

T_w -Fuzzy VIKOR Technique and its Applications

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Abstract : The mechanism of Multi Criteria Decision Making (MCDM) techniques are used to deal with the complex decision making problems, in which fuzzy arithmetic operations play a vital role for analyzing fuzzy matrices. Most of the researchers use α -cut arithmetic operations for constructing the fuzzy models. However, using α -cut arithmetic operations one can realize that the fuzziness of the model makes the calculation complicated due to the accumulating phenomenon of fuzziness. Hence, this paper proposes a novel idea of using the weakest t -norm (T_w) for VIKOR technique. And this technique is illustrated with a real life situation.

1. INTRODUCTION

Multiple Criteria/Attribute Decision Making (MCDM) models are often used to solve various decision making and selection problems. There are well known methods for solving MCDM problems such as Analytical Hierarchy Process (AHP), DEMATEL, TOPSIS, VIKOR, etc. VIKOR (Serbian: Visekriterijumsa Optimizacija I Kompromisno Resenje means: Multi criteria Optimization and compromise solution) is a compromise ranking method initiated by Opricovic (Opricovic., & Tzeng 2007). The method establishes a compromise ranking list, a compromise solution and the weight stability intervals for the compromise solution. Chang C.L proposed a modified VIKOR method to solve multiple criteria decision making (MCDM) problems with contradicting and non-commensurable criteria (Chang, C.L.,2010). Moeinzadeh and Hajfathaliha presented a supply chain risk assessment approach based on the analytic network process (ANP) and VIKOR method under the fuzzy environment where vagueness and subjectivity were handled with linguistic terms parameterized by triangular fuzzy numbers (Moeinzadeh, & Hajfathaliha.,, 2010). Sanayei *et al.* introduced a group decision making process for supplier selection with the VIKOR method under a fuzzy environment (Sanayei, A., et.al, 2010). Kuo and Liang proposed an effective approach by combining VIKOR with GRA techniques for evaluating the service quality of Northeast-Asian international airports by conducting customer surveys under fuzzy environment (Kuo & Liang, 2011). Wan *et al.* developed an extended VIKOR method for multi-attribute group decision making with triangular intuitionistic fuzzy numbers (Wan, SP et.al., 2013). Chang, TH applied fuzzy VIKOR method for the evaluation of hospital service quality in Taiwan (Chang, TH, 2014). Most of the researchers use α -cut arithmetic operations for constructing the fuzzy models. However, using α -cut arithmetic operations one can realize that the fuzziness of the model makes the calculation complicated due to the accumulating phenomenon of fuzziness. Hence, this paper proposes a novel idea of using the weakest t -norm (T_w) for VIKOR technique. And this technique is illustrated by selection of best mobile brand from among three most popular mobile brands.

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2. PRELIMINARIES

Definition 2.1.

A fuzzy set \tilde{A} is a subset of a universe of discourse X , which is characterized by a membership function $\mu_{\tilde{A}}(x)$ representing a mapping $\mu_{\tilde{A}} : X \rightarrow [0, 1]$. The function value of $\mu_{\tilde{A}}(x)$ is called the membership value, which represents the degree of truth that x is an element of fuzzy set \tilde{A} .

Definition 2.2.

A fuzzy set \tilde{A} defined on the set of real numbers R is said to be a fuzzy number and its membership function $\tilde{A} : R \rightarrow [0, 1]$ has the following characteristics,

- \tilde{A} is convex.
- $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \forall x \in [x_1, x_2], \lambda \in [0, 1]$.
- \tilde{A} is normal if $\max \mu_{\tilde{A}}(x) = 1$.
- \tilde{A} is piecewise continuous.

Definition 2.3.

The α -cut of the fuzzy set \tilde{A} of the universe of discourse X is defined as

$$\tilde{A}_\alpha = \{x \in X / \mu_{\tilde{A}} \geq \alpha\}, \text{ where } \alpha \in [0, 1].$$

Definition 2.4.

