IJCTA, 8(3), 2015, pp. 1217-1223

Near Optimal Trajectory Generation of an Ascent Phase Launch Vehicle with Minimum Control Effort

Dileep M V*, and Dr. Surekha Kamath

Abstract: The offline trajectory optimization is carried out for a single stage of a multi stage launch vehicle. The problem is solved using classical steepest descent method by considering the control parameter to be the angle of attack. The behavior of the system is studied, with and without minimum control including constraints. The process has been carried out, starting from mathematical formulation to the offline trajectory generation. The formulation entails nonlinear 2- dimensional launch vehicle flight dynamics with mixed boundary conditions and multiple constraints. The objective is to reduce the terminal error. The problem is stated as a fixed time boundary problem, since the burn duration of the liquid propellant engine is fixed. Numerical results are analyzed with effectiveness of the minimum control problem, and the optimal flight procedure and trajectory were obtained.

***Keywords**:* Trajectory optimization; Minimum control problem; Steepest Descent Method.

1. INTRODUCTION

Designing an ascent phase trajectory of a launch vehicle is a challenging task in space mission problems. To develop numerical algorithms for solving these types of problems is difficult. But in recent decades the researchers developed many numerical methods to solve these problems. Many of this work will concentrate to minimize the launch vehicle mass or operating cost [1]. To ensure this the selection of different propellants and engines in different stages are very important. For formulating staging optimization problem, ideal velocity consideration is important. But in ideal condition the velocity is not achievable due to the environmental force disturbances and there is no way to get the exact values of the velocity reduction occurs in the stage. To reduce these effects the offline trajectory optimization is carried out with separate stages.

Normal launch vehicle trajectory generation process includes two or more different phases to get the effectiveness. The primary phase is open loop guidance for most of the launch vehicle to cover the atmospheric region. Most of the researches are concentrating on this stage to avoid costly launch delay. From the second stage onwards it will be a closed loop control guidance [2]. The final stage is the most crucial stage because the vehicle is approaching the orbit and should satisfies the final orbital conditions more precisely. Most of the time the final injection accuracy decides the success or failure of the mission[3].

Numerical integration of differential equations and optimization are the two main parts of finding solutions of optimal control problems[4,5]. Betts [6] took a detailed survey on these methods and tried to highlight the merits and demerits of each methods. Mainly the numerical methods are divided in to two,

Manipal institute of technology, Manipal University, Manipal 576104, India
* e-mail: dileeppsla@gmail.com

direct and indirect methods. Indirect methods are widely used in early stages because of its high accuracy. But the trend got changed to direct method due to increasing size in higher dimensional problems in indirect method. The size gets doubled in indirect method by the addition of co-state variables.

Trajectory optimization is done for a hypersonic multistage launch vehicle to minimize the terminal error with some control as well as state boundary. The problem is solved with and without minimum control effort using steepest descent method [7]. This is one the standard methods to solve optimization problem based on Gradient methods. The cost function is developed in such a way that will minimize the terminal error as well as the control effort. The cost function has two parts, first one corresponds to the terminal penalty and the second one is for control minimization. In the existing literatures, the trajectory optimization problem is solved by incorporating multiple constraints [8][9][10].

2. MATHEMATICAL MODELING

2.1. Rocket equation of motion

Trajectory optimization of multistage launch vehicle is the one of the complicated optimal control problems. It is formulated by combining several nonlinear equations and various models such as aerodynamic and propulsion model with some path constraints [11,12]. Point-mass equations of motion is considered which represents the dynamic behavior of the launch vehicle. Inorder to simplify the system non-rotating spherical earth approximation is considered. The governing equations are given below:

$$\dot{r} = V \sin\gamma \tag{1}$$

$$\dot{V} = \frac{1}{mV}(T\cos\alpha - D - mg\sin\gamma) \tag{2}$$

$$\dot{\gamma} = \frac{1}{mV}(T\cos\alpha + L) + \left(\frac{V}{r} - \frac{g}{V}\right)\cos\gamma \tag{3}$$

$$\dot{x} = V\cos\gamma \tag{4}$$

Where r ,γ and v represents the altitude from earth center, the flightpath angle and velocity respectively. α ,T and m , is angle of attack, thrust and mass respectively, g is the gravitational of earth, x is the horizontal range. The lift and drag is given by,

$$\left.\begin{aligned} D &= \tfrac{1}{2}\rho S v^2 c_D \\ L &= \tfrac{1}{2}\rho S v^2 c_L \end{aligned}\right\} \tag{5}$$

To include the variation of the earth's gravity with altitude, it is represented as;

$$g = g_0\left(\frac{r_e}{r}\right)^2 \tag{6}$$

Where g_0 the gravity of earth is at sea level and r_e is earth radius.

