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A Comparative Study on Control Techniques of Non-square Matrix Distillation Column

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Abstract: The non-square matrix control is very much important in process industry as most of the plant transfer functions are non-square in nature. The controlling is challenging due to the complexity involved while solving for its performance and robustness. The distillation column transfer function with right half plane zeros is considered here for the comparative study in control techniques. The pseudo inverse of the steady state gain matrix is used here to solve the coupling problem of non-square process. Only few control techniques like Davison - Tanttu and Lieslethto method to control more manipulated and less number of control variables. The Integral Square Error (ISE) performance is calculated to compare the controller design methods, since the error is small and exist for long time. Also settling time, overshoot, interaction and robustness are studied.

Keywords*:* MIMO non-square system; pseudo inverse; centralized controller

1. INTRODUCTION

Majority of the separation work in the chemical industry, amounting to almost 95%, is carried out by distillation and these units consume 3% of the total energy produced in the globe [1]. The distillation column control is important for energy saving and to yield increased profit through improved product recovery. Designing several control methods for the distillation column poses a great challenge in the process control instrumentation field. There are various advantages and disadvantages for using different control methods in the process control industries [2]. In the last two decades, the controller design for high purity distillation column has seen several modifications. This is essential to achieve the desired product purity with minimum cost. A study of the literature shows that several attempts have been made to design a better controller taking into consideration different criteria.

In the chemical process industries the most commonly used systems are Multiple-Input-Multiple-Output (MIMO) systems with multiple time delays or right half plane zeros. The control objective for multivariable systems is to obtain a desirable behaviour of several output variables by simultaneously manipulating several input channels [2]. These systems are further classified into square and non-square systems based on the number of manipulated and controlled variables [3]. The process with equal number of manipulated and controlled variables are referred to as square systems, while those with unequal number of manipulated and controlled variables are non-square systems. Non-square systems are less sensitive to modelling errors, so it has to be controlled in its original form to obtain robust stability and performance [4]. A coupled distillation column by Levein and Morari was considered for the study. The genetic algorithm optimization technique was used here. The simulation results proved that in the presence of uncertainty, the two types of controller structures achieved robust performance [5]. Based on internal model control for non-square systems based on Smith delay compensator control technique also discussed

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in literature. Also decoupling internal model control techniques investigated in literature to test the stability and robustness of the multiple time delay system [6]. The problem of coupling of non-square processes was solved by the pseudo-inverse of the steady state gain matrix. The proposed method dynamically overcame the drawback caused by the static decoupling. Stability and performance of the system was achieved [7]. The controlling of such MIMO system is challenging task due interaction effect on process output. Thus, need to identify the control technique with lesser interaction and better performance such as overshoot, settling time etc.

2. DISTILLATION COLUMN

To purify the end products, chemical and fuel industries generally use distillation as a separation technology. However, the distillation columns present a whole array of control complications because they are the most frequently used process in these industries [1].

Consider a typical column as shown in Fig. 1, which is fed F moles/hr of a feed stream containing two major components. The column separates the feed into D moles/hr of overhead or distillate product, which contains most of the more volatile component and B moles/hr of bottom product, which contains most of the less volatile component. The main job of the control system is to adjust the split between distillate and bottom product by controlling the flow of one or the other of these streams. Many parts make up the distillation columns, and each of these either moves the heat energy or augments mass transfer. Normally, a distillation column is made up of a vertical column, wherein trays or plates are employed to improve the separation of the components; a reboiler positioned at the base of the column, which supplies the heat needed for vaporization; a condenser situated at the top of the column, which cools and condenses the vapor; and lastly, a reflux drum which saves the vapor so condensed, whereby it can be recycled to repeat the process all over again.

