ON γ-IRRESOLUTE FUNCTIONS

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ABSTRACT: *In the year 1987, D. Andrijevic et al. have introduced and studied the concepts of γ-sets (we call them as γ-open sets). Since then many authors have been utilized these sets to define various subsets, separation axioms and functions. In this paper, we introduce and study two new classes of functions called γ-irresolute functions and almost-irresolute functions using γ-open sets inbetween topological spaces.*

Keywords and Phrases: *γ-open sets, β-open sets, α-open sets, γ-interior, γ-closure, γ-irresoluteness*

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1. INTRODUCTION

In 1982, Mashhour *et al.* [12] introduced the notion of preopen sets, also called as locally dense sets by Corson and Michael [5]. The class of preopen sets properly contains the class of open sets. As the intersection of two preopen sets may fail to be preopen, the class of preopen sets does not always form a topology. In a submaximal space i.e. a topological space *X* in which every dense subset is open, collection of all preopen sets form a topology. Indeed, many notions in Topology can be defined in terms of preopen sets (see [4], [7], [8], [13] and [18]. Many researchers also used the notion of preopen sets in fuzzy topological spaces. Professor El-Naschie has recently shown in [8] the importance of the notion of fuzzy topology which may be relevent to quantam particle physics in connection with string theory and $\in^{\infty}$ theory.

Andrijevic [3] desined a new class of open sets called γ-open sets by utilizing preopen sets and proved that the family τ_γ of γ-open sets in a topological space contains the family τ_α of α-open sets. Recently, Abd El-Monsef *et al.* [1] have applied preopen sets in connection with the topological applications of rough set measures in information systems. In 1972, Crossley and Hildebrand [6] introduced the concept of irresolute functions in topological spaces. The class of functions was defined by Maheshwari and Thakur [14] in 1980 and in 2000, Yusuf [20] defined and investigated the almost α-irresolute functions. In this paper, we introduce the γ-irresolute functions and almost γ-irresolute functions and we show that a γ-irresolute function is α-irresolute only when the family of γ-open sets is a subset of the family of all semi open sets [11]. Moreover, we also study these functions comparing with other type of already known functions like, continuous functions, γ-irresolute, pre-irresolute and almost α-irresolute. It turns out that γ-irresoluteness implies almost -irresoluteness.

2. PRELIMINARIES

Throughout this paper, X and Y denote topological spaces (X, τ) and (Y, σ) respectively. The interior and closure of a subset A of X are denoted by $int(A)$ and $cl(A)$ respectively.

Definition 2.1: A subset A of a topological space X is said to be a

(i) α-open set [17] if $A \subset (int(cl(int(A)))$,

(ii). semi-open set [11] if $A \subset (cl(int(A))$,

(iii). pre-open set [12] if $A \subset (int(cl(A))$,

(iv). β-open set [1] if $A \subset (cl(int(cl(A)))$.

In a topological space (X, τ), the β-open sets were defined by Abd El-Monsef *et al.* [1] and Andrijevic [2] called the β-open sets as semi-pre open sets. Njastad [17] proved that the family of α-open sets is a topology. The family of α-open sets, semi-open sets, pre-open sets and β-open sets in X is denoted by $\alpha O(X)$, $SO(X)$, $PO(X)$ and $\beta O(X)$ respectively.

Theorem 2.2: If $X = \{\Pi X_\alpha : \alpha \in I\}$ is the product space, then for any positive integer n, $A = \Pi_{j=1}^{n} A\alpha_j \times \Pi_{\alpha \neq \alpha_j} X_\alpha$ is pre-open in X if and only if A_{α_j} is pre-open in X_{α_j} for each $j = 1, 2, 3, ..., n$ [7].

Definition 2.3: A subset A of a topological space X is said to be a γ-open set [3] if $A \cap S \in PO(X)$ for every $S \in PO(X)$.

We denote the family of γ-open sets in X by $\gamma O(X)$. The complement of a γ-open set in X is called a γ-closed set in X. Andrijevic [3] proved that $\gamma O(X) \subset PO(X) \subset (X)$ [10]. Andrijevic [3] proved that the family of γ-open sets is a topology on X such that $\alpha O(X) \subset \gamma O(X)$. For a topological space X, $\alpha O(X) = \gamma O(X)$ if and only if $\gamma O(X) \subset SO(X)$ [3].

For any subset A of X, $\gamma cl(A)$ [3] and $\gamma int(A)$ [3] stand for the closure of A and the interior of A in the topological space $(X, \gamma O(X))$. Hence for any subset A of X, $\gamma cl(A)$ is the intersection of all the γ-closed sets containing A and $int(A)$ is the union of all the γ-open sets contained in A. Therefore, a subset A of X is γ-closed if and only if $\gamma cl(A) = A$ and γ-open if and only if $\gamma int(A) = A$.

