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MHD FLOW AND HEAT TRANSFER IN POROUS MEDIUM PAST AN INFINITE PLATE IN A ROTATING SYSTEM INCLUDING HALL EFFECTS

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Abstract

In the present Investigation, the Hall effects on the hydromagnetic flow through a porous medium past an infinite plate in a rotating system. Both the fluid and plate are in a state of solid body rotation about an axis normal to the plate. A uniform magnetic field is applied normal to to the plate. The governing equations have been solved using regular perturbation technique. The effects of Hall parameter, porosity of the medium and magnetic field on the velocity and temperature distributions have been discussed and results are displayed graphically.

1. INTRODUCTION

The study of flow through porous media has become of great interest in many scientific and engineering applications. The study of such type of flows is applied to the problems of movement of underground water resources and for filteration and water purification processes. The porous medium is in fact a non-homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid which has dynamical properties equivalent to those of non-homogeneous continuum. Thus, one can study the flow of a hypothetical homogeneous fluid under the action of the properly averaged external forces. Thus, a complicated problem of the flow through a porous medium reduces to the flow problem of a homogeneous fluid with some resistance. This type of flow originates mainly from Darcy's experimental formula.

In recent years, considerable attention has been given to the studies of hydrodynamic and hydromagnetic boundary layer flows with or without Hall current effects in a rotating viscous fluid system. When a conducting fluid moves through a strong magnetic field, the motion of the charged particles across the magnetic field is hindered, there arise currents which are in a direction perpendicular to both electric and magnetic fields, known as Hall currents. If the strength of the applied magnetic field is large, the Hall current effects play a significant role in determining the flow pattern of an incompressible viscous fluid. These Hall currents have physical relevance to several astrophysical situations.

Ahmadi and manvi (1) have derived a general equation of motion and applied the results obtained to some basic flow problems. Agarwal et al. (2) analysed the effects of Hall currents and viscous dissipation on the hydromagnetic free convective flow in the Ekman layer of a conducting liquid past an infinite porous plate. . Datta and Jana (3) have investigated the Hall effects on the oscillatory MHD flow past a flat plate. Debnath (4-6) has made a major contribution to the unsteady hydrodynamic and hydromagnetic boundary layer flows with or without Hall current effects in a rotating, viscous fluid system. His initial value investigations into these problems have provided many new and interesting information on the steady-state and transient flows, the structure of the boundary layers and the propagation of a series of inertial oscillations and diffused hydromagnetic waves. Gulab Ram and mishra (7) applied the equations (derived y Ahmadi and Manvi) to study the MHD flow of a conducting fluid through porous media. Mazumdar et al. (11) and Gupta (8) have studied the hydromagnetic steady flow including Hall current effects. Katagiri (9) has studied the effects of Hall currents on the boundary layer flow past a semi-infinite flat plate. The unsteady viscous flow through a porous medium past an oscillating plate in a rotating system has been discussed by Kumar and Varshney (10). Hall effects in the unsteady cases were discussed by Pop (12), Sakhnovskli (15). Rao and Krishna (12) investigated Hall effects on the non-torsionally generated unsteady hydromagnetic flow in a semi-infinite expanse of an electrically conducting rotating viscous fluid bounded by an infinite non-conducting plate. Yamanishi (17), have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. Z.H. Khan et. al. (18) investigates the unsteady MHD flow and entropy generation in a rotating permeable channel. Kumar S. and Sharma K.(19) studied mathematical modelling of MHD flow and radiative heat transfer past a moving porous rotating disk. M. Veer Krishnna and K. (20) Vajravelu investigates the impact of Hall effects on the unsteady MHD flow of the Rivlin- Ericksen fluid past an infinite vertical porous plate. Benharkat Z. (21) investigates the rotating convective MHD flow over a vertical moving plate in the presence of heat source, radiation and Hall effects.