A triangular fuzzy number \tilde{N} can be defined as a triplet (l, m, u) , and the Membership function $\mu_{\tilde{A}}(x)$ is defined as:

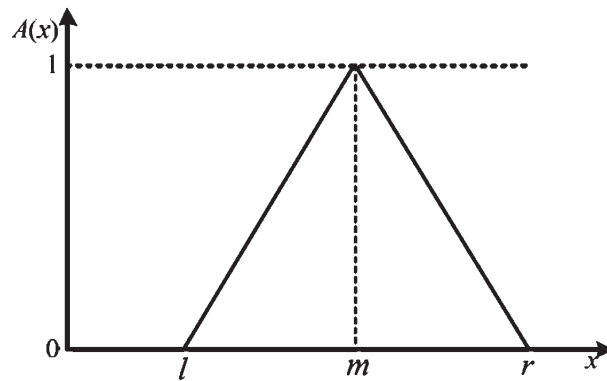


Figure 1: A triangular Fuzzy number \tilde{A}

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l} & l \leq x \leq m \\ \frac{r-x}{r-m} & m \leq x \leq r \\ 0 & \text{otherwise} \end{cases}$$

Where l, m and r are real numbers and $l \leq m \leq r$.

Theorem 2.1.

Let $\tilde{N}_1 = (l_1, m_1, r_1)$ and $\tilde{N}_2 = (l_2, m_2, r_2)$ be two triangle fuzzy numbers. The addition, subtraction, multiplication operations of \tilde{N}_1 and \tilde{N}_2 , denoted by $\tilde{N}_1 \oplus \tilde{N}_2, \tilde{N}_1 \ominus \tilde{N}_2$ and $\tilde{N}_1 \otimes \tilde{N}_2$ respectively, yield another triangular fuzzy number

1. $\tilde{N}_1 \oplus \tilde{N}_2 = (l_1 + l_2, m_1 + m_2, r_1 + r_2)$
2. $\tilde{N}_1 \ominus \tilde{N}_2 = (l_1 - r_2, m_1 - m_2, r_1 - l_2)$
3. $k \otimes \tilde{N}_1 = (kl_1, km_1, kr_1)$, $k > 0$ a crisp number
4. $\tilde{N}_1 \otimes \tilde{N}_2 = (l_1 \times l_2, m_1 \times m_2, r_1 \times r_2)$

Definition 2.4.

A linguistic variable / term is a variable whose values are not crisp numbers but words or sentences expressed in a natural language (Zadeh, 1975).

2.1. T_w Fuzzy Operations

In Zadeh’s extension principle (Zadeh, L.A. 1975), if it is generalized by the binary T norm that replaces the original ‘min’, and the binary T norm on the interval [0, 1], it is said to be a triangular norm (called t-norm) if it is associative, commutative and monotonous in [0, 1]

and
$$T(x, 1) = x \text{ for every } x \in [0, 1]$$

Zadeh’s sup-min operator can be written as,
$$\tilde{A} \circ \tilde{B}(z) = \sup_{x \circ y = z} T(\tilde{A}(x), \tilde{B}(y)),$$

Moreover, each t-norm may be shown to satisfy the following inequalities,

$$T_w(a_2, b_2) = \begin{cases} a_2, & \text{if } b_2 = 1 \\ b_2, & \text{if } a_2 = 1 \\ 0, & \text{otherwise} \end{cases}$$

T_w is the weakest t-norm.

Definition 2.1.1.

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be fuzzy numbers on \mathfrak{R} . Then, addition, subtraction and multiplication operations of T_w can be defined as follows (Hong, D.H., 2006),

1. $\tilde{A} \oplus_{T_w} \tilde{B} = (a_2 + b_2 - \max(a_2 - a_1, b_2 - b_1), a_2 + b_2, a_2 + b_2 + \max(a_3 - a_2, b_3 - b_2))$
2. $\tilde{A} \ominus_{T_w} \tilde{B} = (a_2 - b_2 - \max(a_2 - a_1, b_3 - b_2), a_2 - b_2, a_2 - b_2 + \max(b_2 - b_1, a_3 - a_2))$

$$3. \tilde{A} \otimes_{T_w} \tilde{B} = \begin{cases} (a_2 b_2 - \max((a_2 - a_1)b_2, (b_2 - b_1)a_2), a_2 b_2, & \text{for } a_2 > 0, b_2 > 0 \\ a_2 b_2 + \max((a_3 - a_2)b_2, (b_3 - b_2)a_2)), & \\ (a_2 b_2 - \max((a_3 - a_2)b_2, (b_3 - b_2)a_2), a_2 b_2, & \text{for } a_2 < 0, b_2 < 0 \\ a_2 b_2 + \max((a_2 - a_1)b_2, (b_2 - b_1)a_2)), & \\ (- (a_2 - a_1)b_2, 0, (a_3 - a_2)b_2), & \text{for } a_2 = 0, b_2 > 0 \\ ((a_3 - a_2)b_2, 0, - (a_2 - a_1)b_2), & \text{for } a_2 = 0, b_2 < 0 \\ (0, 0, 0) & \text{for } a_2 = 0, b_2 = 0 \\ (a_2 b_2 - \max((a_2 - a_1)b_2, - (b_3 - b_2)a_2), a_2 b_2, & \text{for } a_2 < 0, b_2 > 0 \\ a_2 b_2 + \max((a_3 - a_2)b_2, - (b_2 - b_1)a_2)), & \end{cases}$$