Definition 2.4: A function $f : X \to Y$ is said to be

(i) α-irresolute [14] if the inverse image of each α-open set of Y is a α-open set in X,

(ii) β-irresolute [15] if the inverse image of each β-open set of Y is a β-open set in X.

(iii) pre-irresolute [19] if the inverse image of each pre-open set of Y is a pre-open set in X.

Definition 2.5: A function $f : X \to Y$ is said to be almost α-irresolute [20] if the inverse image of each α-open set in Y is a β-open set in X.

Definition 2.6: A topological space X is said to be γ-Hausdorff [16] if for any pair of distinct points x, y of X there exist disjoint γ-open sets U and V in X such that $x \in U$ and $y \in V$.

3. IRRESOLUTE FUNCTIONS

In this section we introduce the γ-irresolute function and characterize it.

Definition 3.1: A function $f : X \to Y$ is said to be γ-irresolute if the inverse image of each γ-open set in Y is a γ-open set in X.

Remark 3.2: Continuous functions and γ-irresolute functions are independent to each other as shown in the following examples.

Example 3.3: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $Y = \{p, q, r\}$, $\sigma = \{\emptyset, \{p\}, Y\}$. Define a function $f : X \to Y$ by $f(a) = p$, $f(b) = q$, $f(c) = f(d) = r$. Then f is continuous but not γ-irresolute since $f^{-1}(\{p, r\}) = \{a, c, d\} \notin \gamma O(X)$.

Example 3.4: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $Y = \{p, q, r\}$, $\sigma = \{\emptyset, \{p\}, Y\}$. Define a function $f : X \to Y$ by $f(a) = f(b) = p$, $f(c) = q$ and $f(d) = r$. Then f is γ-irresolute but not continuous since $f^{-1}(\{p\}) = \{a, b\} \notin \tau$.

The following examples illustrate the independence of α-irresoluteness and γ-irresoluteness.

Example 3.5: Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $Y = \{p, q, r\}$, $\sigma = \{\emptyset, \{q, r\}, X\}$. Define a function $f : X \to Y$ by $f(a) = q$, $f(b) = p = f(d)$ and $f(c) = r$. Then f is γ-irresolute but not α-irresolute since $f^{-1}(\{q, r\}) = \{a, c\} \notin \alpha O(X)$.

Example 3.6: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $Y = \{p, q, r\}$, $\sigma = \{\emptyset, \{q, r\}, Y\}$. Define a function $f : X \to Y$ by $f(a) = q$, $f(b) = r$ and $f(c) = p$. Then f is α-irresolute. Since $f^{-1}(\{r\}) = \{\mathrm{b}\} \notin \gamma O(X)$, f is not γ-irresolute.

Theorem 3.7:

(i) A γ-irresolute function $f : X \to Y$ is α-irresolute if $\gamma O(X) \subset SO(X)$.

(ii) An α-irresolute function $f : X \to Y$ is γ-irresolute if $\gamma O(Y) \subset SO(Y)$.

Proof:

(i) Let $f : X \to Y$ be γ-irresolute and $\gamma O(X) \subset SO(X)$. Let $B \subset Y$ be a α-open set in Y. Then B is a γ-open set in Y and by our assumption, $f^{-1}(B)$ is a γ-open set in X, and $f^{-1}(B)$ is a γ-open set in X. Hence f is α-irresolute.

(ii) Let $f : X \to Y$ be α-irresolute and $\gamma O(Y) \subset SO(Y)$. Let $B \subset Y$ be a γ-open set in Y. Then B is a α-open set in X and it follows that f^{-1}(B) is a γ-open set in X. Hence f is γ-irresolute.

Remark 3.8: By Example 3.5 and Example 3.6, it is observed that α-irresoluteness and γ-irresoluteness are independent. However, it is not true if $f : X \to Y$ is a function such that in both the spaces X and Y the family of γ-open sets is contained in the family of semi-open sets.

The following examples illustrate the independence of pre-irresoluteness and γ-irresoluteness.

Example 3.9: Let $X = \{a, b, c\}$, $\tau = \emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $Y = \{p, q, r\}$, $\sigma = \{\emptyset, \{q, r\}, Y\}$. Define a function $f : X \to Y$ by $f(a) = q$, $f(b) = r$ and $f(c) = p$. Then f is γ-irresolute. Since $f^{-1}(\{p, q\}) = \{a, c\} \notin PO(X)$, f is not pre-irresolute.