The propulsion system used is liquid propellant system which produces constant thrust throughout the flight. The thrust produced by the thrusters can be calculated by,

$$T = \dot{m}v_e \tag{7}$$

v_e is the exhaust velocity and $\dot{m}$ is the mass flow rate.

A medium lift launch vehicle is considered. Two types of trajectory optimization is investigated: 1) trajectory optimization to minimize the terminal error, 2) Trajectory optimization using minimum control effort with and without control constraints. The comparison of the two approaches is carried out numerically.

The problem considered in this paper is to estimate the optimum angle of attack to transfer the launch vehicle from an initial condition to the terminal conditions with minimum terminal error. The terminal error [12] is minimized by considering the path constraints [13] throughout the trajectory. Most of the trajectory optimization problem is having high accuracy in some of the variables [14]. This problem can be formulated as a fixed time two point boundary value problem. The objective is to generate an angle of attack which satisfies the following. 1) At final time t_f, the terminal condition has to be achieved as accurately as possible. The terminal condition include final state variables. 2) The system should demand minimum guidance command, which can be ensured by formulating a minimum control problem. The structural load on the vehicle should be minimum, which can be obtained by ensuring minimum control variation in high dynamic pressure region.

To ensure the above objective the following objective function is selected. It has two parts; the first part is for terminal penalty. Terminal conditions can be met by proper tuning the weighting factor for the corresponding terminal term or can select priori articulation of preferences [15] the second part is to ensure minimum control effort. Selection of the weighting factor, S_X depends upon the control minimization. In this paper the selection of R, which is the weighting factor for the control variable is adaptively selected to improve the minimum control nature of the problem.

$$J = \sum S_X \left\| X_f - X_{t_f} \right\|_2^2 + \int_{t_0}^{t_f} Rf(u)dt \tag{8}$$

The objective function X represents the state variables and u represents the control variable. X_f and X_{t_f} represents the desired and actual terminal state variables respectively. The main state variable constraints considered in this paper [16,17] is given below.

$$V_{\min} \le V \le V_{\max} \tag{9}$$

$$\gamma_{\min} \le \gamma \le \gamma_{\max} \tag{10}$$

$$\dot{m} = -m_c \tag{11}$$

Where, $V_{\min}$ and $V_{\max}$ are the velocity boundaries, $\gamma_{\min}$ and $\gamma_{\max}$ are the flightpath angle boundaries, m_c is propellant consumption in unit time.

3. ALGORITHM DEVELOPMENT

The trajectory optimization problem is solved by the classical steepest descent method. The optimal control problem can be formulated as follows [18]: consider $X(t) \in R^n$ and $U(t) \in R^m$, the state variable and the control variable respectively in order the following objective functions.

$$J = \phi[X(t_0), t_0, X(t_f), t_f] + \int_{t_0}^{t_f} L(X(t), U(t), t)dt \tag{12}$$

Subjected to the state differential equations constraints; $\dot{X}(t) = f(X(t), U(t), t) \qquad t \in [t_0, t_f]$

The initial conditions constraints; $\varphi[X(t_0), t_0] = 0$

And the unified form of the equality and inequality path constraints; $C(X(t_0), U(t), t) \le 0$

The terminal time constraints; $\varphi[x(t_f)] = 0$

Thus the Hamiltonian is given by,

$$H = L + \lambda^T (f + \mu C) = 0 \tag{13}$$

Necessary and sufficient conditions of optimality is given by,

$$\frac{\partial H}{\partial U} = 0 \tag{14}$$

$$\dot{\lambda} = -H_x ; \lambda(t_f) = \phi_x \big|_{t_f} \tag{15}$$

4. STEEPEST DESCENT METHOD

The problem is solved using Steepest descent method [12] which is a gradient method. Several steps are involved in this process [4] and it is summarized below.