$$4. \tilde{A} \div_{T_w} \tilde{B} = \begin{cases} (a_2/b_2 - \max((a_2 - a_1)/b_2, (1/b_2 - 1/b_3)a_2), a_2/b_2, & \text{for } a_2 > 0, b_2 > 0 \text{ and } b_1 > 0 \\ a_2/b_2 + \max((a_3 - a_2)/b_2, (1/b_1 - 1/b_2)a_2), & \\ (a_2/b_2 - \max((a_3 - a_2)/b_2, (1/b_2 - 1/b_1)a_2), a_2/b_2, & \text{for } a_2 < 0, b_2 < 0 \text{ and } b_3 < 0 \\ a_2/b_2 + \max((a_2 - a_1)/b_2, (1/b_3 - 1/b_2)a_2), & \\ (-a_2 - a_1)/b_2, 0, (a_3 - a_2)/b_2, & \text{for } a_2 = 0, b_2 > 0 \text{ and } b_1 > 0 \\ (-a_3 - a_2)/b_2, 0, (a_2 - a_1)/b_2, & \text{for } a_2 = 0, b_2 < 0 \text{ and } b_3 < 0 \\ (a_2/b_2 - \max((a_2 - a_3)b_2, (1/b_2 - 1/b_3)a_2), a_2/b_2, & \text{for } a_2 = 0, b_2 = 0 \text{ and } b_3 < 0 \\ a_2/b_2 + \max((a_1 - a_2)/b_2, (1/b_1 - 1/b_2)a_2), & \\ (a_2/b_2 - \max((a_2 - a_1)/b_2, (1/b_2 - 1/b_1)a_2), a_2/b_2, & \text{for } a_2 < 0, b_2 > 0 \text{ and } b_1 > 0 \\ a_2/b_2 + \max((a_3 - a_2)/b_2, -(1/b_3 - 1/b_2)a_2), & \end{cases}$$

3. PROPOSED T_w FUZZY VIKOR ALGORITHM

Fuzzy VIKOR method is a convincing decision approach for solving a complex MADM problem. This technique is used to provide a compromise solution. The compromise solution is an achievable solution, which is closest to the ideal. A compromise solution means an agreement established by mutual adjustment.

For compromise ranking of multi-criteria measurement, VIKOR adopted the following form of LP-metric aggregate function (Yu, 1973):

$$L_{pi} = \left\{ \sum_{j=1}^n [\tilde{w}_j (\tilde{f}_j^+ - \tilde{x}_{ij}) / (\tilde{f}_j^+ - \tilde{f}_j^-)] \right\}^{\frac{1}{p}}$$

Here, $1 \leq p \leq \infty ; j = 1, \dots, n$, with respect to criteria and the variable $i = 1, 2, \dots, m$, represent the number of alternatives such as A_1, A_2, \dots, A_m . For alternative A_p , the evaluated value of the j^{th} criterion is denoted by f_{ij}^p and n is the number of criteria. The measure L_{pi} shows the distance between alternative A_i and the positive-ideal solution. The value obtained by minimum S_i is with a maximum group utility ('majority' rule) and the solution obtained by minimum R_i is with a minimum individual regret of the 'opponent' (Sanayei et al.2010). Here, the extended fuzzy VIKOR method is given in detail.

Step 1: Construct the fuzzy decision matrix and determine the fuzzy weight for each criterion.

Let us consider that there are K persons in the decision group. Then the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as

$$\tilde{x}_{ij} = \frac{1}{K} [x_{ij}^1 \oplus_{T_w} x_{ij}^2 \oplus_{T_w} \dots \oplus_{T_w} x_{ij}^K]$$

$$\tilde{w}_{ij} = \frac{1}{K} [w_{ij}^1 \oplus_{T_w} w_{ij}^2 \oplus_{T_w} \dots \oplus_{T_w} w_{ij}^K]$$

where \tilde{x}_{ij}^K and \tilde{w}_j^K are the rating and the importance weight of the K^{th} decision maker. A fuzzy multi criteria group decision-making problem can be concisely expressed in matrix format as

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{11} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix}$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n]$$

where $\tilde{x}_{ij} \ i = 1, 2, \dots, m, j = 1, 2, \dots$ and \tilde{w}_j are linguistic variables described by triangular fuzzy number in the following table-1 and table-2 respectively.

