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Submarine to Submarine Passive Target Tracking

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Abstract: In maritime environment, surveillance plays a major role. The aim of this paper is to track the target even though the range measurements are not available. Modified gain angles-only extended Kalman filter (MGAEKF) is used for bearing and elevation target tracking. The mathematical modeling and simulation have been carried out. It is shown that MGAEKF algorithm effectively tracks the target in underwater environment.

Keywords: Stochastic theory, statistical signal processing, applied statistics, estimation theory.

1. INTRODUCTION

In underwater, passive target tracking is generally followed to track a submarine target [1].The observer submarine is assumed to be standstill to reduce self-noise for tracking of the targets. In conventional submarines, bearings-only measurements are available and so ownship has to maneuver for observability of the process. But many times, ownship cannot carryout maneuvers due to tactical reasons [2-3].These days; submarines with sonars are coming up having the facility to obtain target elevation measurements. In this paper, research is towards submarine (observer) to track another submarine using elevation and bearing measurements. As angles-only measurements are available, the process is highly nonlinear and hence modified gain angles-only extended Kalman filter (MGAEKF), a nonlinear filter is explored for this application, as shown in Figure 1.[1-4].The contribution in this paper is utilization of elevation measurements in passive target tracking and hence ownship need not maneuver always. The estimated target range, course, bearing and speed are utilized in weapon guidance algorithm (which is not discussed here).

Section 2 deals with modeling of measurements, state vector and MGAEKF. Section 3 describes generalized simulator. Section 4 deals with results obtained for different scenarios in simulation. Finally, the paper is concluded in section 5.

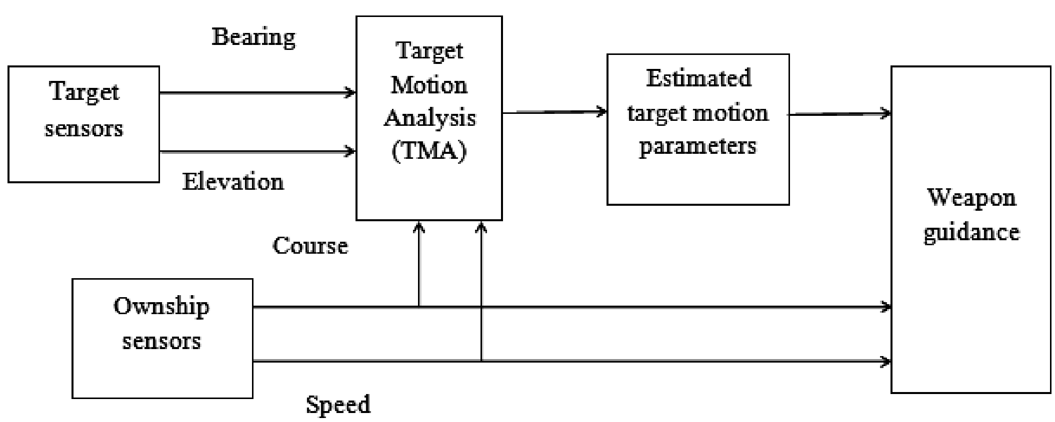

Figure 1: Block diagram of passive target tracking using bearing and elevation measurements

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2. MATHEMATICAL MODELLING

A. Measurements and State Vector

Let $X_s(k)$ be state vector and it is defined as

$$X_S = [\dot{x}(k)\dot{y}(k)\dot{z}(k)R_x(k)R_y(k)R_z(k)]^T \tag{1}$$

where, $\dot{x}(k)$, $\dot{y}(k)$, $\dot{z}(k)$, $\dot{z}(k)$, $R_x(k)$, $R_y(k)$ and $R_z(k)$ are velocity and range components in x, y and z directions respectively. The state equation is

$$X_s(k+1) = \phi(k+1/k)X_s(k) + b(k+1) + \omega(k) \tag{2}$$

where, $\omega(k)$ is noise having zero mean white Gaussian power spectral density. $f(k+1/k)$ is transient matrix and it is

$$\phi(k+1/k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ t & 0 & 0 & 1 & 0 & 0 \\ 0 & t & 0 & 0 & 1 & 0 \\ 0 & 0 & t & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where t is measurement interval. $b(k+1)$, deterministic vector, is

$$b(k+1) = [0 \quad 0 \quad 0 \quad -[x_0(k+1)+x_0(k)] - [y_0(k+1)+y_0(k)] - [z_0(k+1)+y_0(k)]]^T \tag{4}$$

where, $x_0(k)$, $y_0(k)$, $z_0(k)$ are position components of ownship.

