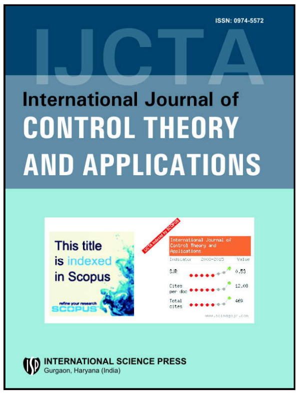

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Robust PID Controller Design for TRMS using Kharitonov Theorem & PSO Technique

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Abstract: In this work, a new approach has been presented based on PSO technique of PID controller to control TRMS within the predetermined searching range which is obtained on the basis of Kharitonov stability theorem. The performance of the compensated system is investigated through experimental results with the laboratory prototype model. The time domain responses for the main and tail rotors are found to be satisfactory. The robustness analysis of the system with proposed control technique has also been analysed.

Keywords: twin rotor multi input multi output system (TRMS), particle swarm optimization (PSO), Kharitonov Theorem

1. INTRODUCTION

TRMS is highly nonlinear strongly coupled system [1-3]. In [4] a decentralized PID controller has been designed to n-link robot manipulators system based on the Kharitonov theorem. In [5, 6] the robust PID controller design has been done for the interval plants. The author describe the Kharitonov theorem for the interval plant for designing the PID controller to stabilize the uncertain plant [7,8]. In [10,12] the author describes how the necessary conditions for stability of Kharitonov theorem is checked.

In this paper two PID controller is designed for main & tail rotor respectively based on the Karitonov theorem. The optimum values of the PID parameters are obtained using PSO technique. A decoupler is also applied to eliminate the cross coupling effects.Next section describes the mathematical modeling of the twin rotor MIMO system. In section III Decoupling technique for the TRMS is discussed. Section IV shows PID controller design for twin rotor MIMO system. Section V covers the results & section VI consist the conclusion section.

2. MATHEMATICAL MODELING

The helicopter has two degree of freedom first the rotation of the helicopter body with respect to the horizontal axis and the rotation around the vertical axis. The TRMS is nonlinear in characteristics.

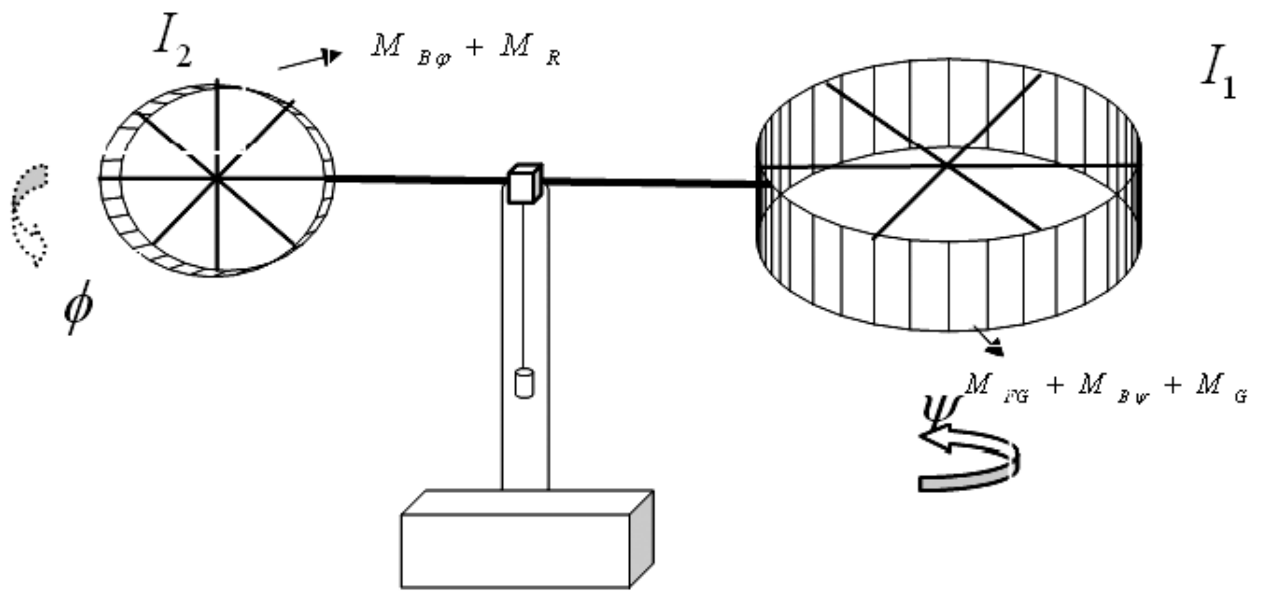

Figure 1: TRMS Phenomenological Model

The non-linear model equations can be derived by the phenomenological model as shown in Fig.-1[7]. Momentum along the pitch angle is derived as below.

$$I_1.\dot{\psi} = M_1 - M_{FG} - M_{B\psi} - M_G \tag{1}$$

Where, M_1 is as given below

$$M_1 = a_1.\tau_1^2 + b_1.\tau_1 \tag{2}$$

The torque equation is derived as below.

