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LEARNING FINANCIAL VOLATILITY

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A theoretical measure of financial volatility in connection with real economic variables is formulated in view of the extensively interactive, integrative and dynamic circular causation (evolutionary) *inter*relationships (also referred to as IIE) between economic and financial variables and relevant policies. The perspective of long-term sustainability of the financial and real economic linkages as positive and normative issues of globalization is reflected in this measure of volatility. The interactive, integrative and dynamic circular causation means the recursive feedback and evolution that continue on in the simulated model involving the normative and positive variables underlying the formulation of the measure of financial volatility. The IIE methodology in the financial volatility model has important implications for the ARCH model. This topic is discussed and formulated. The empirical results on causality and the methods of ARCH, GARCH bring out the importance of our theoretical conclusions respecting the limitation of such methods in statistical inference involving learning-induced methodology as introduced in this paper.

JEL Classification: G15, D63, D83.

I. INTRODUCTION

Volatility in global capital markets is caused by many factors. Some of these can be quantified and others are of the psychological and political nature based on unstable consumer preferences and recently with political repercussions caused by the global terrorism syndrome and wars in various parts of the world. Consumers have revised their preferences upwards for high tech goods, which the IT market with a large stocked up inventory (Jorgenson, 2001) is presently unable to provide. This has led to the sudden over-investment and stagnation in the IT labor market as the industry awaits development of such kinds of goods that will satisfy the refined nature of consumer preferences and of technology in a highly competing industry under globalization. Investment to discover innovative alternatives has thus led to failed experiments. Thus a combination of quantifiable factors and imponderables makes the old measures of financial volatility unusable for estimation of risk and future anticipation. An attention to construct a new measure of volatility is thus in place.

Among the most elusive of the economic variables that defy any predictable relationship between them and the real economic variables like capital movement, capital accounts balances, money supply and output are interest rates and exchange rates. The short-term interest rates have proved to be highly uncertain during the recent financial instability.. The result of such an uncertain relationship between real economic variables and financial variables, principally the interest rate and exchange rate uncertainty has brought into question the viability of the concept of economic fundamentals surrounding any well-determined application of macroeconomic policies in attaining economic and financial stabilization.

On the side of economic and financial relations it is now increasingly difficult to predict the interrelationship between short-run interest rates and exchange rates. Thus the theory and application of Mundell-Fleming model on capital flows under various types of fiscal and monetary policies (Mundell, 1962; Fleming, 1962; Choudhury, 1996) and interest rate-exchange rate mechanism have become questionable.

II. THE PERSPECTIVE OF THIS PAPER

We argue in this paper that a new measure of financial volatility must incorporate not only positivistic relations between variables, which we take as interest rate, exchange rate, money supply, output, capital flow and capital accounts balance. Rather, it must also be normatively determined by policy and information inputs to guide consumer preference-formation along with guarantee of financial security to back up the anticipated sustainable future. A discursive menu between consumers, producers, governments and international development finance institutions is thereby required.

The need for addressing the normative questions of financial globalization has been voiced by Thurow (1996), Korten (1995), Henderson (2000), Pieterse (2000) and Stiglitz (2002). The general argument of these authors is that capitalist globalization has divided the world between a hinterland of cheap inputs for production by transnational companies and the selling of products in expensive markets. Thurow writes on this aspect of global production and trade (op cit p. 115): "Minimizing costs and maximizing revenues is what profit maximization, the heart of capitalism, is all about." Economic integration by trading blocs has not only accentuated such a hinterland capitalist world-system but as the authors point out, it has caused an anticipation of control and management of resources for profit maximization in forward looking formulas constructed by transnational companies, governments and international financial organizations. Such a control over resources has caused the exclusive capitalization of financial assets at the expense of its real economy linkage.

A better understanding and quantitative simulation of volatility of capital flows in terms of the diverse factors that affect them involves a multidimensional objective criterion. The construction and simulation of the objective criterion to stabilize and measure the controllability of volatility between critical real and financial variables in terms of their inherently volatile *inter*relationships is the objective of this paper.