Example 3.10: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $Y = \{p, q, r\}$, $\sigma = \{\emptyset, \{q, r\}, Y\}$. Define a function $f : Y \to X$ by $f(p) = c$, $f(r) = b$ and $f(q) = a$. Then f is pre-irresolute. Since $f^{-1}(\{a, c\}) = \{p, q\} \notin \gamma O(X)$, f is not γ-irresolute.

Theorem 3.11: If $f : X \to Y$ and $g : Y \to Z$ are γ-irresolute functions then $g \circ f$ is γ-irresolute.

Proof: Let $V \subset Z$ be a γ-open set in Z. Then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$. Since f and g are γ-irresolute, it follows that $f^{-1}(g^{-1}(V))$ is a γ-open set in X.

Theorem 3.12: Let $f : X \to Y$ be a function. Then the following are equivalent.

(i) f is γ-irresolute.

(ii) For each $x \in X$ and any γ-open set V of Y containing $f(x)$, there exists $U \in \gamma O(X)$ such that $x \in U$ and $f(U) \subset V$.

(iii) Inverse image of every γ-closed set is γ-closed in X.

Proof:

(i) $\to$ (ii): Let V be a γ-open set in Y containing $f(x)$. Since f is γ-irresolute, $f^{-1}(V)$ is a γ-open set in X and $x \in f^{-1}(V)$. Set $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subset V$.

(ii) $\to$ (i): Let V be a γ-open set in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. By (ii), there exists a γ-open set U_x in X such that $x \in U_x$ and $f(U_x) \subset (V)$. Therefore, $x \in U_x \subset f^{-1}(V)$.

This implies that $f^{-1}(V)$ is a union of γ-open sets in X. Hence $f^{-1}(V)$ is a γ-open set in X, f is γ-irresolute.

(iii) $\to$ (i): Let $V \subset Y$ be a γ-open set in Y. Then $Y \setminus V$ is a γ-closed set in Y and $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is γ-closed in X by (iii) and so $f^{-1}(V)$ is a γ-open set in X. Hence f is γ-irresolute.

(i) $\to$ (iii): It is followed in a similar manner.

Lemma 3.13: If $A \in \gamma O(X)$ and $B \in \gamma O(Y)$ then, $A \times B \in \gamma O(X \times Y)$.

Proof: $A \in \gamma O(X)$ implies that $A \in PO(X)$ and $A \cap U \in PO(X)$ for all $U \in PO(X)$. $B \in \gamma O(Y)$ implies that $B \in PO(Y)$ and $B \cap V \in PO(Y)$ for all $V \in PO(Y)$. $A \in PO(X)$ and $B \in PO(Y)$ implies that $A \times B \in PO(X \times Y)$ by Theorem 2.2 and $U \times V \in PO(X \times Y)$ whenever, $U \in PO(X)$ and $V \in PO(Y)$. Then $(A \times B) \cap (U \times V) = (A \cap U) \times (B \cap V) \in PO(X \times Y)$. Hence $A \times B \in \gamma O(X \times Y)$.

Let $f : X \to Y$ be a function. The subset $\{(x, f(x)) : x \in X\}$ of the product space $X \times Y$ is called the graph of f and is denoted by $G(f)$.

Theorem 3.14: If $f : X \to Y$ is γ-irresolute and Y is γ-Hausdorff then $G(f)$ is a γ-closed set of $X \times$ Y.

Proof: Let $(x, y) \in X \times Y \setminus G(f)$. Then $y \neq f(x)$. Since Y is γ-Hausdorff there exist disjoint γ-open sets U, V of Y such that $f(x) \in U$ and $y \in V$. By (ii) of Theorem 3.12, there exists a γ-open set W in X such that $x \in W$ and $f(W) \subset U$. Hence we get $(x, y) \in W \times V \subset X \times Y \setminus G(f)$. By Lemma 3.13, $W \times V \in \gamma O(X \times Y)$. Thus $X \times Y \setminus \mathrm{G}(f)$ is a union of γ-open sets of $X \times Y$, and so a γ-open set in $X \times Y$. Hence $G(f)$ is γ-closed in $X \times Y$.

Theorem 3.15: If f and g are two γ-irresolute functions from a space X into a γ-Hausdorff space Y then the set $A = \{x / f(x) = g(x)\}$ is a γ-closed set of X.