1) Guess the initial steering control $\alpha^0(t)$ where $t_0 \le t \le t_f$ using
2) The system states is calculated by integrating the dynamic equations from t_0 to t_f. For this the initial control variable is used.
3) Find the values of co-state vectors from t_f to t_0 from initial values of co-state variables. Initial values for co-state variable, $\lambda(t_f)$ can be find out using terminal boundary conditions.
4) Calculate the gradient, $\frac{\partial H}{\partial \alpha}$ from t_0 to $t_f - \Delta t$
5) Control parameter can be updated using $\alpha^{(k+1)}(t) = \alpha^k(t) - \tau \frac{\partial H}{\partial \alpha}$. Where, $\tau \in (0,1)$ is the learning rate and k is the iteration number.
6) Step (2) to step (4) will repeat till the objectives are achieved within a specified tolerance level.

All integration methods can be done by Euler's method. To update control variable the weighting factor τ is adaptively changed with each iteration. This also improves the performance of the problem. Since this an indirect method it needs an initial guess for control variable to start the optimization process. In order to find the feasible steering angle profile, acceleration of the vehicle is assumed to be a linearly increasing quantity.

5. RESULT AND DISCUSSIONS

In this section, the Matlab simulation results are explained. Optimization is done for two cases. The first one is optimization of the trajectory with and without minimum control constraint. The second is by considering minimum control and control constraint.

The launch vehicle parameter are given. Initial mass of the launch vehicle is 5200 *kg*. Total time of flight is 510 seconds. Since a constant thrust vehicle is considered, a constant propellant flow rate is achieved and is 4.881, which can produce a constant thrust of 14599 *N*. The upper and lower limits of control variables are given by, $V_{\min} = 5800 m/s$, $V_{\max} = 7700 m/s$, $\gamma_{\min} = -6\deg$, $\gamma_{\max} = 6\deg$. The boundary conditions of the state variables are given in table 1.

Table 1

State of the vehicle during ascent phase

Initial conditions		*Terminal conditions*	
Parameters	*Value*	*Parameters*	*Value*
Altitude	6805.8545 km	Altitude	6862.4157 m
Velocity	5890.1127 m/s	velocity	7623.5321 m/s
Flightpath angle	5.2229 ^{0}	Flightpath angle	0^0

5.1. Solving problem without minimum control

In this section, the simulation results obtained is explained. The trajectory optimization problem is solved by using steepest descent method with and without minimum control effort. Some control boundaries are also considered in this problem and analyzed the effects on the behavior of the system. The structural load is maintained within limits by forcing angle of attack to a maximum allowable value at that instant. In this no control constraints are imposed. So the angle of attack varies over a wide range. So the angle of attack varies over a wide range. The launch vehicle first reaches an altitude more than the desired value and then achieves the target altitude by changing the angle of attack.

5.2. Solving problem with minimum control

In this section the problem is solved by considering minimum control effort. The control parameter should vary at a minimum rate. To ensure the minimum control a new function other than terminal penalty function is introduced. The below results shows the comparison between trajectory optimization problem solved with and without minimum control effort.