$$M_{FG} = M_g \sin\psi \tag{3}$$

The frictional torque is as below.

$$M_{B\psi} = B_{1\psi}.\dot{\psi} + B_{2\psi}.sign(\dot{\psi}) \tag{4}$$

Due to effect of the carioles force gyroscopic torque is produced in TRMS.

$$M_G = K_{gy}.M_1.\dot{\phi}.\cos\psi \tag{5}$$

The motor momentum is as below.

$$\tau_1 = \frac{K_1}{T_{11}(s) + T_{10}}.u_1 \tag{6}$$

The net torque of tail rotor is as below.

$$I_2.\ddot{\phi} = M_2 - M_{B\phi} - M_R \tag{7}$$

Where,

$$M_2 = a_2.\tau_2^2 + b_2.\tau_2 \tag{8}$$

Frictional torque is shown below.

$$M_{B\psi} = B_{1\phi}.\dot{\psi} + B_{2\phi}.sign(\dot{\phi}) \tag{9}$$

The cross reaction momentum

$$M_R = \frac{K_c.(T_0 s + 1)}{(T_p s + 1)}.\tau_1 \tag{10}$$

$$\tau_2 = \frac{K_2}{T_{21}(s) + T_{20}}.u_2 \tag{11}$$

The linearised TRMS plant system is represented as below.

$$\dot{x} = Ax + Bu\,,\ y = Cx + Du \tag{12}$$

$$A = \begin{bmatrix} -0.8333 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ 1.246 & -4.706 & -0.08824 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \\ 1.482 & 0 & 0 & 3.6 & 0 & -5 & 18.75 & \\ -0.01694 & 0 & 0 & 0 & 0 & 0 & -0.5 & \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \tag{13}$$

Here strong interaction is observed between $u_1 - y_1, u_1 - y_2$ and $u_2 - y_2$ but no interaction is observed between u_2 and y_1. G_{11}, G_{22} are main rotor and tail rotor transfer function in vertical and horizontal plane respectively.

$$G(s) = \begin{bmatrix} \dfrac{1.246}{s^3 + 0.9215s^2 + 4.77s + 3.918} & 0 \\ \dfrac{1.482s + 0.4234}{s^4 + 6.33s^3 + 7.07s^2 + 2.08s} & \dfrac{3.6}{s^3 + 6s^2 + 5s} \end{bmatrix} \tag{14}$$

3. DECOUPLING FOR TRMS

If coupling is present in the plant it is difficult to design controllers for the accurate results, because effect of coupling can be incorporated to the plant performance. The decoupling in the TRMS is introduced to separate the cross coupled path present into the TRMS. Since there are two main paths and two cross coupled path is present in the TRMS only two decoupling functions are required.

$$D(s)=\begin{bmatrix}1 & D_1(s)\\ D_2(s) & 1\end{bmatrix} \qquad D_1(s)=-\frac{G_{12}(s)}{G_{11}(s)}=0 \tag{15}$$

$$D_2(s)=-\frac{G_{21}(s)}{G_{22}(s)} = -\frac{0.41s^4+2.7s^3+2.8s^2+0.59s}{s^4+6.3s^3+9.2s^2+0.2s} \tag{16}$$

4. ROBUST PID CONTROLLER DESIGN

4.1. Application of Kharitonov Theorem in designing of PID Controller

To apply the Kharitonov theorem it is important to understand concept of Kharitonov theorem[13].

$$\delta(s)=\delta_0+\delta_1 s+\delta_2 s^2+\delta_3 s^3+\delta_4 s^4+.........\delta_n s^n \tag{17}$$

Here the coefficient lies within a defined range as, $\delta_0 \in [x_0, y_0], \delta_1 \in [x_1, y_1], ...\delta_n \in [x_n, y_n]$ Here, $\underline{\delta}=[\delta_0+\delta_1,.......,\delta_n]$ and identified a polynomial $\delta(s)$ with its coefficient vector $\underline{\delta}$. The box of coefficient is represented as rectangle shown in Fig. 2.

$$\Delta=\{\underline{\delta} :\in IR^{n+1}, x_i \le \delta_i \le y_i, i=0,1,2,.......n\} \tag{18}$$

By the Kharitonov Theorem,

$$\begin{aligned}
K_1(s)&=x_0+x_1 s+y_2 s^2+y_3 s^3+x_4 s^4+x_5 s^5+y_6 s^6+.......,\\
K_2(s)&=x_0+y_1 s+y_2 s^2+x_3 s^3+x_4 s^4+y_5 s^5+y_6 s^6+.......,\\
K_3(s)&=y_0+x_1 s+x_2 s^2+y_3 s^3+y_4 s^4+x_5 s^5+x_6 s^6+.......,\\
K_4(s)&=y_0+y_1 s+x_2 s^2+x_3 s^3+y_4 s^4+y_5 s^5+x_6 s^6+.......,
\end{aligned} \tag{19}$$