Proof: Let $y \in X \setminus A$. Then $f(y) \neq g(y)$. Since Y is γ-Hausdorff, there exist disjoint γ-open sets U and V of Y such that $f(y) \in U$ and $g(y) \in V$. Then $f^{-1}(U)$ and $f^{-1}(V)$ are γ-open sets in X. Set $B = f^{-1}(U) \setminus g^{-1}(V)$. It is clear that $B \in \gamma O(X)$ and $y \in B$ and $A \cap B = \emptyset$. Therefore, $y \in B \subset X \setminus A$ which shows that $X \setminus A$ is a union of γ-open sets of X. Hence A is γ-closed in X.

Lemma 3.16: Let A be a subset of X. Then $x \in \gamma cl(A)$ if and only if for any γ-open set U containing x, $A \cap U \neq \emptyset$.

Proof:

Necessity: Let $x \in \gamma cl(A)$ and let U be a γ-open set containging x such that $U \cap A = \emptyset$. But $X \setminus U$ is a γ-closed set containing A and hence $\gamma cl(A)$ $X \setminus U$. Since $x \in X \setminus$U, we get $x \notin \gamma cl(A)$, a contradiction.

Suciency: Suppose that for every γ-open set of X containing x meets A. If $x \notin \gamma cl(A)$ then there exists a γ-closed set F of X such that $A \subset F$ and $x \notin F$. Therefore, $x \in X\backslash F$ and $X\backslash F$ is a γ-closed set such that $X\backslash F\backslash A = \emptyset$, a contradiction.

Theorem 3.17: If f is γ-irresolute function of a γ-Hausdorff space X into itself then the set $A = \{x/f(x) = x\}$ is γ-closed.

Proof: Let $a \in \gamma cl(A)$. If $a \notin A$ then $f(a) \neq a$. Since X is γ-Hausdorff, there exist disjoint γ-open sets U, V such that $f(a) \in U$ and $a \in V$. The function f is γ-irresolute, and so $f^{-1}(U)$ is a γ-open set containing a. By the Lemma 3.16, $f^{-1}(U) \cap V \cap A \neq \emptyset$ which is a contradiction. Therefore, $a \in A$ and so $A = \gamma cl(A)$ and A is γ-closed.

4. ALMOST γ-IRRESOLUTE FUNCTIONS

Definition 4.1: A function $f : X \to Y$ is said to be almost γ-irresolute if $f^{-1}(V)$ is β-open in X for every γ-open set of Y.

Theorem 4.2: Every γ-irresolute function is almost γ-irresolute.

Proof: Let $V \subset Y$ be a γ-open set. Then $f^{-1}(V)$ is a γ-open set in X and is a β-open set in X since $\gamma O(X) \subset \beta O(X)$.

Example 4.3: The statement in Theorem 4.2 is not reversible.

Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $Y = \{p, q\}$, $\sigma = \{\emptyset, Y\}$. Define a function $f : X \to Y$ by $f(a) = q = f(c)$ and $f(b) = p$. Then f is almost γ-irresolute but not γ-irresolute since $f^{-1}(\{q\}) = \{a, c\} \notin \gamma O(X)$.

Theorem 4.4: Every γ-irresolute function is almost α-irresolute but the converse is not true.

Proof: Let $V \subset Y$ be a α-open set and so a γ-open open set. Since f is γ-irresolute, $f^{-1}(V)$ is a γ-open set in X and a β-open set in X. Hence f is almost α-irresolute.

Example 4.5: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $Y = \{a, b, c\}$, $\sigma = \{\emptyset, \{a\}, Y\}$. Define a function $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is almost γ-irresolute but not γ-irresolute since $f^{-1}(\{a, c\}) = \{a, c\} \notin \gamma O(X)$.

Theorem 4.6: Every almost γ-irresolute function is almost α-irresolute.

Proof: Follows from the fact that every α-open set is a γ-open set.

The next example shows that an almost α-irresolute function is not almost γ-irresolute.

Example 4.7: In Example 3.5, define a function $f : X \to Y$ by $f(a) = q$, $f(b) = p = f(c)$ and $f(d) = r$. Then f is almost α-irresolute but not almost γ-irresolute since $f^{-1}(\{r\}) = \{a, d\} \notin \beta O(X)$.

Theorem 4.8: The following are equivalent for a function $f: X \to Y$.

(i) f is almost γ-irresolute.

(ii) $f: (X, \tau) \to (Y, \gamma O(Y))$ is β-continuous.

(iii) For each $x \in X$ and each γ-open set V of Y containing $f(x)$, there exists a β-open set U of X containing x such that $f(U) \subset V$.

(iv) For every γ-open set V of Y, $f^{-1}(V) \subset cl(int(f^{-1}(V)))$.

(v) For every γ-closed set F, of Y $f^{-1}(F)$ is γ-closed in X.