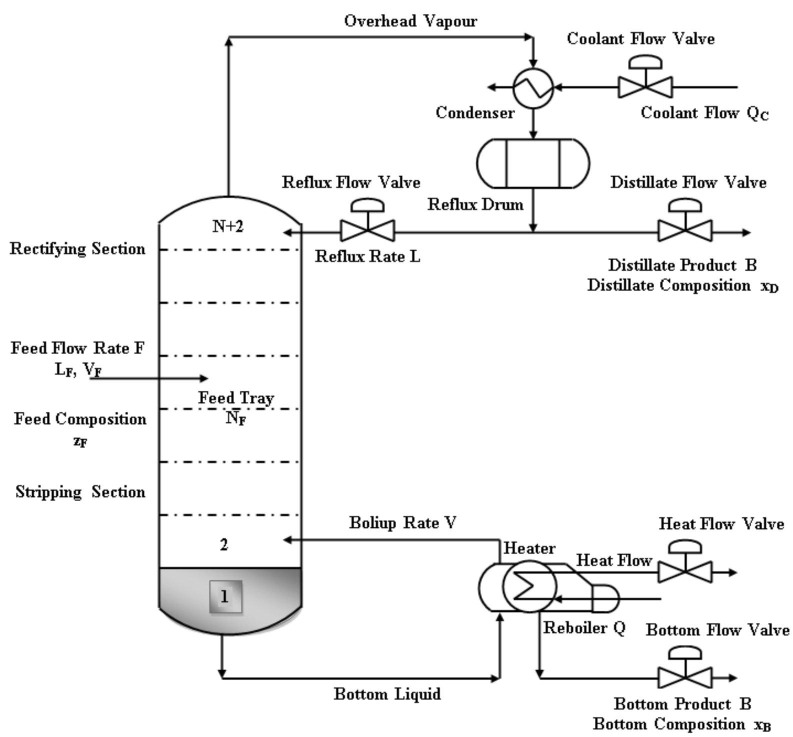

Figure 1: Schematic diagram of distillation column [1]

3. METHODOLOGY

The control block diagram of centralized PI (Proportional-Integral) controller applied to multivariable non-square system is shown in Fig. 2, where $G_p(s)$ is the transfer function of the plant with m inputs and n outputs, $G_c(s)$ is the transfer function of the controller with m outputs and n inputs where m > n. The MIMO transfer function matrix for $G_p(s)$ is considered as

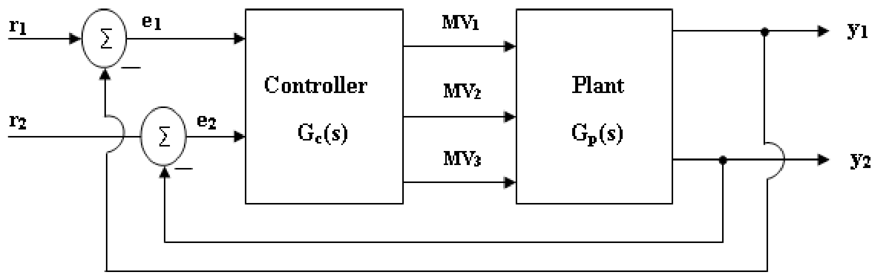

Figure 2: Closed loop multivariable control system [8, 9, 10]

$$G_{p,n\times m}(s)=\begin{bmatrix} G_{11} & G_{12} & \dots & \dots & G_{1m} \\ G_{21} & G_{22} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ G_{n1} & G_{n2} & \dots & \dots & G_{nm} \end{bmatrix} \tag{1}$$

Correspondingly, the controller matrix for the centralized PI controller is considered as

$$G_{c,m\times n}(s)=\begin{bmatrix} K_{11} & K_{12} & \dots & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ K_{m1} & K_{m2} & \dots & \dots & K_{mn} \end{bmatrix} \tag{2}$$

$$K_j = K_{cij} + \left(\frac{K_{Iij}}{s}\right) \tag{3}$$

$$K_{Iij} = \frac{K_{cij}}{\tau_{Iij}} \tag{4}$$

Where, in which K_{cij} is the controller gain and τ_{Iij} is the integral time of the PI controller.

3.1 Comparison criterion for the performance of controller

The performance of the controller on the process can be measure by Integral of Absolute Error (IAE), Integral Square Error (ISE) and Integral Time Absolute Error (ITAE). In MIMO systems, based on different criterion these performance evaluation measures have been considered [5]. The IAE should be used to suppress larger error, while ISE should be used to suppress small errors and error that last for long times, ITAE criterion should be used. In multivariable system, the selection of performance criterion is also based on minimization of both the response and the interaction.

$$IAE = \int_0^\infty \left| r(t) - y(t) \right| dt \tag{5}$$

$$ISE = \int_0^\infty \left[r(t) - y(t) \right]^2 dt \tag{6}$$

$$ITAE = \int_0^\infty t \left| r(t) - y(t) \right| dt \tag{7}$$

4. CONTROLLER DESIGN

The fully cross coupled multivariable controller is necessary for systems with significant interactions [11]. Since the interaction effect can be reduced along with smaller overshoot and faster response [7]. The centralized control system have n×m controllers. There are basically two classical model free tuning methods for centralized multivariable PID controllers. They are i) Davison method and ii) Tanttu and Lieslethto method. These methods are based on the transfer function matrix of the plant [4].

4.1 Division Method

Davison method of centralized multivariable PI controller tuning method is applied for non-square system. There is no inverse exist for non-square system. The Moore-Penrose pseudo inverse is used. For matrix A, it is given by

$$A^+ = A^H \left(A \times A^H \right)^{-1} \tag{8}$$

A^H is the Hermitian matrix of A. The PID controller gains for non-square system is given by

$$K_C = \delta \left[G(s=0) \right]^+ \tag{9}$$

$$K_I = \varepsilon \left[G(s=0) \right]^+ \tag{10}$$

Where $[G(s=0)]^+$ is called rough tuning matrix and fine tuning parameters ε and δ, usually range from 0 to 1.