Table 1

Linguistic variables for the Alternative's Criteria

Linguistic Variable	CODE	Fuzzy Value
Very Poor	VP	(0,0,1)
Poor	P	(0,1,3)
Medium Poor	MP	(1,3,5)
Fair	F	(3,5,7)
Medium Good	MG	(5,7,9)
Good	G	(7,9,10)
Very Good	VG	(9,10,10)

Table 2

Linguistic variables for the Decision Maker's Criteria

Linguistic Variable	CODE	Fuzzy Value
Very Poor	VP	(0,0,0.2)
Poor	P	(0,0.2,0.4)
Moderate	M	(0.3,0.5,0.7)
High	H	(0.8,0.8,1)
Very High	VH	(0.8,1,1)

Step 2 : Determine the Fuzzy Best Value \tilde{f}_j^+ (FBV) and Fuzzy Worst Value \tilde{f}_j^- (FWV) of all criteria functions.

$$\tilde{f}_j^+ = \max_i \tilde{x}_{ij}, \ j \in B$$

$$\tilde{f}_j^- = \min_i \tilde{x}_{ij}, \ j \in C$$

Where B associated with the benefit criteria and C is the cost criteria.

Step 3: Compute the index \tilde{S}_i and \tilde{R}_i

This step computes the separation \tilde{S}_i of A_i from the fuzzy best value \tilde{f}_j^+ . Similarly, the separation of \tilde{R}_i of A_j from the fuzzy worst value \tilde{f}_j^- is also computed.

$$\tilde{S}_i = \sum_{j=1}^n \tilde{w}_j \otimes_{T_w} (\tilde{f}_j^+ \ominus_{T_w} \tilde{x}_{ij}) / (\tilde{f}_j^+ \ominus_{T_w} \tilde{f}_j^-)$$

$$\tilde{R}_i = \max_j [\tilde{w}_j \otimes (\tilde{f}_j^+ - \tilde{x}_{ij}) / (\tilde{f}_j^+ - \tilde{f}_j^-)]$$

Step 4 : Compute the index \tilde{Q}_i

$$\tilde{Q}_i = \frac{\nu(\tilde{S}_i -_{T_w} \tilde{S}_i^+) + (1-\nu)(\tilde{R}_i -_{T_w} \tilde{R}_i^+)}{(\tilde{S}_i^- -_{T_w} \tilde{S}_i^+) +_{T_w} (\tilde{R}_i^- -_{T_w} \tilde{R}_i^+)}$$

Where

$$\tilde{S}_i^+ = \min_i \tilde{S}_i, \tilde{S}_i^- = \max_i \tilde{S}_i$$

$$\tilde{R}_i^+ = \min_i \tilde{R}_i, \tilde{R}_i^- = \max_i \tilde{R}_i$$

The index $\min_i \tilde{S}_i$ is with the maximum majority rule and $\min_i \tilde{R}_i$ is with the minimum individual regret of opponent. ν is taken as weight of the strategy of the maximum utility. $(1-\nu)$ represents the weight of individual majority rule (usually $\nu = 0.5$).

Step 5 : Defuzzifying the index \tilde{Q}_i
 The defuzzification is done through

$$\tilde{Q}_i = \frac{a + 2b + c}{4}$$

Step 6 : According to the crisp value of \tilde{S}_i , \tilde{R}_i and \tilde{Q}_i , the ranking order of all alternatives can be determined.

Step 7: Propose a Compromise solution,

If the following two conditions are satisfied simultaneously, then the scheme with minimum value of Q_i in ranking is considered the optimal compromise solution.

Condition 1: The alternative $Q(A^1)$ has an acceptable advantage

$$Q(A^2) - Q(A^1) \geq \frac{1}{m - 1}$$

Where, A^1 ranked first, A^2 is the alternative with the second position in Q_i , m is the number of alternatives and $1/(m-1)$ is the threshold.

Condition 2: The alternative $Q(A^1)$ is stable within the decision making process; in other words, it is also best ranked in S_i and R_i .