2. BASIC EQUATIONS AND FORMULATION OF THE PROBLEM

Let x'- axis be along the plate in the direction of flow and z'- axis be normal to the plate. The y'- axis is normal to x'- z' plane. Let us consider steady free convection flow of an incompressible viscous electrically conducting fluid bounded by an infinite plate at z'=0. The fluid and the plate rotate in unison with a constant angular velocity Ω about z'- axis which is normal to the plate. The plate is electrically non-conducting. A uniform magnetic field of intensity $\vec{B}_\circ$ is applied along the z'- axis. The induced magnetic field is neglected in comparison to the applied magnetic field $\vec{B} = (0, 0, B_\circ)$. The flow is steady and the plate is infinite in extent, the physical variables are functions of z' only. The equation of continuity $\nabla.\vec{U} = 0$ gives on integration

$$w' = - w_\circ \tag{1}$$

$(w_\circ > 0)$ where $\vec{U} = (u', v', w')$. The equation of conservation of electric charge $\nabla.\vec{J} = 0$, gives J_Z = constant,

Where $\vec{J} = (J_{x'}, J_{y'}, J_z)$. The constant is assumed to be zero since J_z=0 at the plate which is assumed to be electrically non-conducting. Thus J_z=0 everywhere in the flow. In rotating frame of reference, the governing equations are

Momentum equation

$$-w_0 \frac{du'}{dz'} - 2\Omega v' = \nu \frac{d^2u'}{dz'^2} + g\beta(T - T_\infty) - \frac{\nu u'}{K'} + \frac{B_0 J_y}{\rho} \tag{2}$$

$$-w_0 \frac{dv'}{dz'} + 2\Omega u' = \nu \frac{d^2v'}{dz'^2} - \frac{\nu v'}{K'} - \frac{B_0 J_x}{\rho} \tag{3}$$

Energy Equation

$$-w_0 \frac{dT}{dz'} = \frac{k}{\rho C_p} \frac{d^2T}{dz'^2} + \frac{\mu}{\rho C_p}\left[\left(\frac{du'}{dz'}\right)^2 + \left(\frac{dv'}{dz'}\right)^2\right] \tag{4}$$

Where ν is Kinematic viscosity, β the coefficient of volume expansion, k thermal conductivity, K' permeability of the medium.

The Boundary conditions are

$$\begin{aligned} z' = 0 &: u' = 0, v' = 0, T = T_w \\ z' \to \infty &: u' \to 0, v' \to 0, T \to T_\infty \end{aligned} \tag{5}$$

The generalized Ohm's law (Cowling [5]) is

$$\vec{J} + \frac{w_e \tau_e}{H} \vec{J} \times \vec{H} = \sigma \left[\vec{E} + \mu_e \vec{U} \times \vec{H} + \frac{1}{\rho\, n_e} \vec{\nabla} p_e\right] \tag{6}$$

where $\vec{H}$ is magnetic field intensity, $\vec{U}$ is the velocity vector, $\vec{E}$ is the electric field, $\vec{J}$ is electric current density, w_e is the cyclotron frequency, τ_e is the electric collision time, σ is the fluid conductivity, p_e is electron pressure, n_e is the number density of electron, μ_e is the magnetic permeability and e is the electron charge. In writing equation (6) the ionslip and thermoelastic effects are neglected.

We also assume that $\vec{E}$=0 (Meyer [26]). Using the above assumptions, equation (6) gives us

$$J_X + m\, J_Y = \sigma\, B_\circ\, v' \tag{7}$$

$$J_Y - m\, J_X = -\sigma\, B_\circ\, u' \tag{8}$$

Where $m = \omega_e\, \tau_e$ and $\overrightarrow{B_\circ} = \mu_e\, \overrightarrow{H_\circ}$ are respectively the Hall parameter and electromagnetic induction. On solving (7) and (8), we have

$$J_X = \frac{\sigma\, B_\circ}{1+m^2}\,(v' + mu') \tag{9}$$

$$J_Y = \frac{\sigma\, B_\circ}{1+m^2}\,(mv' - u') \tag{10}$$

Substituting the values of J_X and J_Y in (2) and (3), we obtain

$$-w_\circ \frac{du'}{dz'} - 2\Omega v' = \nu \frac{d^2u'}{dz'^2} + g\beta(T - T_\infty) - \nu\frac{u'}{K'} + \frac{\sigma B_\circ{}^2(mv'-u')}{\rho(1+m^2)} \tag{11}$$

$$-w_\circ \frac{dv'}{dz'} + 2\Omega u' = \nu \frac{d^2v}{dz'^2} - \nu\frac{v'}{K'} - \frac{\sigma B_\circ{}^2(mu'+v')}{\rho(1+m^2)} \tag{12}$$