The vertices of the Kharitonov polynomial are as shown in the fig.2

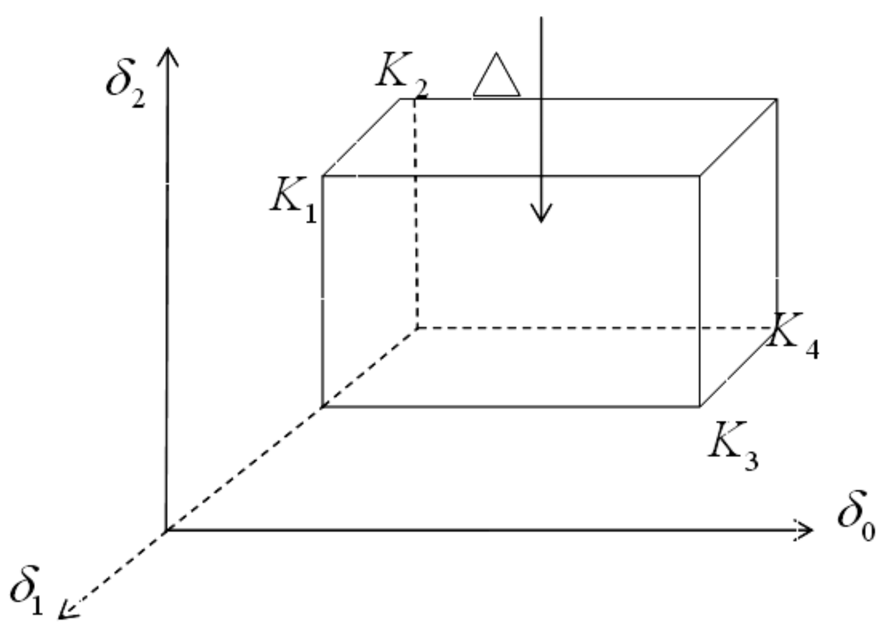

Figure 2: Four Kharitonov Vertices

The controller output of conventional PID controller is given as in equation below.

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{20}$$

$$G_c(s) = K_p + K_i/s + K_d s \tag{21}$$

The resultant transfer function of the main rotor with the PID controller can be written as below.

$$G_{C11} = \left[\frac{K_P s + K_i + K_d s^2}{s}\right] * \left[\frac{1.246}{s^3 + 0.9215s^2 + 4.77s + 3.918}\right]$$

$$= \left[\frac{1.246s^2 K_d + 1.246sK_p + 1.246K_i}{s^4 + 0.9215s^3 + 4.77s^2 + 3.918s}\right] \tag{22}$$

With the resultant transfer function the characteristics equation is determined as below

$$s^4 + 0.9215s^3 + 3.918s + 4.77s^2 + \left[1.246s^2 K_d + 1.246sK_p + 1.246K_i\right] * \left[A\cos\gamma - jA\sin\gamma\right] \tag{23}$$

The objective is to find the set of possible value of K_p, K_i & K_d such that the system became stable. Now by putting $s = j\omega$ the above equation can be expressed in real and imaginary parts.

$$\omega^4 - 4.77\omega^2 - 1.246\omega^2 K_d A\cos\gamma + 1.246K_i A\cos\gamma + 1.246K_p \omega A\sin\gamma = 0 \tag{24}$$

$$-0.9215\omega^3 + 1.246\omega^2 K_d A\sin\gamma + 1.246K_p A\cos\gamma + 3.918\omega = 0 \tag{25}$$

Here the control parameter is found by assigning the value of K_i first than solving the equation 24 and 25 with the condition of marginal stability. !.e. $A = 1$ & $\gamma = 0$.

The ranges of the parameters are found as K_p = [0.08 0.25], K_i = [1 3], K_d = [0.40 0.65].

Similarly the resultant transfer function of the tail rotor with the PID controller can be written as below.

$$G_{C22} = \left[\frac{K_P s + K_i + K_d s^2}{s}\right] * \left[\frac{3.6}{s^3 + 6s^2 + 5s}\right] = \left[\frac{3.6s^2 K_d + 3.6sK_p + 3.6K_i}{s^4 + 6s^3 + 5s^2}\right] \tag{26}$$

Similarly, by the resultant transfer function of the tail rotor with controller the following characteristic equation is being obtained.

$$s^4 + 6s^3 + 5s^2 + \left[3.6s^2 K_d + 3.6sK_p + 3.6K_i\right] * \left[A\cos\gamma - jA\sin\gamma\right] \tag{27}$$

Here also the objective is to find the set of possible value of K_p, K_i & K_d such that the system became stable. Now by putting $s = j\omega$ the above equation can be expressed as real and imaginary parts.

$$\omega^4 - 3.6\omega^2 K_d A\cos\gamma + 3.6K_i A\cos\gamma + 3.6K_p \omega A\sin\gamma = 0 \tag{28}$$

$$-6\omega^3 + 3.6\omega^2 K_d A \sin\gamma + 3.6 K_p \omega A \cos\gamma - 3.6 K_i A \sin\gamma = 0 \quad (29)$$

Here also the value of K_i is assigned first and then K_p & K_d is obtained by solving eqn. 28 & 29 with the condition of marginal stability. The ranges of the controller parameter gain is obtained as K_p = [1.6 3], K_i = [0.5 1.6], K_d = [0.7 1.7].