If Condition-1 is not satisfied, that is if $Q(A^2) - Q(A^1) < \frac{1}{m - 1}$, then alternatives A^1, A^2, \dots, A^m are all the same compromise solution; there is no comparative advantage of A^1 from others. But for the case of maximum value, the corresponding alternative is the compromise (closeness) solution. If condition -2 is not satisfied, the stability in decision making is deficient while A^1 has a comparative advantage. Therefore, A^1 and A^2 are the same compromise solutions.

Step 8: Select the best alternatives as a compromise solution.

3. ILLUSTRATION

As an illustration three most popular mobile brands are taken as the alternatives namely Apple, Samsung and Nokia. Six characteristics namely, cost, memory, camera, battery life, screen size and weight are taken as the criteria for selection.

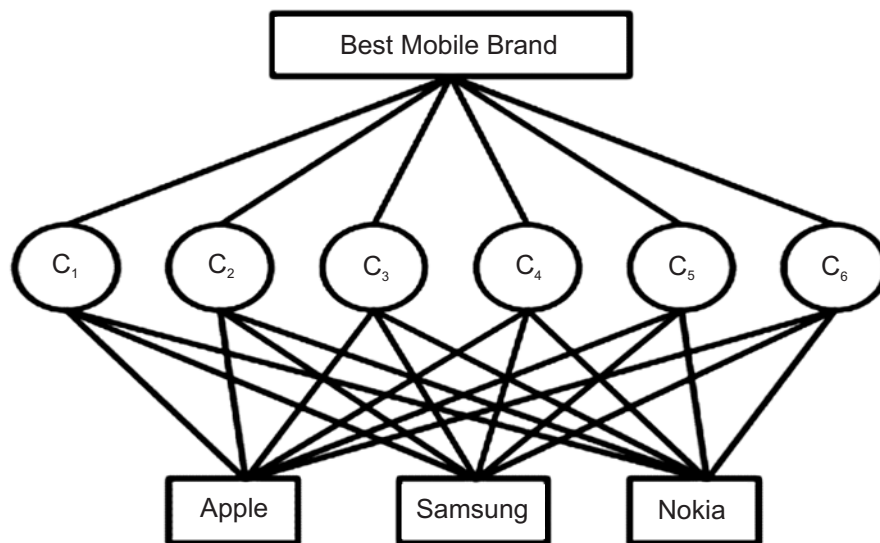


Figure 2: The hierarchical structure

Step-1 Constructing the Decision Matrix : The decision-makers use linguistic rating variables and linguistic weighting variables (shown in Table 1 and Table 2) to assess the importance of the criteria which are then transformed into triangular fuzzy number and take the average of them.

Table 3
T_w Fuzzy Decision Matrix and weights of the three alternatives

	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C4</i>	<i>C5</i>	<i>C6</i>
<i>APPLE</i>	(5,5.4,5.8)	(4,4.4,4.8)	(7.8,8.2,8.6)	(7.6,8,8.4)	(5.8,6.2,6.6)	(6.6,7,7.4)
<i>SAMSUNG</i>	(5,5.4,5.8)	(5.2,5.6,6)	(3.2,3.6,4)	(3,3.4,3.8)	(4.8,5.2,5.6)	(2.2,2.6,3)
<i>NOKIA</i>	(5.4,5.8,6.2)	(3.8,4.2,4.6)	(4.6,5,5.4)	(6.4,6.8,7.2)	(5.2,5.6,6)	(5,5.4,5.8)
<i>Weight</i>	(2.3,2.5,2.7)	(2.2,2.4,2.6)	(2.3,2.5,2.7)	(2.9,3.1,3.3)	(2.5,2.7,2.9)	(1.8,2,2.2)

Step 2: Determine the Fuzzy Best Value \tilde{f}_j^+ (FBV) and Fuzzy Worst Value \tilde{f}_j^- (FWV) of all criteria functions from table 5.