Introducing the new variable $q' = u' + iv'$, we get from (11), (12), and (4)

$$-w_\circ \frac{dq'}{dz'} + 2i\Omega q' = \nu \frac{d^2q'}{dz'^2} + g\beta(T - T_\infty) - \nu\frac{q'}{K'} - \frac{\sigma B_\circ{}^2(1+im)q'}{\rho(1+m^2)} \tag{13}$$

$$-w_\circ \frac{dT}{dz'} = \frac{k}{\rho\, c_p}\frac{d^2T}{dz'^2} + \frac{\mu}{\rho\, c_p}\frac{dq'}{dz'}\frac{d\overline{q'}}{dz'} \tag{14}$$

Where $\overline{q'}$ is the complex conjugate of q'.

The corresponding boundary conditions are

$$\begin{aligned} z' = 0 &: \; q' = 0, \quad T = T_w \\ z' \to \infty &: \; q' \to 0, \; T \to T_\infty \end{aligned} \tag{15}$$

3. SOLUTION OF THE PROBLEM

Let us introduce the following non- dimensional variables:

$$q = \frac{q'}{w_\circ},\; z = \frac{z' w_\circ}{\nu},\;\; \theta = \frac{T - T_\infty}{T_w - T_\infty},\, K = \frac{K'\, w_0^2}{\nu^2},\, A = \frac{\Omega\, \nu}{w_0^2}$$

E (Eckert number) = $\frac{w_o^2}{C_p(T_w - T_\infty)}$

G (grashoff number) = $\frac{g\nu\beta(T_w - T_\infty)}{w_o^3}$

M (Hartmann number) = $\sqrt{\frac{\sigma B_o^2 \nu}{\rho w_o^2}}$

Pr(Prandtl number) = $\frac{\mu C_p}{\kappa}$ (16)

Then equation (13) and (14) reduce to

$$\frac{d^2q}{dz^2} + \frac{dq}{dz} - \left(2iA + \frac{M^2}{1-im} + \frac{1}{K}\right) q = -G\,\theta \tag{17}$$

$$\frac{d^2\theta}{dz^2} + Pr\frac{d\theta}{dz} = -E\,Pr\frac{dq}{dz}\frac{d\bar{q}}{dz} \tag{18}$$

Where $\bar{q}$ is the complex conjugate of $\bar{q}$.

The corresponding boundary conditions are

$$z = 0\ :\ q = 0\,, \qquad \theta = 1$$
$$z \to \infty\ :\ q \to 0\,,\ \ \theta \to 0 \tag{19}$$

To solve the non-linear coupled equations (17) and (18), we consider a series solution in power of Eckert number. (Eckert numbers are assumed to be very small), then

$$\mathrm{q} = q_o + E\,q_1 \tag{20}$$

$$\theta = \theta_o + E\,\theta_1 \tag{21}$$

Substituting (20) and (21) in equations (17) and (18) and equating the coefficients of E^o, E^1, we have

$$\frac{d^2q_o}{dz^2} + \frac{dq_o}{dz} - \left(2iA + \frac{M^2}{1-im} + \frac{1}{K}\right) q_o = -G\,\theta_o \tag{22}$$

$$\frac{d^2\theta_o}{dz^2} + \mathrm{Pr}\frac{d\theta_o}{dz} = 0 \tag{23}$$

$$\frac{d^2q_1}{dz^2} + \frac{dq_1}{dz} - \left(2iA + \frac{M^2}{1-im} + \frac{1}{K}\right) q_1 = -G\,\theta_1 \tag{24}$$

$$\frac{d^2\theta_1}{dz^2} + \mathrm{Pr}\frac{d\theta_1}{dz} = -Pr\frac{dq_o}{dz}\frac{d\bar{q}_\circ}{dz} \tag{25}$$

The corresponding boundary conditions are

$$\begin{aligned} z = 0 \ &: \ q_o = 0 = q_1 \ , \theta_o = 1 \, , \theta_1 = 0 \\ z \to \infty \ &: \ q_o \to 0, \ q_1 \to 0 \, , \ \theta_o \to 0 \, , \ \theta_1 \to 0 \end{aligned} \tag{26}$$

The solution q_o and θ_o corresponds to the case without viscous dissipation, while q_1 and θ_1 are perturbed quantities when the viscous dissipation is taken into account.

