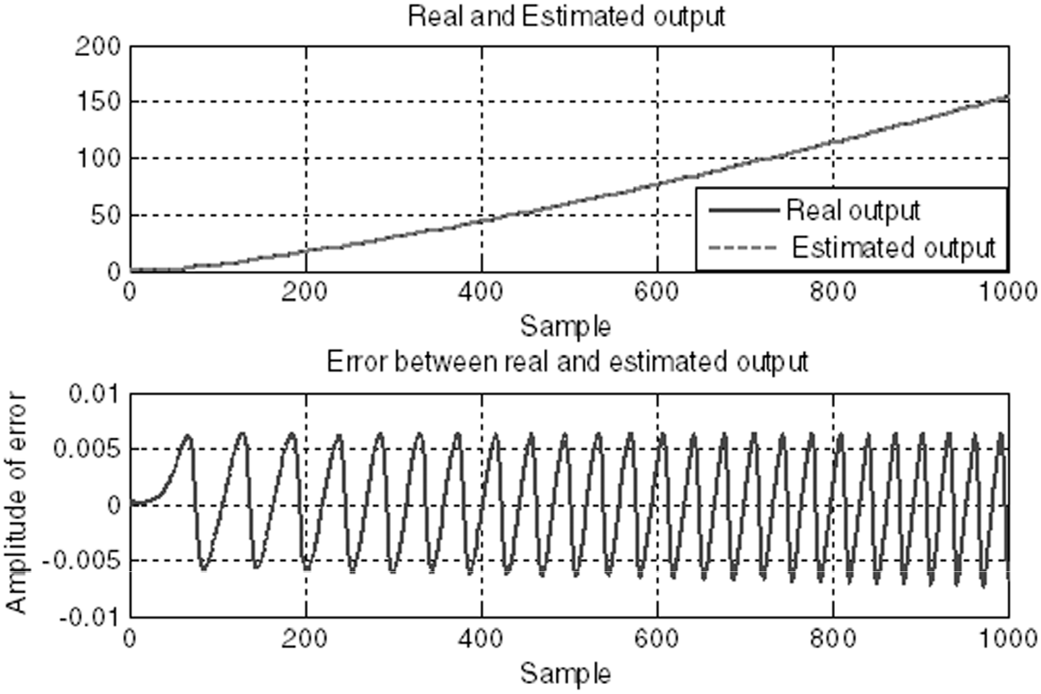

Figure 2: Identification results of FCM-APSO algorithm

6.3. Model validation

In this part, we discussed the efficiency of the FCM-APSO algorithm. For this reason, we presented statistical performance indexes formulas which are Entropy Classification (CE), Root Mean Square Error (RMSE) and Variance Accounting For (VAF) the results offered by each criterion are mentioned by tables (I and II).

6.3.1. Entropy classification

In the beginning, we start by identifying the selection of the clusters numbers. Based in the literature, the criteria Entropy classification is the best compared to several criteria [4].

$$C_{EC}(C)=\frac{1}{N}\sum_{k=1}^{N}\sum_{i=1}^{c}\mu_{ik}\log(\mu_{ik})$$
$$\forall\, 1\le i\le c, 1\le K\le N \tag{31}$$

The criterion condition is mentioned as follows:

$$C^{*}=\min_{C=2,\dots.N-1}\left[C_{EC}(C)\right] \tag{32}$$

Therefore, the number of fuzzy rules is summarized in the Table 1.

Table 1
Clusters numbersresults

Clusters numbers	C=2	C=3	C=4	C=5
$C_{CE}(10^{-4})$	-3.3657	e3.1620	e3.4657	3.2189

6.3.2. Root Mean Square Error (RMSE)

In this test, we use the average value of the error between real output and estimated output of the system based the T-S model to compute the Root Mean Square Error.

$$RMSE=\frac{1}{N}\sqrt{\sum_{k=1}^{N}(y_k-\hat{y}_k)^2} \tag{33}$$

where

N is number of observations, y_k is actual output and $\hat{y}_k$ is the estimated output.

6.3.3. Variance Accounting For (VAF)

The VAF criterion can be given by

$$VAF=100\%.\left[1-\frac{\operatorname{var}(y_k-\hat{y}_k)}{\operatorname{var}(y_k)}\right] \tag{34}$$

The model will be validated, if the VAF criterion is around. The VAF test and the RMSE test are summarized in Table 2. The obtained results of the VAF and RMSE tests provide a good performance.