4.2 Tanttu and Lieslehto Method

The tuning method for the design of PI controller is based on internal model control principles. For a first order time delay system

$$K_{cij} = \frac{\left(2\tau_{ij} + L_{ij} \right)}{2\lambda k_{ij}} \tag{11}$$

$$\tau_{Iij} = \tau_{ij} + 0.5 L_{ij} \tag{12}$$

$$K_{Iij} = \frac{K_{cij}}{\tau_{Iij}} \tag{13}$$

Where k_{ij} and L_{ij} are the gain and time delay of an element in the process model for the i^{th} output and j^{th} input. K_{cij} and τ_{Iij} are the proportional gain and integral time constant of the internal model controller of the ij^{th} loop.

Then the multivariable PID controllers are designed by taking the pseudo-inverse.

$$K_c = \left[\frac{1}{K_{cij}}\right]^{+} \tag{14}$$

$$K_I = \left[\frac{1}{K_{Iij}}\right]^{+} \tag{15}$$

Where K_{Iij} is the integral gain constant of the ijth loop.

5. SIMULATION STUDIES

To analyze the comparative study on the above control technique, we considered a coupled distillation column with 3-input and 2-output studies by Levein and Morari is described by [5],

$$G_p(s) = \begin{bmatrix} \frac{0.052e^{-8s}}{19.8s+1} & \frac{-0.03(1-15.8s))}{108s^2+63s+1} & \frac{0.012(1-47s)}{181s^2+29s+1} \\ \frac{0.0725}{890s^2+64s+1} & \frac{-0.0029(1-560s)}{293s^2+51s+1} & \frac{0.0078}{42.3s+1} \end{bmatrix} \tag{16}$$

Here the mole fraction of ethanol in distillate (y_1) and mole fraction of water in bottoms (y_2) are the controlled variables. The distillate flow rate (MV1), steam flow rate (MV2), product fraction from the side column (MV3) are the manipulated variables.

For the above plant transfer function model, steady-state gain matrix is given by:

$$G(0) = \begin{bmatrix} 0.052 & -0.03 & 0.012 \\ 0.0725 & -0.0029 & 0.0078 \end{bmatrix} \tag{17}$$

The centralized PI controller for the pseudo-inverse of the above matrix is calculated using the equation. The tuning parameter ε=0 to 0.1 and δ=0.7 to 1, at which the system is stable. Thus, the centralized PI controller matrix based on Davison method is obtained as:

$$G_C(s) = \begin{bmatrix} -2.0845 + \frac{-0.0625}{s} & 15.1612 + \frac{0.4548}{s} \\ -33.0046 + \frac{-0.9901}{s} & 23.9571 + \frac{0.7187}{s} \\ 7.1044 + \frac{0.213}{s} & -3.8095 + \frac{-0.1142}{s} \end{bmatrix} \tag{18}$$

Similarly, using Tanttu and Lieslehto method the PI controller is obtained as:

$$G_C(s) = \begin{bmatrix} 9.6833 + \frac{0.3145}{s} & 3.464 + \frac{0.1084}{s} \\ -3.1519 + \frac{-0.1221}{s} & 14.5772 + \frac{0.2666}{s} \\ 14.0520 + \frac{0.5107}{s} & 0.1198 + \frac{0.0390}{s} \end{bmatrix} \tag{19}$$

It is observed that, in both the control technique, the controller setting signs are different. (eq. (18) and (19)). The presence of zeros in the plant transfer function lead to changes sign in some of the individual controllers.

5.1 Simulation Results

The simulation study is carried out using Simulink. For a unit step change in r_1 Fig. 3 shows the response in y_1. Also, for a unit step change in r_2 Fig. 4 shows the response in y_2. The Fig. 5 and Fig. 6 show the interactive responses in y_2 or y_1. The settling time and ISE values are compared for both the methods. It is found that the Tanttu and Lieslehto control techniques need more time to settle compare to Davison method. Also Tanttu and Lieslehto method gives sluggish response and ISE values are 2-3 times as compared to Davison's method. The sum of the ISE values corresponding to the responses in y_1 and y_2 for the step change in r_1 and r_2 are given in Table 1. The sum of ISE for both perfect parameter and perturbed system is noted. It is observed that the Davison method gives ISE values closer to that of the perfect parameter system.

5.2 Robustness Studies for the Distillation Column

The robustness of distillation column is carried out by changing the individual element time constant by ±10%. Also by changing the system process gain by ±10% [12]. In both the case the same controller is used which is obtained for original process transfer function. The Davison's method is found to be more robust than Tanttu and Lieslehto method as tabulated in Table 2.