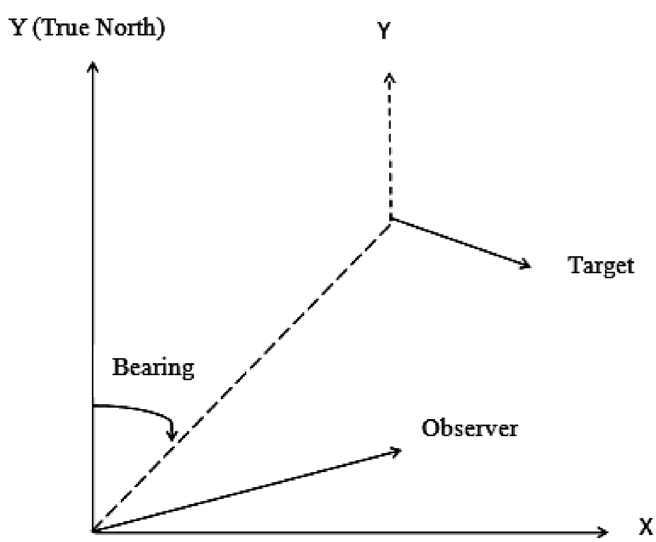

Figure 2: Target and observer Encounter

For clarity of concepts, the observer and target encounter in horizontal plane is shown in Figure 2. The observer is standstill and at the origin. The target is moving at a constant velocity. To reduce the mathematical complexity, all angles are measured with respect to True North. $Z(k)$ is measurement vector and it is

$$Z(k) = \begin{bmatrix} B_m(k) \\ \theta_m(k) \end{bmatrix}$$

where, $B_m(k)$ and $\theta_m(k)$ are bearing and elevation measurements respectively and are

$$B_m(k) = B(k) + \gamma(k) \tag{6}$$

$$\theta_m(k) = \theta(k) + \eta(k) \tag{7}$$

Actual bearing and elevation angles are $B(k)$ and $\theta(k)$ respectively and these are

$$B(k) = \tan^{-1}\left(\frac{R_x(k)}{R_y(k)}\right) \tag{8}$$

$$\theta(k) = \tan^{-1}\left(\frac{R_{xy}(k)}{R_z(k)}\right) \tag{9}$$

The noises $\eta(k)$ and $\gamma(k)$ are uncorrelated Gaussian noises. The measurement equation is written as

$$Z(k) = H(k)\, X_s(k) + \xi(k) \tag{10}$$

$$\text{where,}\quad H(k) = \begin{bmatrix} 0 & 0 & 0 & \dfrac{\cos(\hat{B}(k))}{\hat{R}_{xy}} & \dfrac{-\sin(\hat{B}(k))}{\hat{R}_{xy}} & 0 \\ 0 & 0 & 0 & \dfrac{\cos(\hat{\theta}(k)) \times \sin(\hat{B}(k))}{\hat{R}} & \dfrac{\cos(\hat{\theta}(k)) \times \cos(\hat{B}(k))}{\hat{R}} & \dfrac{-\sin(\hat{\theta}(k))}{\hat{R}} \end{bmatrix} \tag{11}$$

where, $\hat{B}(k), \hat{R}(k), \theta(k)$ are estimated bearing, range and elevation respectively.

B. MGAEKF Algorithm

[1-4] MGAEKF algorithm is given in Table 1.