4.2. Application of Interlacing Property of Kharitonov Theorem in Designing of PID Controller

By considering the Hurwitz stability the family of interval polynomial is stable if it satisfies the interlacing property as sown in Fig.3 [12]. It states that the polynomial $K^e_{\max}(\omega), K^e_{\min}(\omega), K^o_{\max}(\omega), K^o_{\min}(\omega)$ have only real roots and set of positive must interlace as follows.

$$0 \langle \omega^{\min}_{e1} \langle \omega^{\max}_{e1} \langle \omega^{\min}_{o1} \langle \omega^{\max}_{o1} \langle \omega^{\min}_{e2} \langle \omega^{\max}_{e2} \quad (30)$$

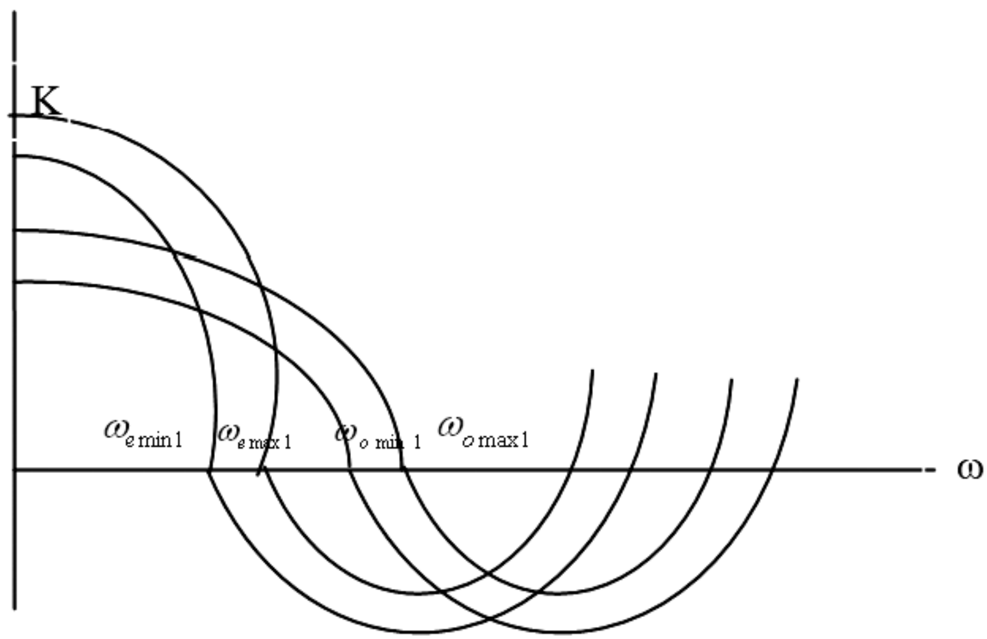

Figure 3: Interlacing odd and even parts

The obtained positive roots are as below which clearly satisfies the equation (30).as below. Hence it is said that main rotor part of the TRMS is robust stable.

$$\omega_{e\min 1} = 0.5, \omega_{e\max 1} = 0.83, \omega_{o\min 1} = 2.09, \omega_{o\max 1} = 2.12, \omega_{e\min 2} = 2.17, \omega_{e\max 2} = 2.24,$$

$$0.5<0.83<2.09<2.12<2.17<2.24 \quad (31)$$

Similarly for the tail rotor the obtained positive roots of the odd and even interval polynomial is written as below, which also satisfy equation (30).

$$\omega_{e\min 1} = 0.48, \omega_{e\max 1} = 0.73, \omega_{o\min 1} = 0.97, \omega_{o\max 1} = 1.34, \omega_{e\min 2} = 2.69, \omega_{e\max 2} = 3.25,$$

$$0.48<0.73<0.97<1.34<2.69<3.25 \quad (32)$$

4.3. Application of particle swarm optimization to achieve optimum value of PID Controller parameter

The dimension used here is three because there is three parameter of PID controller is tuned by PSO as shown in figure 4.

The flowchart is shown in Fig. 5.