Table 2
Results of model validation

Tests	*RMSE*	*VAF*
FCM-APSO	0.0043	99.99

6.4. Quadratic Optimal Control

Using the FCM-APSO algorithm, local models are obtained as follows:

$$A_1 = \begin{bmatrix} -1.9876 & -0.9876 \\ 1 & 0 \end{bmatrix};\ A_2 = \begin{bmatrix} -1.9934 & -0.9934 \\ 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1.9954 & -0.9954 \\ 1 & 0 \end{bmatrix};\ A_4 = \begin{bmatrix} -1.9964 & -0.9964 \\ 1 & 0 \end{bmatrix}$$

$$B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 0.0312 \\ 0 \end{bmatrix}$$

$$C_1 = C_2 = C_3 = C_4 = [1 \quad 0]$$

To obtain the parameters of each model, we solve the system (27). Indeed, the control gains are:

$$F_1 = [\text{-50.0143-27.9569}]\ ;\ F_2 = [\text{-50.0633-28.0154}]$$
$$F_3 = [\text{-50.0797-28.0350}]\ ;\ F_4 = [\text{-50.0879-28.0447}]$$

In addition, the positive definite matrices are given as below:

$$P_1 = \begin{bmatrix} 241.98 & 70.75 \\ 70.75 & 91.22 \end{bmatrix};\ P_2 = \begin{bmatrix} 242.98 & 71.39 \\ 71.39 & 91.73 \end{bmatrix}$$
$$P_3 = \begin{bmatrix} 243.19 & 71.61 \\ 71.61 & 91.91 \end{bmatrix};\ P_4 = \begin{bmatrix} 243.34 & 71.72 \\ 71.72 & 92.00 \end{bmatrix}$$

The performance of the developed control approach is illustrated by the numerical simulation. In fact, Figure (3) and Figure (4) show the evolution of angular position and the evolution of angular velocity,