III. FORMALIZING A VOLATILITY MEASURE OF CAPITAL FLOWS IN TERMS OF LEARNING-INDUCED FINANCIAL AND ECONOMIC VARIABLES

The normative and quantitative study of the above-mentioned volatility index implies a constantly learning environment that can increasingly enable information flow and controllability of an ever-expanding domain of interaction among the critical variables and the relevant institutions. But interaction among the variables alone is not enough to grapple with the problem of volatility. The expanding learning experience, control, organization and predictability of the interacting variables influencing capital flows must be well determined within given bounds, which though remain dynamic in themselves. This means attaining temporary integration (consensus) in the interactive nature of the variables and learning generated in the discursive polity-market system. The expanding nature of learning of the interactively integrating variables and policy-making is of an evolutionary nature as learning and adjustment continues by information flow, organization and controllability both in the system of variables and institutions.

Such a quantitative interactive, integrative and evolutionary learning-measure of financial volatility is now formalized. The volatility measure combines the positivistic and normative features of the underlying institutional and politico-economic learning process in the form of a multidimensional objective criterion.

1. Formalism of the Learning Model of Volatility

Let, $x(\theta)$ denote a particular variable in the vector $\mathbf{x}(\theta) = \{K, e, i, M, Y, KAB\}[\theta]$, that we will use to develop a multidimensional measure of volatility of net capital flows. The square bracket means that all internal variables are q-induced.

$\mathbf{x}(\theta)$ is time-dependent and influenced by the common learning variable denoted by $\{\theta\}$. $\{\theta\}$ is subject to convergence to a limiting value arrived at by a participatory process of learning across agents and systems of relations and their variables. The variables of $\mathbf{x}(\theta)$ are defined as follows below.

K denotes net capital flow over time. e denotes real effective exchange rate of the US-dollar, defined as the ratio of the normalized unit labor costs in the manufacturing sector measured in US-dollar to the weighted average of those of the industrial country trading partners of the U.S. i denotes an average of the short-term interest rates. M denotes the broad monetary aggregate of the developing countries. Y denotes the Gross Domestic Product. KAB denotes the balance-of-payments on capital account.

*Inter*relations among the variables occur as a result of multi-domain interactions. These constantly arise by human participation and increasingly expand by the emerging learning process. An inter-systemic idea of integrative interaction in the dynamic sense of systemic evolution is thus conveyed in the endogenous learning process *inter*relating θ with $\mathbf{x}(\theta)$.

A positive relationship between K and e and i could be due to the predominance of speculative capital flows, whereas long-term FDIs in the real sector investments would be negatively influenced by increasing e and i. Consequently, a negative relationship of i with K and e will increase the use of monetary balance and decrease the use of KAB in the balance-of-payments resolution.

The monetary balance (use of M) and capital accounts balance (KAB) substitute each other according to theory when the current accounts remain in deficit. This is shown by the negative relationship between these variables. But in the intertemporal case, since e and i can increase both M and KAB, therefore, the role of M and KAB can become complementary towards financing current accounts deficits.

The output Y is affected by multidimensional movements of e, K and M variables, depending upon the impact of K on Y in reference to the short-run and long-run relationship between i, e, M and KAB. Thus we cannot presume the results of the Mundell-Fleming theorems on e and i vis-à-vis the impact of K on monetary policy.

The measure of financial volatility in such an uncertain and unpredictable domain is influenced by the underlying nature of movements in the $\mathbf{x}(\theta)$-variables. The argument we are presenting on the normative side is that a volatile situation can be arrested not by recourse to a presumed acceptance of 'economic fundamentals' but by heuristic decision-making that simulates the underlying financial and economic relations through participatory discourse in the light of policy and market realities.

2. Critical definitions of indicators

We now define the volatility measure of net capital flows as,

$$\text{VOL (K)} = (d/dx)[\text{Var}(K(q,\mathbf{x}(q))] \tag{1}$$

$\mathbf{x}(\theta)$ denotes the complementing vector of variables influencing K and which excludes K. $x = x(\theta)$ is a variable of this vector.

Controllability of $\text{VOL}(K(\theta, \mathbf{x}(\theta)))$ is defined by,

$$\text{CVOL(K)} = \lim (\theta \to \theta^*) [\text{VOL}(K(\theta, \mathbf{x}(\theta))) - \text{VOL}^*] \tag{2}$$

where, $\text{VOL}^* = \text{VOL}(K^*(\theta^*, \mathbf{x}(\theta^*)))$.