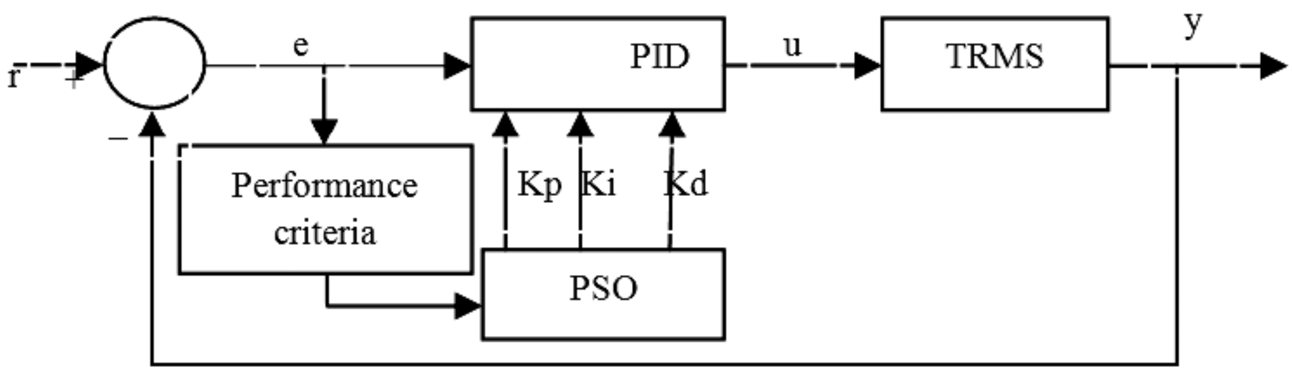

Figure 4: PID controller based on PSO

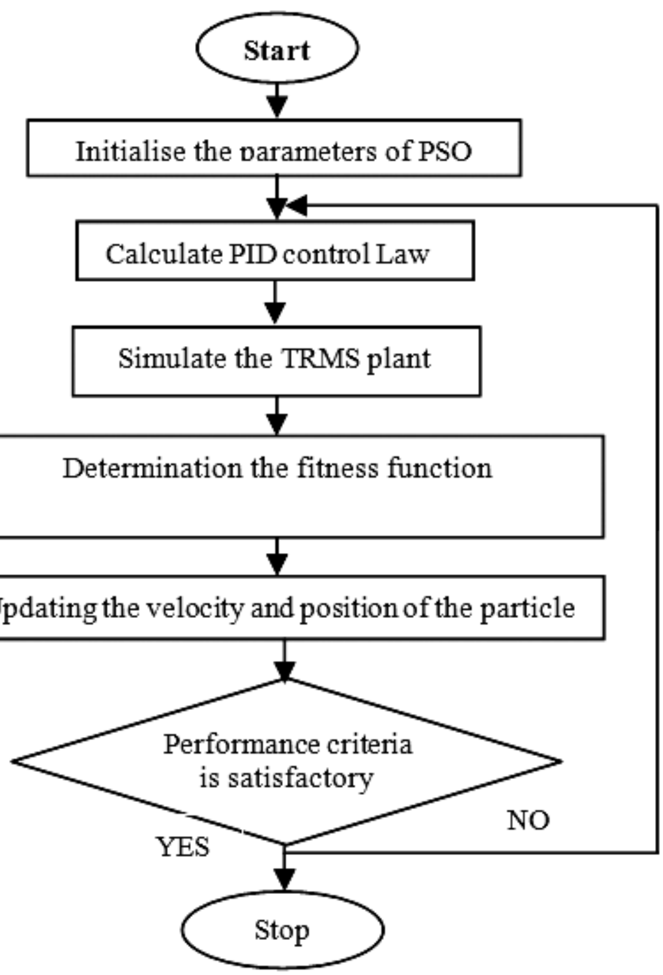

Figure 5: Flow Chart of PSO algorithm

The velocity formula is calculated here is as below.

$$V_{id}^{n+1} = V_{id}^{n} + c_1 rand().(P_{id}^{n} - X_{id}^{n}) + c_2 rand().(P_{gd}^{n} - X_{id}^{n}) \quad (33)$$

$$X_{id}^{n+1} = X_{id}^{n} + V_{id}^{n+1} \quad (34)$$

The values for the PSO parameters are given in Table 1. Fig. 6 shows the plot of minimum values of the objective function (IAE) for PID controller for both main & tail rotor of TRMS.

Table 1
Parameters of PSO

Sl. No.	*Parameter*	*Value*
1.	Cognitive Constant (C_1)	1.36
2.	Group Constant(C_2)	1.36
3.	Number of initial population	10
4.	Number of iteration	30

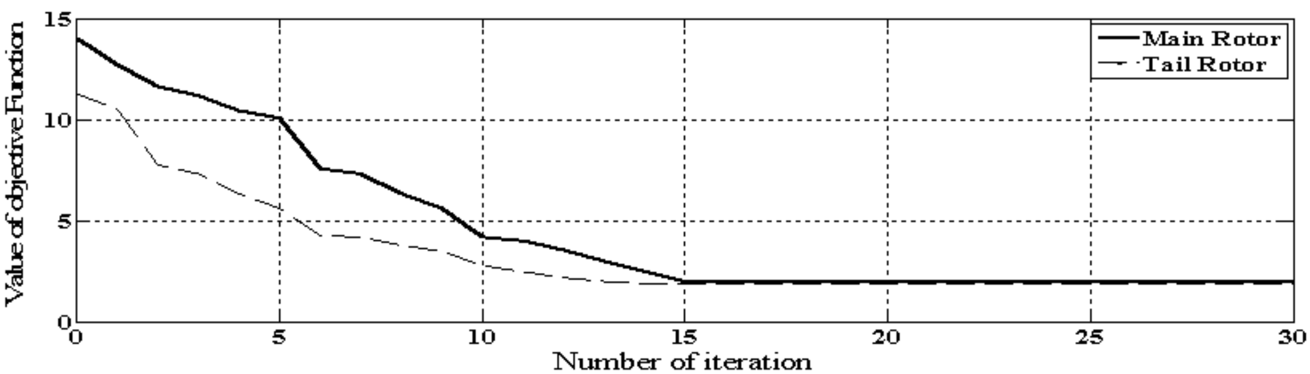

Figure 6: Convergence curves of objective function (IAE) for optimized PID controller

5. EXPERIMENTAL RESULTS

Figure 7: Experimental setup of twin rotor MIMO system

5.1. Time domain analysis

The value of PID controllers for main rotor and tail rotor found out by using PSO technique within the ranges as obtained by Kharitonov theorem.The reference value for step input is taken here as 0.5 for horizontal plane & 0.4 for vertical plane, respectively. The step response and the control signal are shown in Fig. 8 (a) & 8(b) and 9(a) & 9(b), respectively, for main and tail rotors.