Under the boundary conditions (26), the solutions of the equations (22) and (25) are given by

$$q_o = \frac{G}{\alpha_2 - i\beta_2}\left[e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z} - e^{-Pr\,z}\right] \tag{27}$$

$$\theta_o = e^{-\mathrm{Pr}\,z} \tag{28}$$

$$\begin{aligned} q_1 = \frac{\mathrm{Pr}\ G^3}{\alpha_2^2+\beta_2^2}\Bigg[& \frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2 + \frac{\beta_1^2}{4}\right\}(\alpha_2+i\beta_2)}{(\alpha_2^2+\beta_2^2)a_1} e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z} + \frac{(\alpha_2+i\beta_2)\, e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z}}{2\,(\alpha_2^2+\beta_2^2)} - \\ & \frac{\mathrm{Pr}\ \alpha_4(\alpha_2+i\beta_2)\, e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z}}{(\alpha_2^2+\beta_2^2)(\alpha_3^2+\beta_3^2)} - \frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2 + \frac{\beta_1^2}{4}\right\}(\alpha_2+i\beta_2)\, e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z}}{a_1(\alpha_2^2+\beta_2^2)} - \frac{(\alpha_3+i\beta_2)\, e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z}}{2\,(\alpha_3^2+\beta_2^2)} + \\ & \frac{Pr\,(a_4+ia_5)e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z}}{(\alpha_3^2+\beta_3^2)\left\{(\alpha_5^2+\beta_5^2-\beta_2^2)^2+4\alpha_5^2\beta_2^2\right\}} - \frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2 + \frac{\beta_1^2}{4}\right\}(\alpha_2+i\beta_2)}{(\alpha_2^2+\beta_2^2)a_1} e^{-\mathrm{Pr}\,z} - \frac{(\alpha_2+i\beta_2)}{2\,(\alpha_2^2+\beta_2^2)} e^{-\mathrm{Pr}\,z} + \\ & \frac{Pr\,(\alpha_2+i\beta_2)}{(\alpha_2^2+\beta_2^2)(\alpha_3^2+\beta_3^2)} e^{-\mathrm{Pr}\,z} + \frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2 + \frac{\beta_1^2}{4}\right\}(a_2+i\beta_2)}{a_1(\alpha_2^2+\beta_2^2)} e^{-(1+\alpha_1)z} + \frac{(a_3+i\beta_2)}{2(\alpha_3^2+\beta_2^2)}\ e^{-2\,\mathrm{Pr}\,z} - \\ & \frac{Pr}{(\alpha_3^2+\beta_3^2)} e^{-(\frac{1+\alpha_1}{2}+\mathrm{Pr})z}\left\{(a_4+ia_5)\cos\frac{\beta_1 z}{2} + (a_6+ia_7)\sin\frac{\beta_1 z}{2}\right\}\Bigg] \end{aligned} \tag{29}$$

$$\begin{aligned} \theta_1 = \frac{\mathrm{Pr}\ G^2}{\alpha_2^2+\beta_2^2}\Bigg[& \left\{\frac{\left(\frac{1+\alpha_1}{2}\right)^2 + \frac{\beta_1^2}{4}}{a_1} + \frac{1}{2} - \frac{\mathrm{Pr}\,\alpha_4}{(\alpha_3^2+\beta_3^2)}\right\} e^{-\mathrm{Pr}\,z} - \frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2 + \frac{\beta_1^2}{4}\right\}}{a_1} e^{-(1+\alpha_1)z} - \frac{e^{-2\,\mathrm{Pr}\,z}}{2} + \\ & \frac{Pr}{(\alpha_3^2+\beta_3^2)} e^{-(\frac{1+\alpha_1}{2}+\mathrm{Pr})z}\left(\alpha_4\cos\frac{\beta_1 z}{2} + \beta_4\sin\frac{\beta_1 z}{2}\right)\Bigg] \end{aligned} \tag{30}$$

The constants involved in above expressions are given in the appendix.