Table 4
The Fuzzy Best Value \tilde{f}_j^+ (FBV) and Fuzzy Worst Value \tilde{f}_j^- (FWV)

	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C4</i>	<i>C5</i>	<i>C6</i>
\tilde{f}_j^+	(5.4,5.8,6.2)	(5.2,5.6,6)	(7.8,8.2,8.6)	(7.6,8,8.4)	(5.8,6.2,6.6)	(6.6,7,7.4)
\tilde{f}_j^-	(5,5.4,5.8)	(3.8,4.2,4.6)	(3.2,3.6,4)	(3,3.4,3.8)	(4.8,5.2,5.6)	(2.2,2.6,3)

Step 3 : Computing the index \tilde{S}_i and \tilde{R}_i

Table 5
The fuzzy index \tilde{S}_i and \tilde{R}_i

	<i>Apple</i>	<i>Samsung</i>	<i>Nokia</i>
\tilde{S}_i	(-0.38,7.06,15.46)	(-39.00,27.45,45.90)	(-12.07,2.44,31.86)
\tilde{R}_i	(1.37,2.50,5.00)	(2.28,28.75,28.75)	(1.71,2.40,20.00)

Step 4 : Compute the index \tilde{Q}_i

Table 6
The fuzzy index $\tilde{S}_i^+, \tilde{S}_i^-$ and $\tilde{R}_i^+, \tilde{R}_i^-$

\tilde{S}_i^+	\tilde{R}_i^+	\tilde{S}_i^-	\tilde{R}_i^-
(-39.00,2.44,15.46)	(-0.38,27.45,45.90)	(1.37,2.40,5.00)	(2.28,28.75,28.75)

Table 7
The fuzzy index \tilde{Q}_i

	<i>Apple</i>	<i>Samsung</i>	<i>Nokia</i>
\tilde{Q}_i	(-0.17,0.09,0.04)	(-1.33,0.00,-0.50)	(-0.33,0.00,-0.05)

Step 5 : Defuzzifying the index \tilde{S}_i , \tilde{R}_i and \tilde{Q}_i

Table 8
The values of S_i , R_i and Q_i

	A_1 -Apple	A_2 -Samsung	A_3 -Nokia
\tilde{S}_i	7.299	15.449	5.558
\tilde{R}_i	2.843	22.133	6.029
\tilde{Q}_i	0.052	1.831	0.096

Step 6 : According to the crisp value of \tilde{Q}_i , the ranking order of all alternatives can be determined.

Table 9

Ranking the values by S, R and Q in ascending order

The ranking order	
By S	$A_1 > A_3 > A_2$
By R	$A_1 > A_3 > A_2$
By Q	$A_1 > A_3 > A_2$

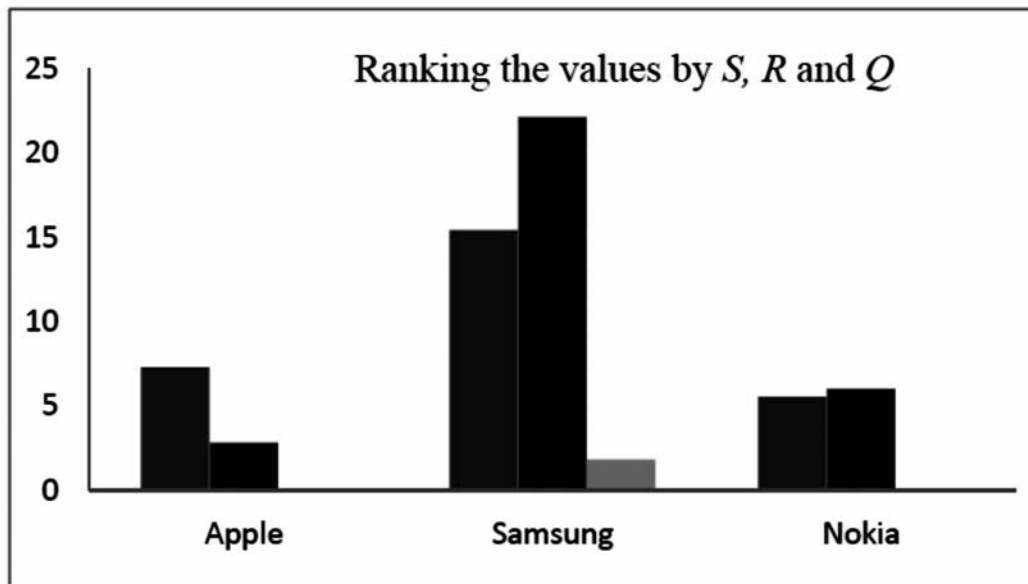


Figure 3: The values of S_i , R_i and Q_i

5. CONCLUSION

In this study, arithmetic operations of the weakest t -norm (T_w) have been used to construct Tw-VIKOR technique. It becomes an effective and simple tool to process the imprecise, vague, subtle information for MCDM problem. Using this fuzzy Tw-VIKOR system, it is observed that the ranking order of the three mobiles is A_1, A_3 , and A_2 .

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