Table 1
MGAEKF algorithm

1. To start with X(0/0) and P(0/0), initial state vector and its covariance matrix respectively are chosen.
2. Kalman gain is given as

$$K(k+1)/(k)) = P((k+1)/(k))\, h^T(k+1/k)(h((k+1)/k)\, P((k+1/k)h^T((k+1)/k) + r(k))^{-1} \tag{12}$$

   here, $r(k)$ is measurement covariance matrix.
3. Updated state vector is

$$X(k+1/k+1) = X(k+1/k) + K(k+1)(z(k+1) - h(k+1)\, X(k+1/k) \tag{13}$$

4. Updated state covariance matrix is

$$\begin{aligned} P(k+1/k+1) = {} & (I - K(k+1)g(z(k1), X(k+1/k))\, .\, P(k+1/k)\, (I - (k+1)g(z(k+1), X(k+1/k))^T \\ & + K(k+1)r(k)G(k+1)^T \end{aligned} \tag{14}$$

$$G(k) = \begin{bmatrix} 0\;0\;0 & \dfrac{\cos(\hat{B}(k))}{R_{xy}} & \dfrac{-\sin(\hat{B}(k))}{R_{xy}} & 0 \\ 0\;0\;0 & \dfrac{\cos(\varphi_m(k)) \times \sin\left(\dfrac{\hat{B}(k) + B_m(k)}{2}\right)}{\hat{R} \times \cos\left(\dfrac{B_m(k) - \hat{B}(k)}{2}\right)} & \dfrac{\cos\left(\dfrac{\hat{B}(k) + B_m(k)}{2}\right) \times \cos(\varphi_m(k))}{\hat{R} \times \cos\left(\dfrac{B_m(k) - \hat{B}(k)}{2}\right)} & \dfrac{-\sin(\varphi_m(k))}{\hat{R}} \end{bmatrix} \tag{15}$$

5. For next iteration

$$X(k/k) = X(k+1)/(k+1)) \tag{16}$$

$$P(k/k) = P(k+1/k+1)) \tag{17}$$

6. Predicted state vector is

$$X(k+1)/k) = \phi(k+1)/k)X(k/k) \tag{18}$$

7. Predicted state covariance matrix is

$$P(k+1)/k) = \phi(k+1/k)\,\phi^{T}(k+1/k) + Q(k+1/k) \tag{19}$$

Here $Q(k)$ is plant noise covariance matrix

Algorithm flow is shown in Figure 3. [4-10].

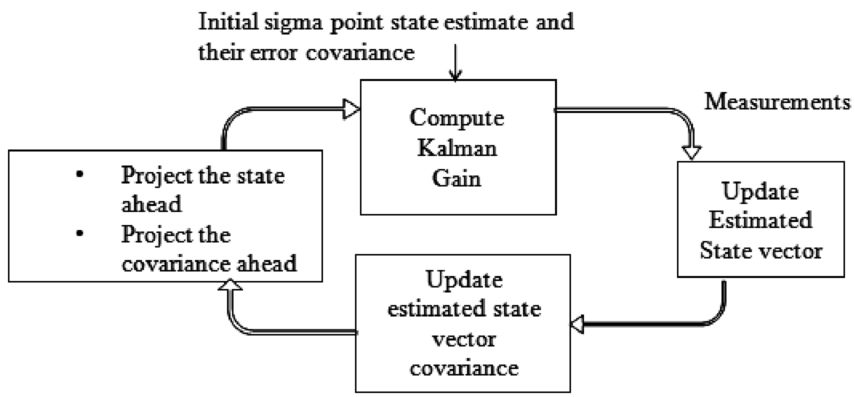

Figure 3: Modified gain angles-only extended Kalman filter

3. GENERALISED SIMULATOR

Let initial position of the target be (x_t, y_t, z_t) and the target moves with velocity v_t. After time t seconds, observer position changes. Change in the observer position is given by

$$dx_0 = v_0 \times \sin(ocr) \times \sin(oph) \times t \tag{20}$$

$$dy_0 = v_0 \times \cos(ocr) \times \sin(oph) \times t \tag{21}$$

$$dz_0 = v_0 \times \cos(oph) \times t \tag{22}$$

where *ocr* and *oph* are observer course and pitch respectively. Now the new observer position becomes