Since

$$\mathrm{q} = q_o + E\,q_1 = \mathrm{u} + \mathrm{iv}$$

Therefore, u and v are given by

u = Real Part of q

$$u=\frac{G}{\alpha_2^2+\beta_2^2}\left[(\alpha_2\cos\frac{\beta_1 z}{2}+\beta_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}-\alpha_2 e^{-\mathrm{Pr}\,z}\right]+\frac{E\,Pr\,G^3}{\alpha_2^2+\beta_2^2}\left[\left\{\frac{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}}{a_1(\alpha_2^2+\beta_2^2)}+\frac{1}{2(\alpha_2^2+\beta_2^2)}-\right.\right.$$

$$\left.\frac{\mathrm{Pr}\ \alpha_4}{(\alpha_2^2+\beta_2^2)(\alpha_3^2+\beta_3^2)}\right\}(\alpha_2\cos\frac{\beta_1 z}{2}+\beta_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}-\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}(a_2\cos\frac{\beta_1 z}{2}+\beta_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{a_1(a_2^2+\beta_2^2)}-$$

$$\frac{(a_3\cos\frac{\beta_1 z}{2}+\beta_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2(a_3^2+\beta_2^2)}+\frac{\mathrm{Pr}(a_4\cos\frac{\beta_1 z}{2}+a_5\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{(\alpha_3^2+\beta_3^2)\left\{(\alpha_5^2+\beta_5^2-\beta_2^2)^2+4\alpha_5^2\beta_2^2\right\}}-\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\alpha_2 e^{-\mathrm{Pr}\,z}}{a_1(\alpha_2^2+\beta_2^2)}-\frac{\alpha_2 e^{-\mathrm{Pr}\,z}}{2(\alpha_2^2+\beta_2^2)}+$$

$$\left.\frac{\mathrm{Pr}\ \alpha_4\alpha_2 e^{-\mathrm{Pr}\,z}}{(\alpha_2^2+\beta_2^2)(\alpha_3^2+\beta_3^2)}+\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\alpha_2 e^{-(1+\alpha_1)z}}{a_1(a_2^2+\beta_2^2)}+\frac{a_3 e^{-2\,\mathrm{Pr}\,z}}{2(a_3^2+\beta_2^2)}-\frac{\mathrm{Pr}(a_4\cos\frac{\beta_1 z}{2}+a_6\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}+Pr\right)z}}{(\alpha_3^2+\beta_3^2)\left\{(\alpha_5^2+\beta_5^2-\beta_2^2)^2+4\alpha_5^2\beta_2^2\right\}}\right] \quad (31)$$

v = imaginary part of q

$$v=\frac{G}{\alpha_2^2+\beta_2^2}\left[(\beta_2\cos\frac{\beta_1 z}{2}-\alpha_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}-\beta_2 e^{-\mathrm{Pr}\,z}\right]+\frac{E\,Pr\,G^3}{\alpha_2^2+\beta_2^2}\left[\left\{\frac{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}}{a_1(\alpha_2^2+\beta_2^2)}+\frac{1}{2(\alpha_2^2+\beta_2^2)}-\right.\right.$$

$$\left.\frac{\mathrm{Pr}\ \alpha_4}{(\alpha_2^2+\beta_2^2)(\alpha_3^2+\beta_3^2)}\right\}(\beta_2\cos\frac{\beta_1 z}{2}-\alpha_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}-\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}(\beta_2\cos\frac{\beta_1 z}{2}-a_2\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{a_1(a_2^2+\beta_2^2)}-$$

$$\frac{(\beta_2\cos\frac{\beta_1 z}{2}-a_3\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2(a_3^2+\beta_2^2)}+\frac{\mathrm{Pr}(a_5\cos\frac{\beta_1 z}{2}-a_4\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{(a_3^2+\beta_3^2)\left\{(\alpha_5^2+\beta_5^2-\beta_2^2)^2+4\alpha_5^2\beta_2^2\right\}}-\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\beta_2 e^{-\mathrm{Pr}\,z}}{a_1(\alpha_2^2+\beta_2^2)}-\frac{\beta_2 e^{-\mathrm{Pr}\,z}}{2(\alpha_2^2+\beta_2^2)}+$$

$$\left.\frac{\mathrm{Pr}\ \alpha_4\beta_2 e^{-\mathrm{Pr}\,z}}{(\alpha_2^2+\beta_2^2)(\alpha_3^2+\beta_3^2)}+\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\beta_2 e^{-(1+\alpha_1)z}}{a_1(a_2^2+\beta_2^2)}+\frac{\beta_2 e^{-2\,\mathrm{Pr}\,z}}{2(a_3^2+\beta_2^2)}-\frac{\mathrm{Pr}(a_5\cos\frac{\beta_1 z}{2}+a_7\sin\frac{\beta_1 z}{2})\,e^{-\left(\frac{1+\alpha_1}{2}+Pr\right)z}}{(\alpha_3^2+\beta_3^2)\left\{(\alpha_5^2+\beta_5^2-\beta_2^2)^2+4\alpha_5^2\beta_2^2\right\}}\right] \quad (32)$$