Since θ^* and thereby $K^*(\theta^*)$ are both evolutionary in the learning sense of θ-values beyond certain limiting values of θ^*, therefore, expression (2) can be written as, d[VOL*]. From this expression we obtain,

$$(d/d\theta) [\text{VOL}^*] = [(d^2/d\theta.dx)[\text{Var}(K^*(\theta^*, \mathbf{x}(\theta^*)))]. \tag{3}$$

For the limit of controllability of volatility under conditions of complementary relations among variables and interactive and integrative institutional participation, expression (2) must remain bounded by $B(\theta^*)$. That is,

$$(d/dq) [\text{VOL}^*] = (d^2/d\theta.dx) [\text{Var}(K^*(\theta^*, x(\theta^*)))] \leq B(\theta^*,x(\theta^*)) \tag{4}$$

Expression (3) gives an evolutionary function with $B(\theta^*)$ responding to increasing values of $\{\theta^*\}$ as the institutional learning process advances. Such a participatory and complementary advance in systemic learning of real and capital market interaction causes $B(\theta^*)$ to remain stable. That is,

$$\text{Var}(K^*(\theta^*, x(\theta^*))) \leq \int_{\theta^*} \int_{x(\theta^*)} B(\theta^*, x(\theta^*)) dx(\theta^*).d\theta^* \tag{5}$$

Expression (5) means that the risk of net capital flow is bounded by the accumulated value of learning in the system through participation and complementarities among variables, their relations and the discoursing agents on controllability of financial volatility. This experience stabilizes $B(\theta^*, x(\theta^*))$, as $\{\theta^*\}$-values advance across evolutionary domains of learning through interaction and integration in the system-institutional discursive domain. In the special case when $B(\theta^*, x(\theta^*))$ becomes a given stable value, then,

$$\text{Var}(K^*(\theta^*, x(\theta^*))) \leq \int_{\theta^*} X(\theta^*) d\theta^* = f(\theta^*), \text{ where, } X(\theta^*) = \int_{x(\theta^*)} x(\theta^*) dx(\theta^*) \tag{6}$$

Expression (6) points out that the upper bound of variance depends intrinsically upon the learning parameters, $\{\theta^*\}$ in extensively evolutionary phases of interactive and integrative domains.

3. Circular Causation Regression Equations between Critical Variables

The attribute of circular *inter*relationship between the variables explains their endogenous complementarities. This system of circular *inter*relationships with q-induction (implied) is taken up in the formalism given below.

$k = f_1$ (e, i, m, y, z); $e = f_2$ (k, i, m, y, z); $i = f_3$ (e, k, m, y, z); $m = f_4$ (e, i, k, y, z);
$y = f_5$(e, i, m, k, z); $z = f_5$ (e, i, m, y, k). (7)

f's denote forms of the circular *inter*relations between the variables shown in the sense of knowledge production in the discursive system. k = percentage rate of change in K; m = percentage rate of change in M; y = percentage rate of change in output; z = percentage rate of change in KAB.

From the structural form of the above-mentioned relations we can write down our measure of financial volatility of k with respect to the other variables.

$$k = A_0 + A_1.e + A_2.i + A_3.m + A_4.y + A_5.z + u \quad (8)$$

$$Var(k) = A_1^2.Var(e) + A_2^2.Var(i) + A_3^2.Var(m) + A_4^2.Var(y) + A_4^2.Var(z) \quad (9)$$

$$VOL(k) = (d/dx)[Var(k)] = \Sigma_x (d/dx)[Var(x)] = \Sigma_x A_x^2 [\partial Var(x)/\partial i].di/dx, \quad (10)$$

This expression takes i and x to be interrelated in order to show the sensitivity of variations in i on the volatility measure generated by circular recursion.

x = {e, i, m, y, z}; the A's are certain regression coefficients of the reduced form of the circular *inter*relations. u is a random variable. The above expressions are to be combined with (1)-(6). Differentiation of Var(x) is taken over different states of the variable in the vector x.