$$x_0 = x_0 + dx_0 \tag{23}$$

$$y_0 = y_0 + dy_0 \tag{24}$$

$$z_0 = z_0 + dz_0 \tag{25}$$

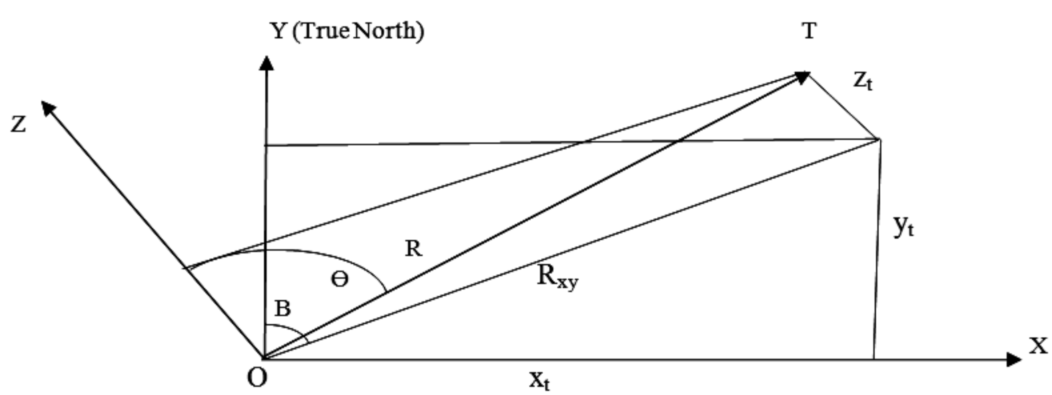

Figure 4: Target and Observer Positions

From Figure 4

$$x_t = R_{xy} \times \sin(B) \tag{26}$$

$$y_t = R_{xy} \times \cos(B) \tag{27}$$

$$\sin(\theta) = R_{xy}/R \tag{28}$$

Substituting equations (28) in (26) and (27)

$$x_t = R \times \sin(\theta) \times \sin(B) \tag{29}$$

$$y_t = R \times \sin(\theta) \times \cos(B) \tag{30}$$

$$z_t = R \times \cos(\theta) \tag{31}$$

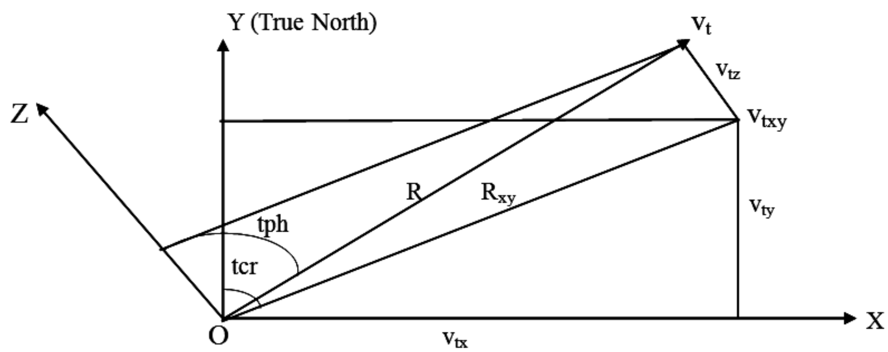

Figure 5: Target and Observer velocities

$$dx_t = v_t \times \sin(tcr) \times \sin(tph) \times t \tag{32}$$

$$dy_t = v_t \times \cos(tcr) \times \sin(tph) \times t \tag{33}$$

$$dz_t = v_t \times \cos(tph) \times t \tag{34}$$

where *tcr* and *tph* are target course and pitch respectively.

Now the new target position is

$$x_t = x_t + dx_t \tag{35}$$

$$y_t = y_t + dy_t \tag{36}$$

$$z_t = z_t + dz_t \tag{37}$$

Target true bearing, range and elevation are

$$\text{True bearing} = \tan^{-1}\left(\frac{x_t - x_0}{y_t - y_0}\right) \tag{38}$$

$$\text{True range} = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2 + (z_t - z_0)^2} \tag{39}$$

$$\text{True elevation} = \tan^{-1}\left(\frac{R_{xy}}{z_t - z_0}\right) \tag{40}$$

Since the measurements are affected by noise in real situations, noise is added to these measurements.