Non- dimensional temperature is given by

$$\theta=e^{-\mathrm{Pr}\,z}+\frac{E\,Pr\,G^2}{\alpha_2^2+\beta_2^2}\left[\left\{\frac{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}}{a_1}+\frac{1}{2}-\frac{\mathrm{Pr}\,\alpha_4}{(\alpha_3^2+\beta_3^2)}\right\}e^{-\mathrm{Pr}\,z}-\left\{\frac{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}}{a_1}\right\}e^{-(1+\alpha_1)z}-\frac{e^{-2\,\mathrm{Pr}\,z}}{2}+\right.$$

$$\left.\frac{Pr}{(\alpha_3^2+\beta_3^2)}e^{-(\frac{1+\alpha_1}{2}+\mathrm{Pr})\,z}(\alpha_4\cos\frac{\beta_1 z}{2}+\beta_4\sin\frac{\beta_1 z}{2})\right] \quad (33)$$

The rate of heat transfer at the wall is given by

$$\mathrm{Q}=-\mathrm{k}\left(\frac{\partial \mathrm{T}}{\partial z'}\right)_{z'=0}$$

$$=-\mathrm{k}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{z}}\right)_{z=0}\frac{w_o}{\nu}$$

$$=-\mathrm{k}\frac{w_o}{\nu}(T_w-T_\infty)\left(\frac{\partial \theta}{\partial \mathrm{z}}\right)_{z=0}$$

$$= \mathrm{k}\,\frac{\mathrm{w_o}}{\nu}(\mathrm{T_w}-\mathrm{T_\infty})\mathrm{Pr}+\mathrm{k}\frac{w_o}{\nu}(T_w-T_\infty)\frac{E\,Pr\,G^2}{\alpha_2^2+\beta_2^2}\left[-\frac{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}}{1+\alpha_1}-\frac{\mathrm{Pr}}{2}+\frac{\mathrm{Pr}\{\alpha_4(1+\alpha_1)-\beta_1\beta_4\}}{2(\alpha_3^2+\beta_3^2)}\right] \quad (34)$$

Where

$$a=1+\frac{4M^2}{1+m^2}+\frac{4}{K}$$

$$b=8A+\frac{4M^2m}{1+m^2}$$

$$\alpha_1=\frac{1}{\sqrt{2}}\left[\sqrt{a^2+b^2}+a\right]^{\frac{1}{2}}$$

$$\beta_1=\frac{1}{\sqrt{2}}\left[\sqrt{a^2+b^2}-a\right]^{\frac{1}{2}}$$

$$\alpha_2=Pr^2-Pr-\frac{1}{K}-\frac{M^2}{1+m^2}$$

$$\beta_2=2A+\frac{M^2m}{1+m^2}$$

$$\alpha_3=\left(\frac{1+\alpha_1}{2}\right)\left(\frac{1+\alpha_1}{2}+\mathrm{Pr}\right)-\frac{\beta_1^2}{4}$$

$$\beta_3=(1+\alpha_1+Pr)\frac{\beta_1}{2}$$

$$\alpha_4=(1+\alpha_1)\alpha_3+\beta_1\beta_3$$

$$\beta_4=\beta_1\alpha_3-(1+\alpha_1)\beta_3$$

$$\alpha_5=\left(\frac{1+\alpha_1}{2}+Pr\right)^2-\frac{\beta_1^2}{4}-\frac{1+\alpha_1}{2}-Pr-\frac{M^2}{1+m^2}-\frac{1}{K}$$

$$\beta_5=(\alpha_1+2Pr)\frac{\beta_1}{2}$$

$$a_1=(1+\alpha_1)(1+\alpha_1-Pr)$$

$$a_2=\alpha_1^2+\alpha_1-\frac{1}{K}-\frac{M^2}{1+m^2}$$

$$a_3=4\mathrm{Pr}^2-2\mathrm{Pr}-\frac{1}{K}-\frac{M^2}{1+m^2}$$

$$a_4=(\alpha_5^2+\beta_5^2-\beta_2^2)(\alpha_4\alpha_5+\beta_4\beta_5)+2\alpha_4\alpha_5\beta_2^2$$