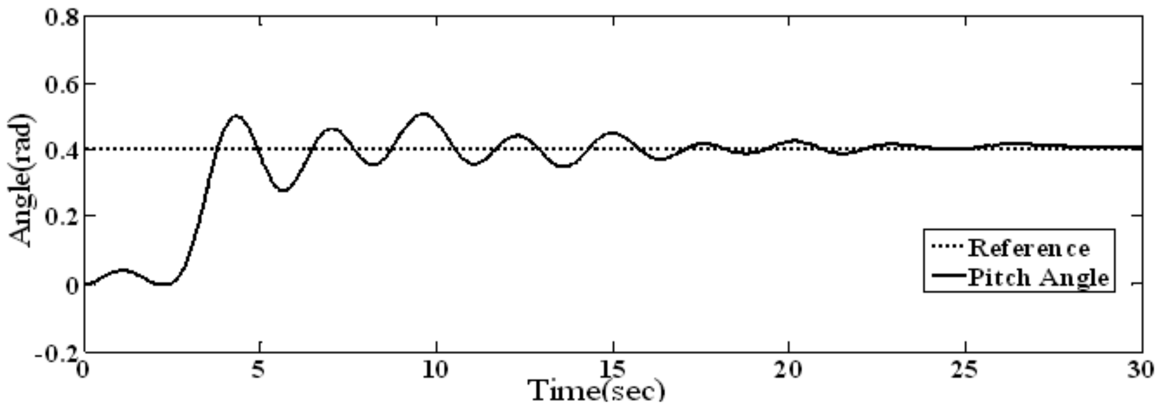

Figure 8: (a) Step response of Main rotor

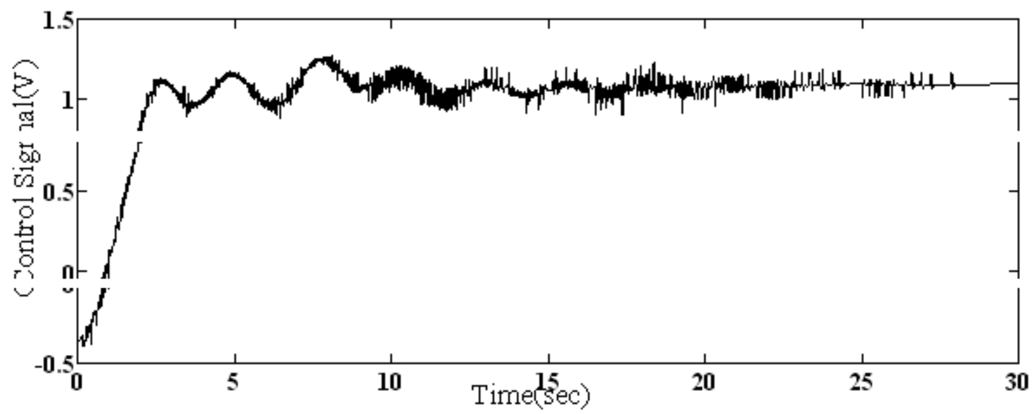

Figure 8: (b) Control signal of Main rotor

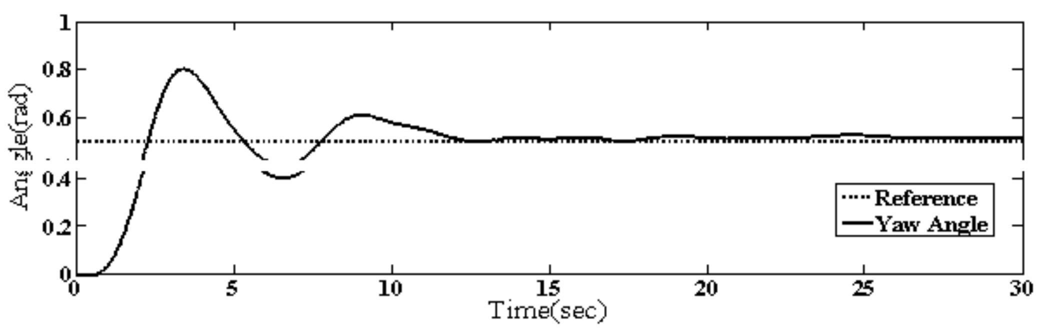

Figure 9: (a) Step response of Tail rotor

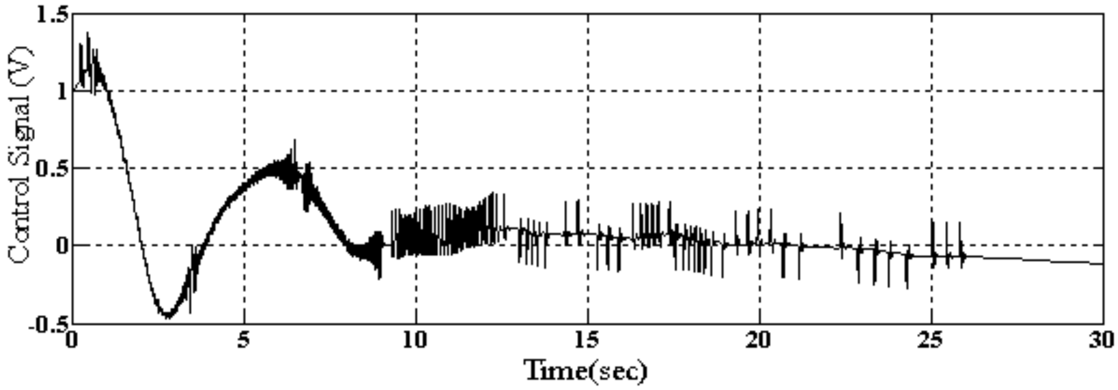

Figure 9: (b) Control signal of Tail rotor

5.2. Robustness analysis

The robustness of the compensated system with the PID controllers has been investigated with respect to the variations of the multiplicative gains which are associated with the control signals applied to the plant by the two PID controllers designed in Section IV. For the main rotor, the maximum value of the gain for which the system remains stable is found out to be 1.8 and minimum value is 0.5. The step input responses are shown in Fig. 10 and 11, respectively, for maximum & minimum gains.

Similarly, for the tail rotor the maximum value of the gain is 1.4 and minimum value is 0.4. The step input responses are shown in Fig. 12 and 13, respectively, for maximum & minimum gains.