IV. ILLUSTRATIVE EMPIRICAL RESULTS FOR CIRCULAR RECURSIVE REGRESSIONS

A shortened version of expressions (7) involving average annual U.S. rate growth of investment income = k, average annual rate of change in U.S. exchange rates = e, and average annual rate of change in interest rates = i, is estimated. The purpose here is to show what the recursive structural regressions equations imply in respect to circular interrelations between these highly volatile variables. The data are shown in Tables A.11, A.12 and A.20 of the United Nations publication (2002). The results of the circular regressions are detailed in tables at the end of this paper in an attached appendix.

1. Estimated Circular Structural Regression Equations are given below

k =	**-1.848**	**+ 6.468.i**	**+ 5.571.e**	(11)
t-statistics	(-1.848)	(1.115)	(1.279)	
levels of significance	(0.124)	(0.315)	(0.257)	
DW=1.935				
$R^2 = 0.256$				
F= 0.861				
i =	**0.144**	**+ 0.031.k**	**-0.599.e**	(12)
t-statistics	(2.310)	(1.115)	(-2.733)	
levels of significance	(0.069)	(0.315)	(0.041)	
DW=1.352				
$R^2 = 0.604$				
F= 3.814				

e =	**0.183**	**+ 0.044.k**	**-1.000.i**	(13)
t-statistics	(2.238)	(1.279)	(-2.733)	
levels of significance	(0.075)	(0.257)	(0.041)	

DW=1.682

$R^2 = 0.627$

F= 4.24

V. INTERPRETATION OF THE ESTIMATED REGRESSION EQUATIONS

The R^2-values, F and Durbin-Watson Statistics and the low levels of statistical significance in all the estimated equations imply that log-linear (that is the conversion into growth rates in the linear form) does not appropriately specify the volatile inter-relations between (k,e,i), in this case for U.S.A. This is expected to be the nature of the structural relations in the face of high volatility. The estimated results establish this nature of the circular recursive *inter*-relations.

We note also that the circular direction of the *inter*-relations between the dependent and independent variables interchanged sequentially gives different coefficients (relations) though the nature of the t-probability distribution keeps the levels of significance the same. In random fields the recursive *inter*-relations evolve with different effects on the economy as variables interact once a given direction of the relationship is initiated.

It is clear now from expression (9) that the estimated variances (V^(.)) of the variables are different from the computed variances (V(.)) in the data.

$$V(k) = 3.8029;\ V(e) = 0.0321;\ V(i) = 0.0603$$

$$V\hat{}(k) = 7.793;\ V\hat{}(e) = 0.164;\ V\hat{}(i) = 0.303$$

The implication here is that the variance differentials of the most of the variables are substantial than the actual, especially for k in the case of the US. Consequently, even small variations in the rate of interest are causing sensitive volatility in k, i and e.

Finally, expression (10) shows variations in the variables due to endogenous interrelationships in (k,e,i). This is matter of learning and policy revisions that affect subsequent rounds of interrelationships of the estimated types. But unless such policy and learning recursions are in place no recursive rounds of continued relationships of the type shown in the estimated forms can take place. This missing point in this empirical analysis is much more an institutional matter promoting firstly policy discourse followed by their market implementations affecting new values of (k,e,i). This part of the theoretical dispensation of this paper has profound implications in the estimation methods on volatility. We point out below that the problems of volatility addressed by institutional relations in learning and policy making and implementations as endogenous forces affecting changing patterns of (k,e,i) etc. cannot be taken up by the ARCH and GARCH methods.

VI. REVISIONS RELATING TO THE ARCH MODEL OF CAPITAL MARKET VOLATILITY (ROSSI, 1996)

Expressions (9) and (10) in conjunction with the circular causation interrelationships of the system of equations (7) show that a principal assumption of the ARCH model, namely respecting the independently and identically distributed residual variables of the volatility equations is violated. See Appendix.