Measured bearing = true bearing + sigma *b*

Measured range = true range + sigma *r*

Measured elevation = true elevation + sigma *e*

where sigma *b*, sigma *r* and sigma *e* are 1σ values of white Gaussian process. The details are shown in Figure 5.

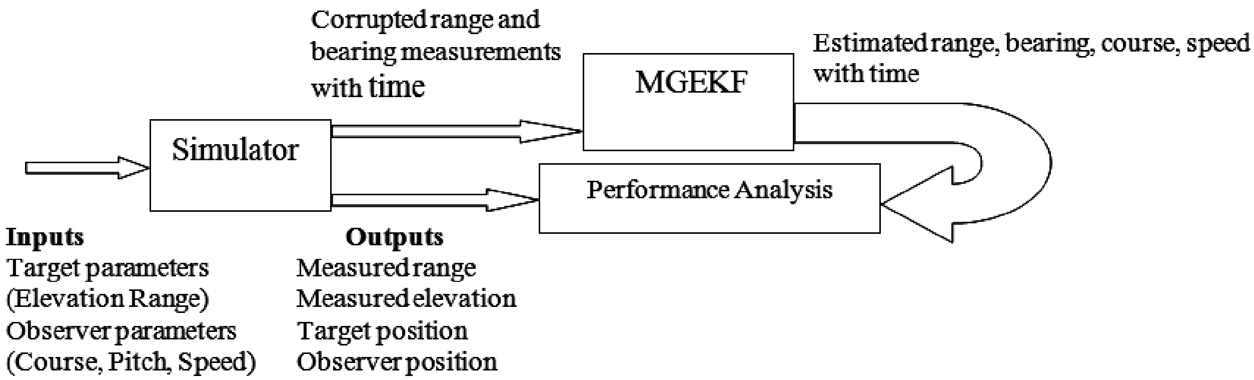

Figure 6: Block diagram of TMA in simulation mode

4. SIMULATION AND RESULTS

It is assumed that experiment is conducted at favorable environmental conditions and hence the angle measurements are available continuously. Simulation is realized on a personal computer using Matlab. The scenarios chosen for evaluation of algorithm are shown in Table 2. For example, scenario1 describes a target moving with bearing of 45° with course and speeds of 255° and 10 m/s respectively. The elevation angle is 45°. The bearing and elevation measurements are corrupted with 0.33° (1σ) and 0.33° (1σ) respectively.

In simulation mode, estimated and actual values are available and hence the validity of the solution based on certain acceptance criterion is possible. The following acceptance criterion is chosen. The solution is converged when error in course estimate <= 3° and error in speed estimate <= 5 m/s and range estimate <= 8%.

The errors in estimated range, speed and course for scenario 1 are represented in Figure 7, Figure 8 and Figure 9 respectively for clarity of the concepts.

The solution is converged when the course, speed and range are converged. The convergence time (seconds) for the scenarios is given in Table 3.

In simulation, it is observed that the estimated course of the target, speed and range of the target are converged at 66^{th} sample, 3rd sample and 97^{th} sample respectively for scenario 1. So, for scenario 1, the total solution is obtained at 97^{th} sample. Similarly for the other scenario the convergence time is shown in Table 3.

