New Advances in Physics, Vol. 13, No. 1, January 2019, pp. 27-31

Report: A delta function contribution in linearized gravity

B.G. Sidharth*
Abhishek Das**

Abstract: *In this paper, we innovate a delta function contribution in the metric of linearized gravity and investigate the situation. The motivation for this is that even though the linearized approach has been found to be useful in certain contexts, in the case of phenomenal events like the collision of massive objects, for example black holes, they leave a footprint which we can study.*

INTRODUCTION

It is very well known that the field equations of general relativity are practically impossible to be solved exactly, owing to the high degree of non-linearity and the number of variables. Only, very simple cases have been tackled which in turn have yielded important insights. For example, the Schwarzchild metric and it's solution for a single heavy particle. Another fruitful approximation has been that of the so called linearized gravity. On the other hand, the modern trend has been to go straight to numerical solutions using softwares. Here, as we will see below gravitation is treated as a perturbation of the metric to a much simple linearized metric [1]. This too has modelled some realistic situations. The linearized field equations are used primarily in the theory of gravitational radiation apposite to gravitational waves, where the gravitational field far from the source is approximated by these equations.

There is a plethora of literature on the topic of linearized gravity [2, 3, 6, 5, 4, 7], that deal with different aspects. However, in the current paper we consider a linearized situation where a delta function contribution is introduced in the perturbed metric. We will see that this indeed yields some very interesting results, pertinent to gravitational waves that were recently detected [8, 9, 10, 11].

*birlasc@gmail.com
**parbihtih3@gmail.com

MODIFIED LINEARIZED GRAVITY

In this section, we shall derive the modified form of the linearized Einstein field equations. As we know, linearized gravity is simply a perturbation theory around Minkowski spacetime, where the metric tensor is assumed to be given by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

where, $\eta_{\mu\nu}$ is the nondynamical background metric that is being perturbed and $h_{\mu\nu}$ represents the deviation of the original metric $g_{\mu\nu}$. Now, from this using the Ricci tensor $(R_{\mu\nu})$ and it's contracted form of Ricci scalar *(R)*, the equation of linearized gravity is derived as [1]

$$-\Box h_{\mu\nu} + h_{\nu\ ,\mu\alpha}^{\ \alpha} + h_{\mu\alpha\ \nu}^{\ ;\alpha} - h_{,\mu\nu} - h^{\alpha\beta}_{\ ,\alpha\beta}\eta_{\mu\nu} + \eta_{\mu\nu}\Box h = 16\pi T_{\mu\nu} \quad (1)$$

Now, considering the trace-reversed perturbation variable, dened as

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$$

and the Lorentz gauge given by

$$\partial_\mu \bar{h}_{\mu\nu} = 0$$

one gets from equation (1)

$$\Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu} \quad (2)$$

It must be borne in the mind that the Lorentz gauge is fundamental here, just as in case of electromagnetism. Now, we would like to modify the metric tensor $g_{\mu\nu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta(X) h_{\mu\nu}$$

where, X denotes the spatial and time coordinates. Now, the rationale behind this modification is that we want to investigate the effects introduced by the delta function which amplifies the perturbation $h_{\mu\nu}$. Resorting to the same methodology and the same Lorentz gauge, we will derive the modified version of equation (2) as

$$\Box[\delta(X)\bar{h}_{\mu\nu}] = -16\pi T_{\mu\nu} \quad (3)$$

From this we get

$$\Box\delta(X)\bar{h}_{\mu\nu} + \delta(X)\,\Box\bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}$$

Here, in the second term of the left hand side, we consider the $h_{\mu\nu}$ to embody the Newtonian potential [1], such that

$$\bar{h}_{\mu\nu} \approx \bar{h}_{00} \approx -4\Phi$$

where, Φ the Newtonian potential. Thus, equation (3) becomes

$$-4\Phi\,\Box\delta(X) + \delta(X)\,\Box\bar{h}_{\mu\nu} = -16\pi T_{\mu\nu} \tag{4}$$