Table 1

ISE values for the distillation column using centralized PI controller

Technique	*Step Change in*	*ISE value of y1*	*ISE value of y2*	*Sum of ISE*
Davison	r1	32.19	12.38	44.57
	r2	3.136	10.26	13.4
Tanttu & Lieslehto	r1	32.99	45.27	78.26
	r2	0.385	52.53	52.92

Table 2

ISE values for robustness comparison for the distillation column

Technique	*Step change in*	*Sum of ISE for original plant*	*+10% change in process gain*	*+10% change in time constant*
Davison	r1	44.57	43.25	45.85
	r2	13.4	12.075	14.47
Tanttu & Lieslehto	r1	78.26	72.84	81.05
	r2	52.92	47.57	55.48

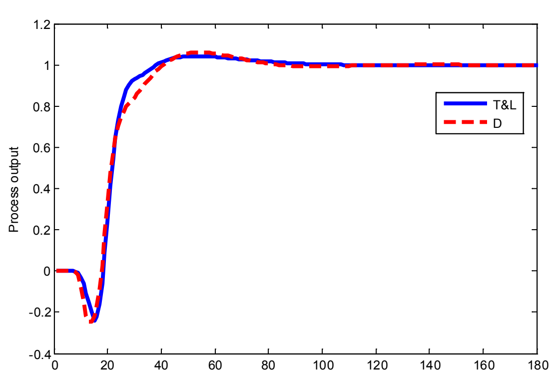

Figure 3: The unit step response of y1 for a step in r_1=1 and r_2=0

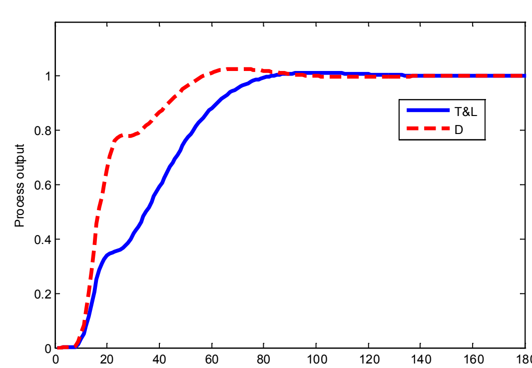

Figure 4: The unit step response of y2 for a step in r_1=0 when r_2=1

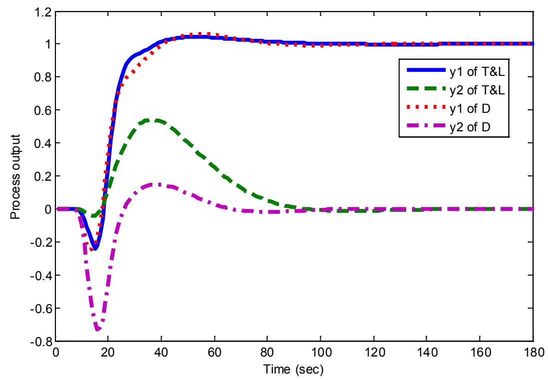

Figure 5: The interactive response for r_1=1 and r_2=0

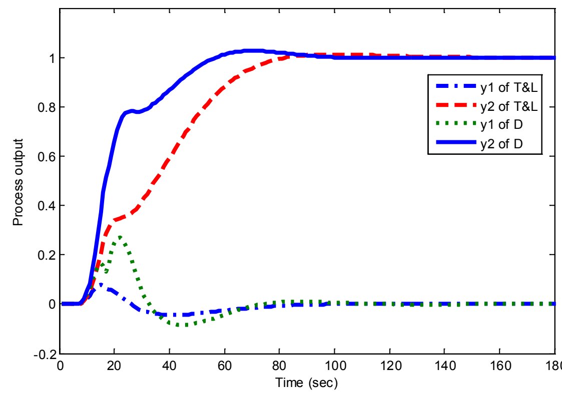

Figure 6: The interactive response for r_1=0 and r_2=1

6. CONCLUSIONS

The Moore-Penrose pseudo-inverse method is used to find the inverse of the non-square system. The Davison method is having the benefits of small overshoot, faster tracking features, and less interaction compared to Tanttu and Lieslethto method. Better robustness could be achieved when the process plant mismatched with the process transfer function model. The simulation result also showed better operational stability and performance than the conventional control strategy in the literature.

There is a sign change in the individual controller designed due to the zeros in the process transfer function. But the controller sign setting is different in both the method for the same process transfer function. Also the Davison method fails when the system has integral term. Also with load disturbance the response need to be controlled to attain robustness and performance. Further research required to be done in this direction.

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