Table 2
Input parameters chosen for the algorithm

Scenario	*Initial range (m)*	*Bearing (deg)*	*Elevation (deg)*	*Pitch (deg)*	*Course (deg)*
Scenario 1	3000	45	135	135	255
Scenario 2	6000	45	135	135	255
Scenario 3	19000	350	45	45	320
Scenario 4	6000	45	135	135	275

Table 3
Convergence time in samples for the chosen scenarios

Scenario	*Course*	*Elevation*	*Range*	*Speed*	*Total solution*
1	66	3	97	3	97
2	90	3	77	290	290
3	250	3	3	169	250
4	65	3	33	209	209

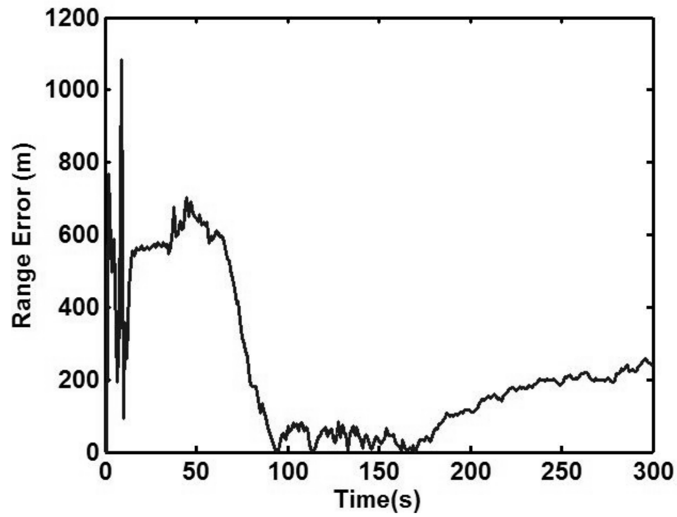

Figure 7: Error in range estimate

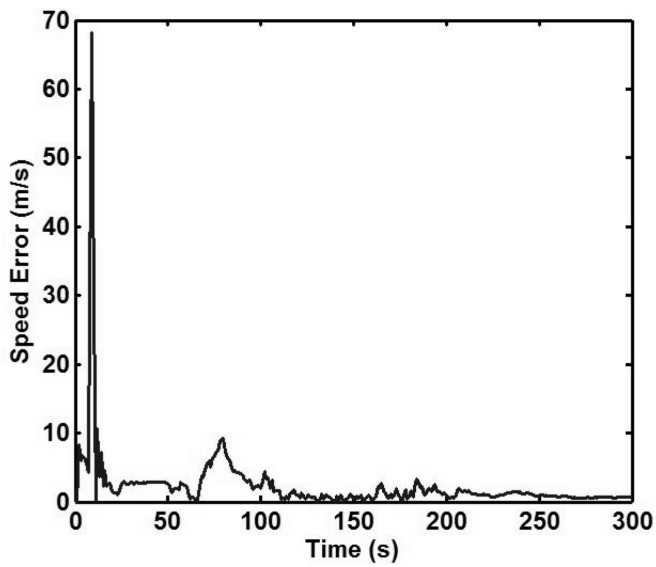

Figure 8: Error in speed estimate

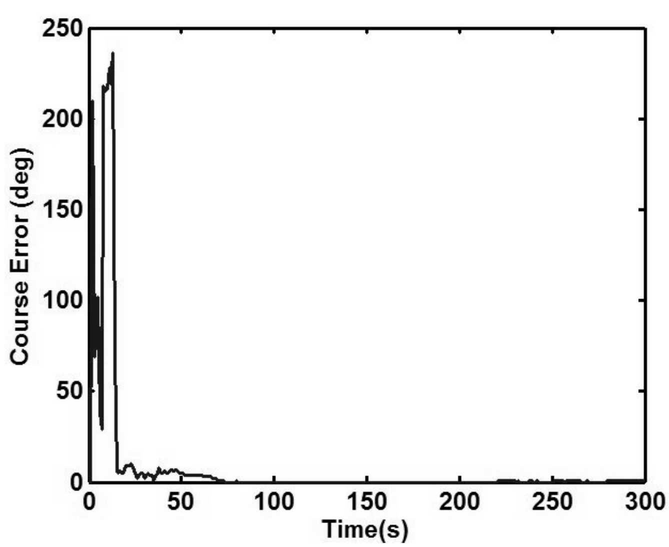

Figure 9: Error in course estimate

5. CONCLUSION

Based on these results, MGAEKF is recommended for passive target tracking and in particular, submarine to submarine scenario, when elevation measurements are also available along with bearing measurements.

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