A new formulation arises. We briefly examine this here. A problem arises particularly from the side of the residual variables, $\{Z_t\}$ in the relationships (Engle, 1982; Nelson, 1996),

$$\sigma_t^2 = \sigma^2(\xi_{t-1}, \xi_{t-2}, \ldots, t, \mathbf{x}_t, \mathbf{b}), \tag{14}$$

$$\text{with, } \xi_t = \sigma_t Z_t, \tag{15}$$

where, $Z_t \sim$ independently and identically distributed with $E(Z_t) = 0$, $E(Z_t^2) = 1$. (16)

σ_t^2 are the variances of the error term in the stock market regression equation.

σ^2 is a variance function of the bracket variables.

ξ_{t-i} are lagged prediction errors, i = 1,2,…

t is the time variable.

$\mathbf{x}_t$ is the vector of predetermined variables including exogenous and lagged endogenous variables.

b is the vector of parameters of the regression equation for estimating stock market volatility.

Z_t are normalized residual terms in the regression equation of the stock market volatility.

The requirement of independence and identical probability distribution, most often as lognormal distribution, causes cessation of interaction between the multi-equations and multi-variable consequences of $\{Z_t\}$. At the end, the stock market volatility equations turn out to be independently constructed, based only on the time sequence of changes in the residual variables within a given equation and not inter-equations. The complexity of policy basis is thus rendered benign in the estimation of the stock market volatility equations. The coefficients and predictability power of such estimated equations lose meaning.

In the case of the IIE character of the learning-induced mathematical formulation the expressions (14)-(16) will assume interactive features and the property of the independently and identically distributed probability distributions for the $\{Z_t\}$-variables will be replaced by the joint probability distribution of these variables. Consequently, expressions (14) – (16) would be replaced by,

$$\sigma_t^2 = \sigma^2(\xi_{t-1}, \xi_{t-2}, \ldots, t, \mathbf{x}_t, b)[q], \tag{17}$$

meaning that all variables are influenced by q-values

with, $$\xi_i = \sigma_i Z_i = fi(\sigma_1 Z_1, \ldots, \sigma_{i-1} Z_{i-1}, \sigma_{i+1} Z_{i+1}, \ldots, t, \mathbf{x}_t, b)[q] \tag{18}$$

$$x_i = gi(\sigma_1 Z_1, \ldots, \sigma_{i-1} Z_{i-1}, \sigma_{i-1} Z_{i-1}, \sigma_{i+1} Z_{i+1}, \ldots, t, x_1, x_2, ,, x_{i-1}, x_{i+1}, \ldots, b)\ [q] \tag{19}$$

i = 1,2,…..

The joint probability distribution of $\{Z_t\}$ is,

$$F(\mathbf{Z}_t) = F_1(Z_1).F_2(Z_2 \mid Z_1).F_3(Z_3 \mid Z_1 \cap Z_2)\ldots\ldots \tag{20}$$

where, Z_t are jointly distributed with probability distribution $F(\mathbf{Z}_t)$,

with $$E(\mathbf{Z}_t) = \Sigma_i \mathrm{Prob}(Z_i \mid Z_{i-1} \cap Z_{i-2} \cap \ldots \cap Z_1).Z_i,\ E(Z_t - E(Z_t))^2 = f(\sigma^2). \tag{21}$$

Since each of the variables is a function of the underling q-variable, which is institutionally sensitive through the IIE-process, and because $\{\theta_t\} \leftrightarrow \{\mathbf{Z}_t\}$ isomorphically (Maddox, 1871), therefore there is a similar conditional probability and variance relationship interrelating the various $\{\theta_t\}$-values corresponding to the $\{\mathbf{Z}_t\}$-variables as shown.

It is noted now that there exist no exogenous variable in the system (14)-(18). All variables are lagged endogenous ones interrelated to each other through the principle of complementarities in the IIE process. Such complementarities are not automatic. The market will not yield them by sheer market forces alone. Thus we note the central role of θ-values in guiding the market process according to a fundamental epistemology of unity of knowledge among the entities of diverse systems.

We have thus transformed the relations of the ARCH model of expression (14)-(16) (Engel, 1982) by the learning sensitivity of the system (17)-(20) in the framework of the learning-induced market-institution IIE process.