$a_5 = 2\alpha_5\beta_2(\alpha_4\alpha_5 + \beta_4\beta_5) - \alpha_4\beta_2(\alpha_5^2 + \beta_5^2 - \beta_2^2)$

$a_6 = (\alpha_5^2 + \beta_5^2 - \beta_2^2)(\alpha_5\beta_4 - \alpha_4\beta_5) + 2\alpha_5\beta_2^2\beta_4$

$a_7 = 2\alpha_5\beta_2(\alpha_5\beta_4 - \alpha_4\beta_5) - \beta_2\beta_4(\alpha_5^2 + \beta_5^2 - \beta_2^2)$

4. DISCUSSIONS

From the solutions, it can be observed that the steady state distribution of the velocity components in the plane of rotation, are in the form of logarithmic spiral similar to the Ekman velocity spiral for the rotating flow over a disk. Since the magnetic field is strong, the exponential $e^{-\mathrm{Pr}\,z}$ decays slowly than the other exponential terms hence the thickness of the boundary layer is of order $1/Pr$.

To explore the effects of Hall parameter on hydro-magnetic free convective flow in a rotating fluid, we have carried out the numerical calculation for different values of m, M, K. The non-dimensional primary and secondary velocities have been plotted against z and θ_1 is potted against z in figures 1, 2 and 3 respectively.

From figures 1 and 2 it is seen that with increase in m, the primary velocity u increases and secondary velocity decreases. However, for fixed m, primary velocity decreases and secondary velocity increase with increase in M. As permeability of the medium increases primary velocity increases and secondary velocity decreases. Figure 3.6 shows that θ_1 increases with increases in m for fixed M and decreases with increase in M.