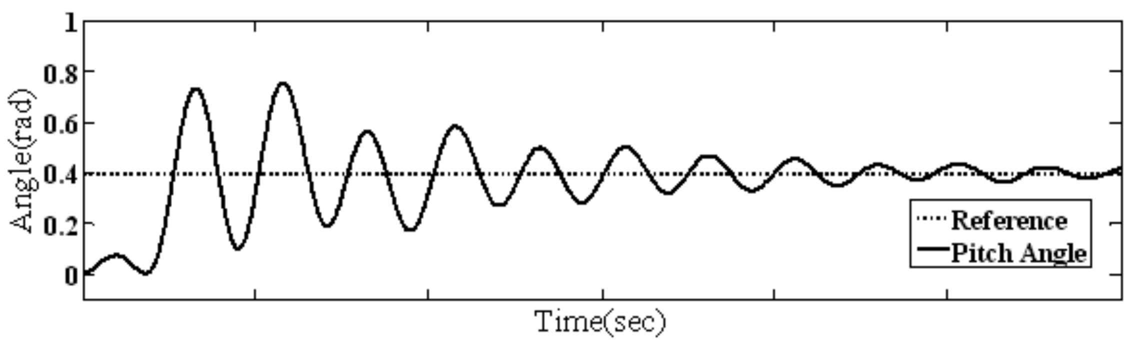

Figure 10: Step response for Main rotor with multiplicative gain 1.8

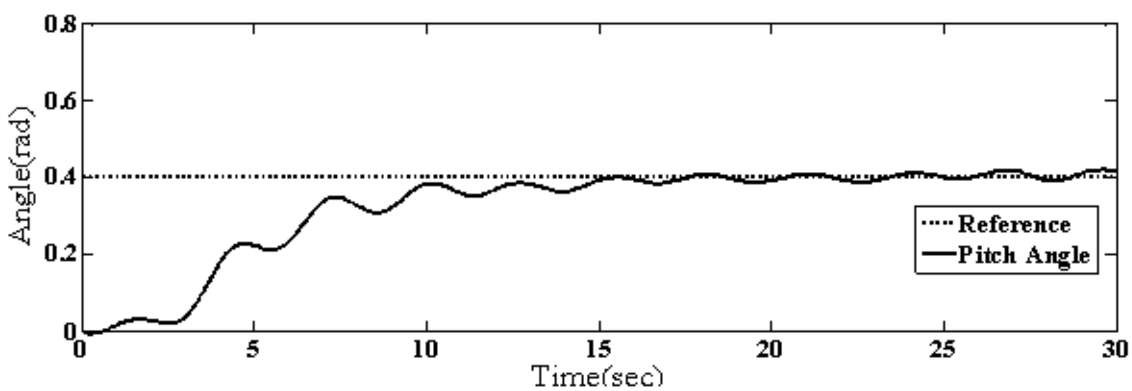

Figure 11: Step response for Main rotor with multiplicative gain 0.5

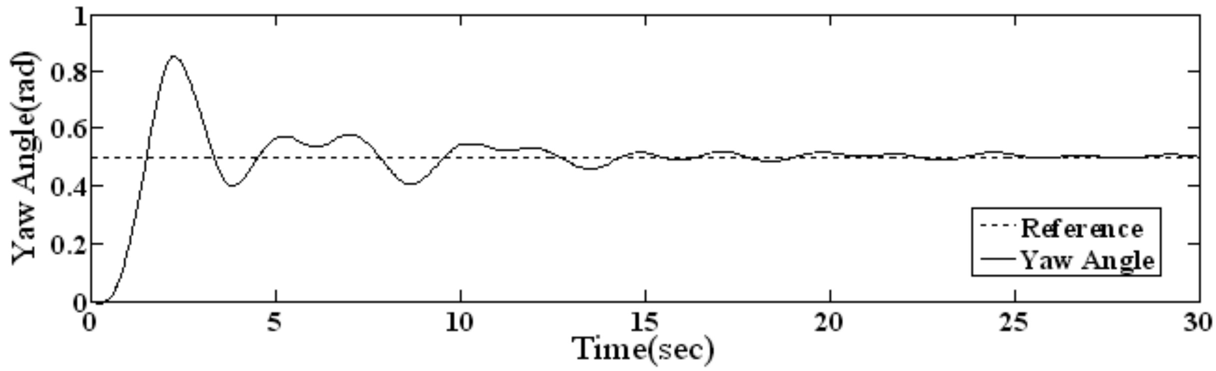

Figure 12: Step response for Tail rotor with multiplicative gain 1.4

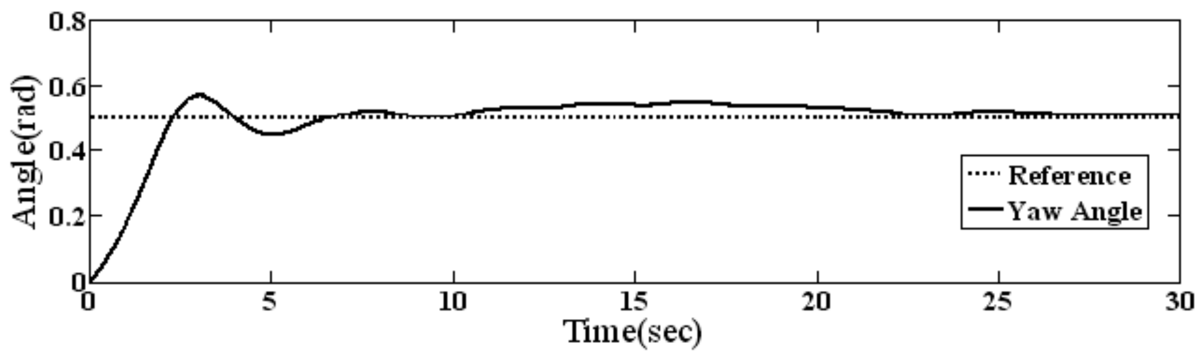