Now, the solution for equation (2) is known as

$$\bar{h}_{\mu\nu} = \int \mathrm{d}^4 x T_{\mu\nu}(x) G(x - x')$$

where, the $G(x-x')$ is the retarded Green's function [12] generally taken into consideration, in order to obtain the solution. Thus, in this modified scenario, integrating equation (4) we obtain

$$-4\Phi\delta(X) + \bar{h}_{\mu\nu} = 4G\int \mathrm{d}^4 x \frac{T_{\mu\nu}(x)G(x-x')}{\delta(X)} \tag{5}$$

We choose the retarded Green's function as

$$G(x-x') = \frac{1}{4\pi(x-x')}\delta(X)[\delta\{|x-x'| - (t-t')\}v(t-t')]$$

to have

$$\frac{G(x-x')}{\delta(X)} = \frac{1}{4\pi(x-x')}[\delta\{|x-x'| - (t-t')\}v(t-t')]$$

Therefore, using this function in equation (5) and then performing an integration with the dt, one derives the following solution

$$\bar{h}_{\mu\nu} = 4G\int \mathrm{d}^3 x \frac{T_{\mu\nu}(t-|x-x'|, x)}{x-x'} + 4\Phi\delta(X) \tag{6}$$

As expected, we have obtained an extra term in the solution of $\bar{h}_{\mu\nu}$. Now, this extra term can consists of a delta function contribution. As it is known that a delta function has infinite contribution at the origin. So, when $X = 0$, i.e., where a gravitational wave is created the effects and intensity are maximum. This reduces with increasing time and distance.

DISCUSSIONS AND CONCLUSIONS

Essentially, we have used the fundamental property of a delta function to explain the nature of gravitational waves at their origin, resorting to a Newtonian like potential. Ostensibly, different results will be obtained by considering $\bar{h}_{00}$ to be different. Now, since gravitational waves transport energy as gravitational radiation - a form of radiant energy in near likeness with electromagnetic radiation, one cane calculate the extra contribution given in (6), by means of recent observational results [8, 9, 10, 11].

It would be worth investigating in future research whether such delta-function contributions can yield further novel insights apropos of gravitational waves or something new that has not yet been discovered.

So, the contribution of such violent and phenomenal events like collision of black holes is thrown up by the extra term in equation (6) comprised of a Newtonian potential and the delta function.

References

[1] Charles W. Misner, Kip S. Thorne and John Archibald Wheeler, Gravitation, W. H. Freeman and Company, 1973.

[2] Marc Henneaux and Claudio Teitelboim, Duality in linearized gravity, Phys. Rev. D 71, 024018, 2005.

[3] S. Aksteiner and L. Andersson, Linearized gravity and gauge conditions, Class. Quant.Grav. 28:065001, 2011.

[4] Christopher J. Fewster and David S. Hunt, Quantization of Linearized Gravity in Cos- mological Vacuum Spacetimes, Reviews in Mathematical Physics, Vol. 25 (2), 1330003 (2013).

[5] S. Aksteiner, L. Andersson, and T. Backdahl, New identities for linearized gravity on the kerr spacetime, Phys. Rev. D 99, 044043 (2019).

[6] John T. Giblin Jr et. al., The Limited Accuracy of Linearized Gravity, Phys. Rev. D 99, 023527 (2019).

[7] B. G. Sidharth, The Thermodynamic Universe, World Scientic, Singapore, ISBN-13: 978-9812812346, 2008.

[8] B. P. Abbott et al., Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116, 061102, 2016.

[9] B. P. Abbott et al., GW151226: Observation of Gravitational Waves from a 22 Solar Mass Binary Black Hole Coalescence, Phys. Rev. Lett. 116, 241103, 2016.

[10] B. P. Abbott et al., GW170104: Observation of a 50 Solar Mass Binary Black Hole Coalescence at Redshift 0.2, Phys. Rev. Lett. 118, 221101, 2017.

[11] B. P. Abbott et al., GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral, Phys. Rev. Lett. 119, 161101, 2017.

[12] Yi-Zen Chu and Glenn D. Starkmany, Retarded Green's Functions In Perturbed Space- times For Cosmology and Gravitational Physics, Phys. Rev. D 84, 124020, 2011.