(vi) For every subset B of Y, $int(cl(f^{-1}(B))) \subset f^{-1}(\gamma cl(B)$.

(vii) For every subset A of X, $f(int(cl(A))) \subset \gamma cl(f(A))$.

Proof:

(i) $\to$ (ii): Let $x \in X$ and V be any γ-open set of Y containing $f(x)$. By (i), $f^{-1}(V)$ is a β-open set in X and it contains x. Hence $f: (X, \tau) \to (Y, \gamma O(Y))$ is β-continuous.

(ii) $\to$ (iii): Let $x \in X$ and let V be any γ-open set of Y containing $f(x)$. Let $U = f^{-1}(V)$. Then by (ii), U is a β-open set in X containing x and $f(U) \subset V$.

(iii) $\to$ (iv): Let V be any γ-open set of Y and $x \in f^{-1}(V)$. By (iii), there exists a β-open set U of X containing x such that $f(U) \subset V$. We have $x \in U \subset (cl(int(cl(U))) \subset (cl(int(cl(f^{-1}(V))))$ and hence $f^{-1}(V) \subset cl(int(cl(f^{-1}(V))))$.

(iv) $\to$ (v): Let F be any γ-closed set of Y. Then $V = Y\backslash F$ is a γ-open set in Y. By (iv), $f^{-1}(V) \subset cl(int(cl(f^{-1}(V))))$ and hence $f^{-1}(F) = X\backslash f^{-1}(Y\backslash F) = X\backslash f^{-1}(V)$ is β-closed in X.

(v) $\to$ (vi): Let B be any subset of Y. Since $\gamma cl(B)$ is γ-closed in Y, $f^{-1}(cl(B))$ is β-closed in X and hence $int(\mathrm{cl}(int((\gamma cl(B)))) \subset f^{-1}(\gamma cl(B))$. Therefore, $int(cl(int(f^{-1}(B)))) \subset f^{-1}(cl(B))$.

(vi) $\to$ (vii): Let A be any subset of X. By (vi), $int(cl(int(A))) \subset int(cl(int(f^{-1}(f(A)))))$ $\subset \mathrm{f}^{-1}(\gamma cl(f(A)))$ and hence $f(int(cl(int(A)))) \subset \gamma cl(f(A))$.

(vii) $\to$ (i): Let V be any γ-open set in Y. Then $f^{-1}(Y\backslash V) = X\backslash f^{-1}(V)$ is a subset of X and so by (vii), $f(int(cl(int(f^{-1}(Y\backslash V))))) \subset (\gamma cl(f(f^{-1}(Y\backslash V))) \subset \gamma(cl(Y\backslash V) = Y\backslash \gamma int(V) = Y\backslash V$. Hence $X\backslash cl(int(cl(f^{-1}(V)))) = int(cl(int(\mathrm{X}\backslash f^{-1}(V))) = int(cl(int(f^{-1}(Y \backslash V)))) \subset f^{-1}(f(int(cl(int(f^{-1}(Y\backslash V))))) \subset f^{-1}(Y\backslash V) = X\backslash f^{-1}(V)$. Therefore, $f^{-1}(V) \subset cl(int(cl(f^{-1}(V))))$ which shows that $f^{-1}(V)$ is a β-set in X. Hence f is α-irresolute.

Theorem 4.9: A function $f: X \to Y$ is almost γ-irresolute if the graph function $g: X \to X \times Y$ defined by $g(x) = (x, f(x))$ for each $x \in X$, is almost γ-irresolute.

Proof: Let $x \in X$ and V be any γ-open set of Y containing $f(x)$. Then by Lemma 3.13, $X \times V$ is a γ-open set of $X \times Y$ containing $g(x)$. Since g is almost γ-irresolute, there exists

β-open set U of X containing x such that $g(U)$ $X \times V$. Then $g(x) = (x, f(x)) \in X \times V$ and hence $f(x) \in V$ implies that $f(U) \subset V$. Thus the function f is almost γ-irresolute.

Theorem 4.10: Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition $g \circ f$: $X \to Z$ is almost γ-irresolute if f and g satisfy one of the following conditions.

(i) f is almost γ-irresolute and g is γ-irresolute.

(ii) f is β-irresolute and g is almost γ-irresolute.

Proof: (i) Let V be a γ-open set in Z. Since g is γ-irresolute $g^{-1}(V)$ is γ-open in Y and $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is β-open in X since f is almost γ-irresolute.

(ii) Let V be a γ-open set in Z. Since g is almost γ-irresolute $g^{-1}(V)$ is β-open in Y and $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is β-open in X since f is β-irresolute.

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