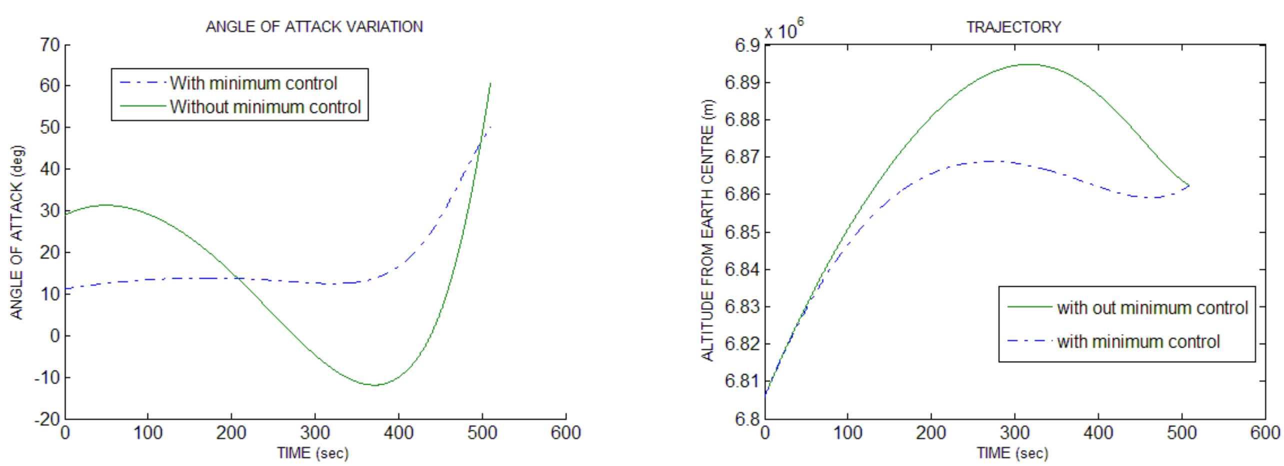

Figure 1: (a) Control parameter variation with time, 1(b) Altitude variation with time

Figure (1) and (2) are the graphical representation of both control and state variables with time. It is observed that the vehicle maneuvering is less in minimum control problem. The vehicle takes higher altitude in the case of non-minimum control problem and with minimum control it is comparatively less. The control parameter variation is limited in minimum control problem so it can be used in high atmospheric density regions. This is method has an added advantage that it will help in smooth functioning to the actuators as compared to the other.

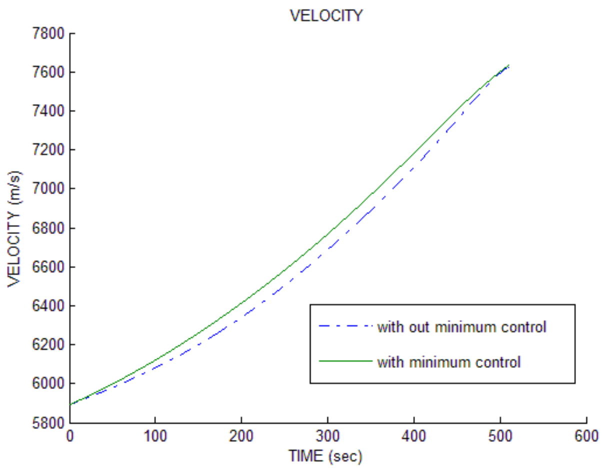

Figure 2 (a): Velocity variation with time

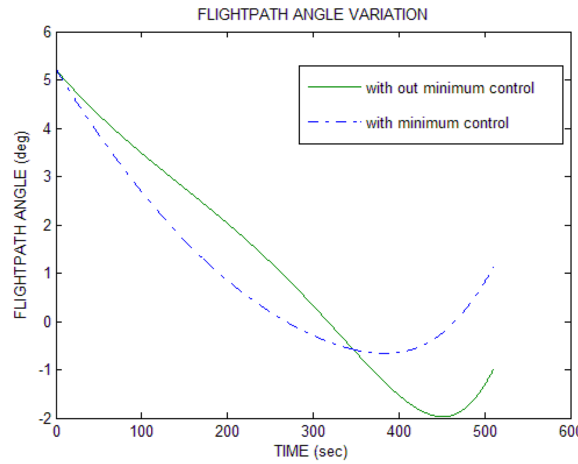

Figure 2(b): Flightpath angle variation with time

Figure. 2(a), 2(b) shows the velocity and flightpath angle variation. It has been observed from the above figures that the launch vehicle achieves the target within acceptable tolerance level. Table. 2 shows the error occurred while achieving the terminal conditions. From the table it is clear that the accuracy is more in case (1) ie, without minimum control. For practical application the control variation should be minimum.

Table 2

Terminal condition error for single stage launch vehicle

Methods	*Error in altitude*	*Error in velocity*	*Error in flightpath angle*
Without minimum control	–0.02216	0.00747	–0.9912
With minimum control	–0.0373	8.0565	1.1688

6. CONCLUSION

The offline trajectory optimization for a single stage of a multistage launch vehicle is presented. Classical steepest descent method, considering angle of attack as a control variable is used for solving the optimization problem. Nonlinear 2- dimensional launch vehicle flight dynamics with mixed boundary conditions and multiple constraints is taken for the study. Numerical results are analyzed with and with applying minimum control problem and the optimal flight procedure and trajectory were obtained. In minimum control problem the control variation limits are less. Future research will include the application of various combinations of evolutionary algorithms.