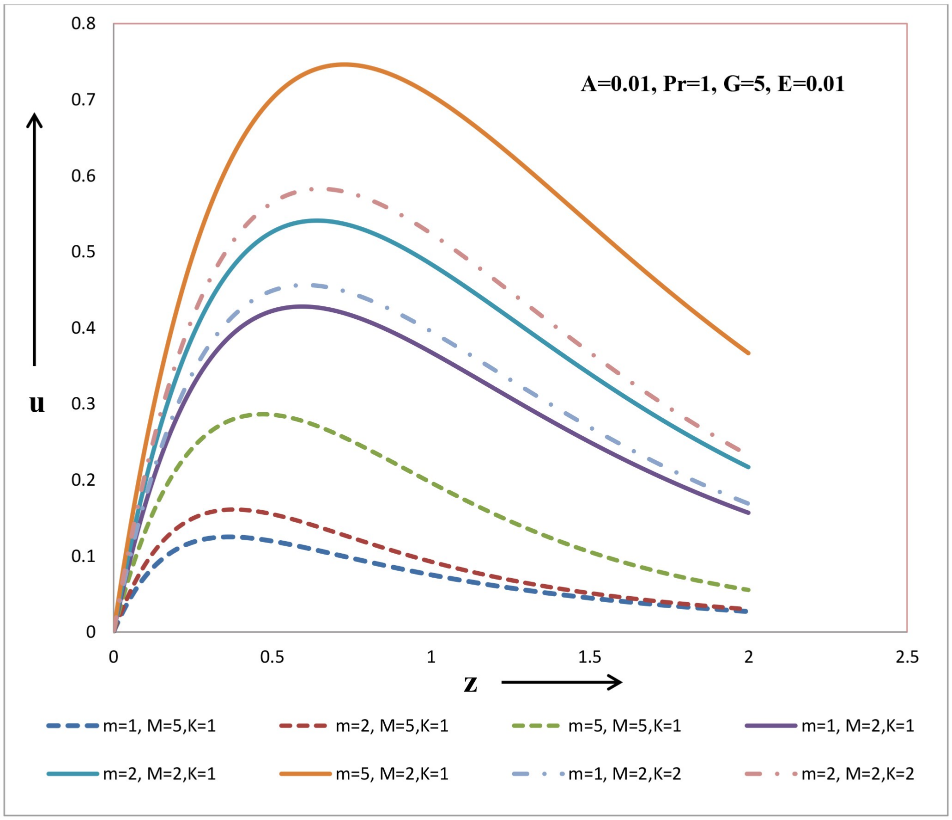

Fig. 1 u *vs* z

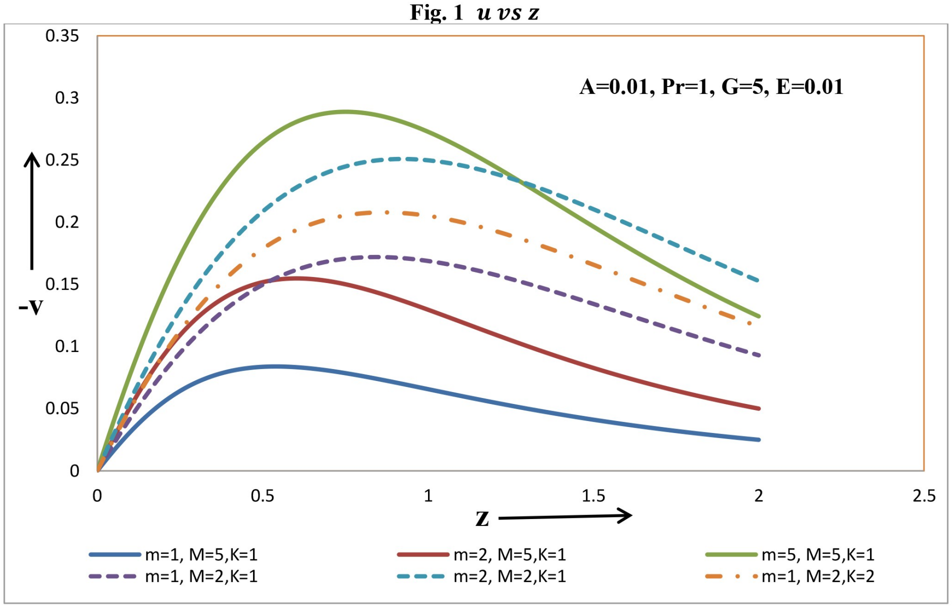

Fig.2 $-v$ *vs* z

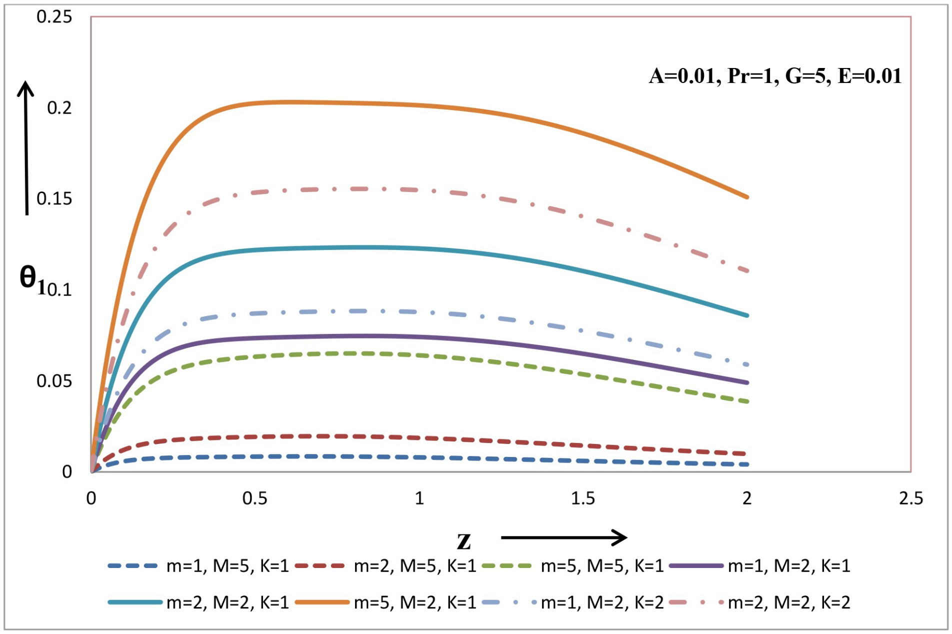

Fig.3 θ_1 *vs z*

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