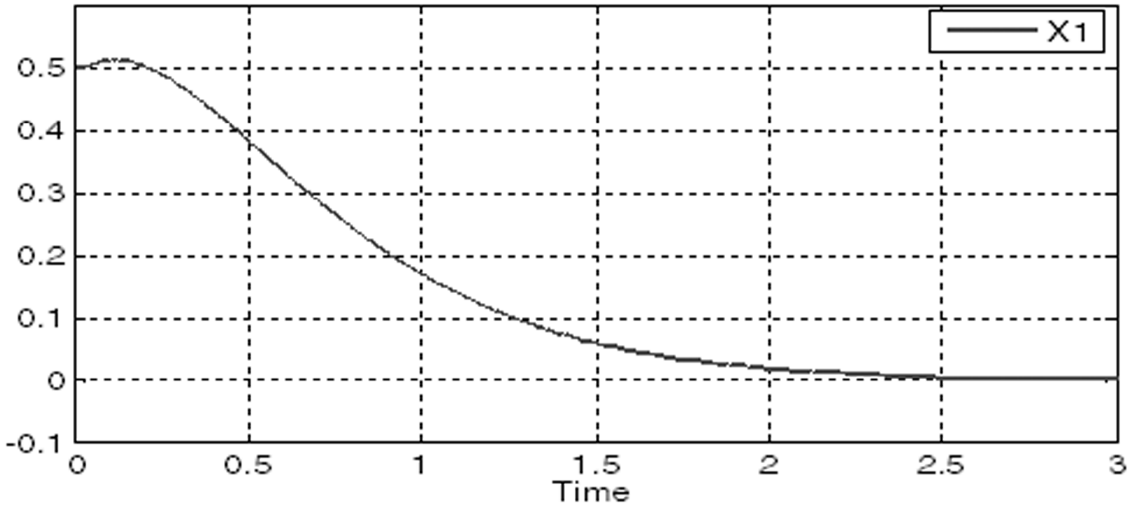

Figure 3: Angular position

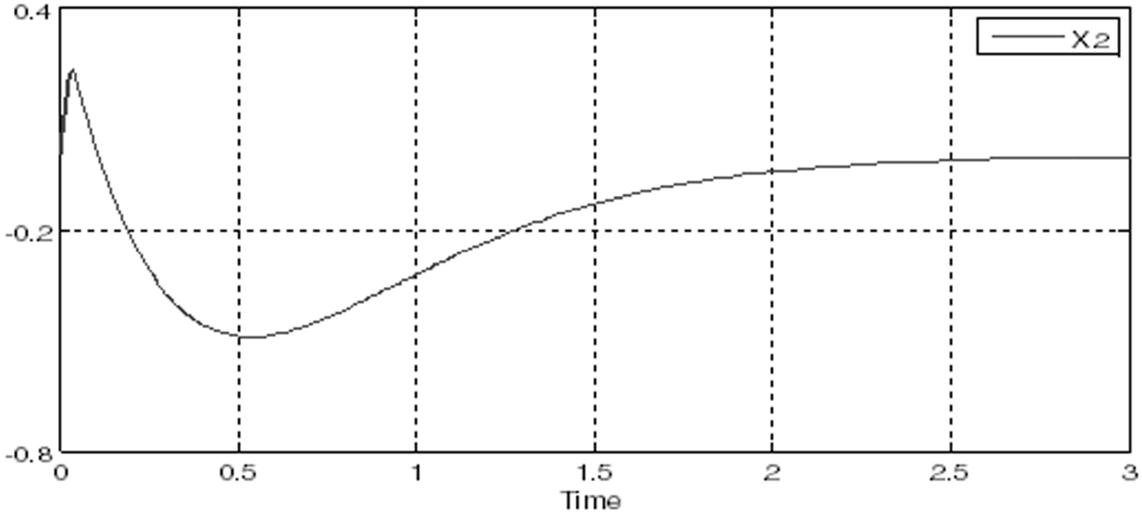

Figure 4: Angular velocity

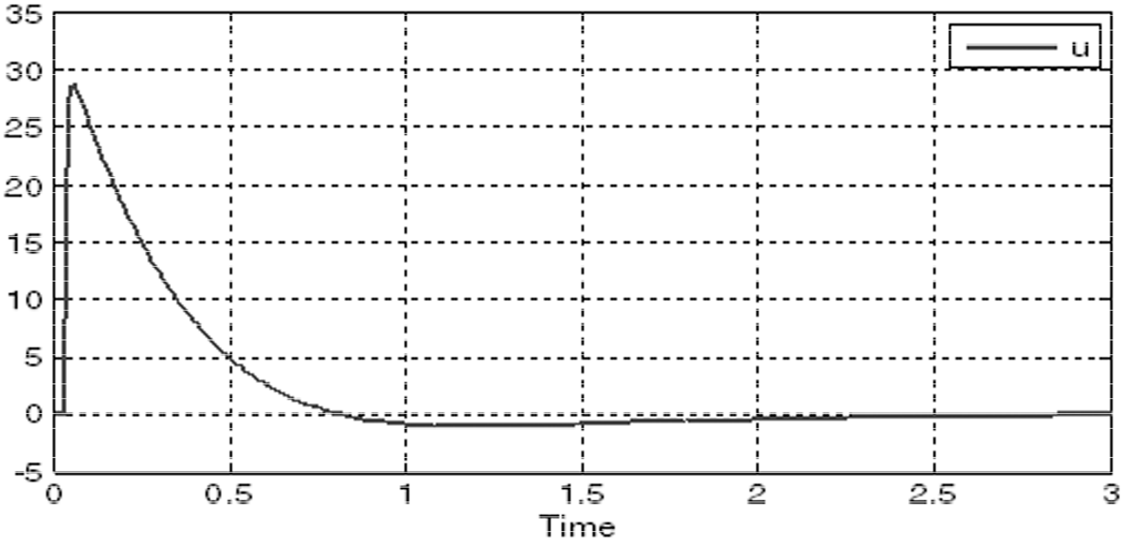

Figure 5: The global control law

respectively. According to the fusion of different optimal gains computed for each local model, the evolution of the global control law is shown in Figure (5).

7. CONCLUSIONS

In this paper, a fuzzy C-means clustering algorithm combined with adaptive particle swarm optimization algorithm for T-S fuzzy model identification is firstly presented. The second level is devoted to the synthesis of an optimal control law in order to ensure the global stability of the closed loop system. The developed approach is implemented via an inverted pendulum system. The simulation showed favorable results for the modeling of pendulum system. Further, the stability of the system's states is successfully achieved by the proposed control low with satisfactory performance which proves the effectiveness and the validity of the used approach.

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