References

[1] Reza Jamilnia, Abolghasem Naghash. “Simultaneous optimization of staging and trajectory of launch vehicles using two different approaches” *Aerospace Science and Technology* 23 (2012) 85–92.

[2] Leung, M. S. K., and Calise, A. J., "Hybrid Approach to Near-Optimal Launch Vehicle Guidance," *Journal of Guidance, Control, and Dynamics,* Vol. 17, No. 5, 1994, pp. 881-888.

[3] Michael Dellnitz, Sina Ober-Blöbaum, Marcus Post, Oliver Schütze, Bianca Thiere, "A multi-objective approach to the design of low thrust space trajectories using optimal control" *Celest Mech Dyn Astr* (2009) 105:33–59 Springer Science+Business Media B.V. 2009.

[4] Bruce A. Conway―A Survey of Methods Available for the Numerical Optimization of Continuous Dynamic Systems‖ *Journal Optimum Theory Application* (2012) 152:271–306.

[5] Huang G Q, Lu Y P, Nan Y. "A survey of numerical algorithms for trajectory optimization of flight vehicles" *Science China Tech Sci, 2012, 55:2538-2560*, doi: 10.1007/s11431-012-4946-y.

[6] John T. Betts "Survey of Numerical Methods for Trajectory Optimization" *Journal of Guidance, Control, and Dynamics* Vol. 21, No. 2, AIAA 1998.

[7] Lukkana Varaprasad, Radhakant Padhi "Ascent Phase Trajectory Optimization of a Generic Launch Vehicle" *Xxxii National Systems Conference,* IIT-Roorkee. *NSC* 2008.

[8] Shen H X, Zhou J P, Peng Q B, et al. "Multi-objective interplanetary trajectory optimization combining low-thrust propulsion and gravity-assist maneuvers" *Science China Tech Sci*, 2012, 55: 841847, doi: 10.1007/s11431-011-4705-5

[9] James D. Thorne, Christopher D. Hall, "Approximate Initial Lagrange Costates for Continuous-Thrust Spacecraft" *Journal of Guidance, Control, and Dynamics* Vol. 19, No. 2, March-April 1996.

[10] Li Huifeng, Zhang Ran, Li Zhaoying, Zhang Rui, "New Method to Enforce Inequality Constraints of Entry Trajectories" *Journal of Guidance, Control, and Dynamics* Vol. 35, No. 5, September–October 2012.

[11] C.-H. Chuang, Hitoshi Morimoto, "Periodic Optimal Cruise for a Hypersonic Vehicle with Constraints*" Journal of Spacecraft and Rockets* Vol. 34, No. 2, March–April 1997.

[12] B. Sudhir Kumar1 and Dr. Ashok Joshi, "Effect of initial flight path angle error and control constraint on the optimized ascent trajectory of a typical launch vehicle", Proceeding of 19th AIAA ISPH ST Conference. 16-20 June 2014, Atlanta, GA.

[13] Marco Antonio Leonel Caetano, Takashi Yoneyama, "New Iterative Method to Solve Optimal Control Problems with Terminal Constraints" *J. Guidance*, Vol. 19, No. 1(1996), Engineering Notes.

[14] Hui Yan, Qi Gong, Christopher N. D'Souza, "High-Accuracy Trajectory Optimization for a Trans-Earth Lunar Mission" *Journal of Guidance, Control, and Dynamics* Vol. 34, No. 4, July–August 2011.

[15] R. Timothy Marler, Jasbir S. Arora "Survey of multi-objective optimization methods for engineering" *Structural and Multi-disciplinary Optimization* 26, 369–395 (2004).

[16] I. Michael Ross, "Extremal Angle of Attack over a Singular Thrust Arc in Rocket Flight" *Journal of Guidance, Control, and Dynamics* Vol. 20, No. 2, March–April 1997.

[17] Zuu-Chang Hong, Fu-Chi Hsu, Jeng-Shing Chern "Vertical Ascent to Geosynchronous Orbit with Constrained Thrust Angle" *Journal of Spacecraft and Rockets* Vol. 37, No. 1, January–February 2000.

[18] Martin S. K. Leung, Anthony J. Calise, "Hybrid Approach to Near-Optimal Launch Vehicle Guidance" *Journal of Guidance, Control, and Dynamics* Vol. 17, No. 5, September-October 1994.