VII. CONCLUSION

A combination of the positivistic and normative factors in the construction of the volatility measure presents a model of polity-market interaction, integration (consensus) and simulation on the basis of the evolutionary knowledge flows that arise from discursive processes between polity, markets and policy variables. The anticipation of the future trends of global resources and their development effects, particularly the impact that financial flows can make on organizing sustainable regimes of economic growth and alleviation of poverty, is receiving serious attention by the International Monetary Fund (25 Feb. 2002) and the World Bank (2000). Consequently, a measure of financial-cum-economic volatility is a much-needed perspective of development planning for the future. This paper shows that such a new comprehensive measure of volatility and its controllability depends centrally on learning as systemic interaction, integration and dynamic evolutionary responses by institutions.

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STATISTICAL APPENDIX

Estimation of the Circular Causation Model System

Table 1
Regression k on i & e

	Model	*Coefficients*	*B*	*Standardized Coefficients* *Std. Error*	*t* *Beta*	*Sig.*	*Collinearity Statistics* *Tolerance*	*VIF*
1	(Constant)	-1.848	1.000		-1.848	.124		
	VAR00002	6.468	5.798	.612	1.115	.315	.494	2.023
	VAR00003	5.571	4.355	.702	1.279	.257	.494	2.023

a Dependent Variable: k
DW=1.935
$R^2 = 0.256$
F= 0.861

Table 2
Regression i on k & e

	Model	*Coefficients*	*B*	*Standardized Coefficients* *Std. Error*	*t* *Beta*	*Sig.*	*Collinearity Statistics* *Tolerance*	*VIF*
1	(Constant)	.144	.062		2.310	.069		
	VAR00001	3.081E-02	.028	.326	1.115	.315	.929	1.077
	VAR00003	-.599	.219	-.798	-2.733	.041	.929	1.077

a Dependent Variable: i
DW=1.352
$R^2 = 0.604$
F= 3.814

Table 3
Regression e on k & i

Model		*Coefficients* *B*	*Std. Error*	*Standardized Coefficients* *Beta*	*t*	*Sig.*	*Collinearity Statistics* *Tolerance*	*VIF*
1	(Constant)	.183	.082		2.238	.075		
	VAR00001	4.426E-02	.035	.351	1.279	.257	.987	1.013
	VAR00002	-1.000	.366	-.751	-2.733	.041	.987	1.013

a Dependent Variable: e
DW=1.682
$R^2 = 0.627$
F= 4.24

MODEL SELECTION

Model Selection Based on ARCH Approach of Information Criteria

Methods	*K depend*	*E depend*	*I depend*
Akaike info criterion	4.916817	0.353394	-0.144002
Schwarz criterion	4.946608	0.383185	-0.114211
Durbin-Watson stat	**2.013333**	**1.433944**	**1.041483**

Model Selection Based on GARCH Approach

Methods	**K depend**	**E depend**	**I depend**
Akaike info criterion	4.279697	0.404990	-0.557988
Schwarz criterion	4.319418	0.444711	-0.518268
Durbin-Watson stat	1.723493	1.184732	1.059224

Using model selection process we find that equation (11) is the most suitable for modeling this regression**.**

As mentioned in the text of the paper, the singular selection of equation (11) leads to dropping off the important results of circular causation of equations (12) and (13) along with equation (11). The measure of the normal distribution of the error term probability distribution required for the ARCH and GARCH estimation is thus a limitation in the estimation of the circular causation regression system.

Because of the limitation of the ARCH and GARCH methods in estimating the phenomenon of circular causation in learning-induced system (q-induced) presented in the text of the paper, we must now turn to causality tests for the different variables entered in the system of equations (11)–(13).

CAUSALITY TESTS

Pairwise Granger Causality Tests for lag 1 and lag 2

	Lags: 2		*Lags: 1*	
	F-Statistics	*Probability*	*F-Statistic*	*Probability*
Null Hypothesis:				
K does not Granger Cause E	8.75886	0.23238	1.04335	0.36479
E does not Granger Cause K	0.16624	0.86630	1.14366	0.34512
T does not Granger Cause E	0.14412	0.88105	0.72669	0.44199
E does not Granger Cause T	0.42513	0.73516	0.10134	0.76616
T does not Granger Cause K	13.1024	0.19172	3.77856	0.12383
K does not Granger Cause T	14.2042	0.18440	0.13632	0.73068