Figure 13: Step response for Tail rotor with multiplicative gain 0.4

5.3. Disturbance Rejection

Fig. 14 & 15 shows the experimental results when an external disturbance is applied.The disturbance taken here is 20% of the magnitude of reference signal.

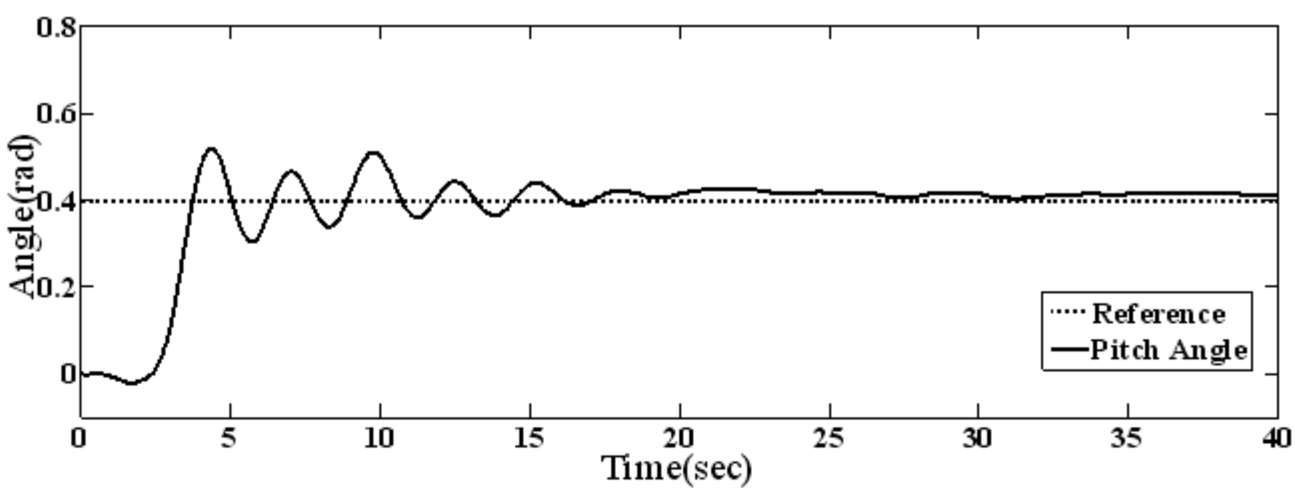

Figure 14: Step response for Main rotor disturbance applied at 20 sec.

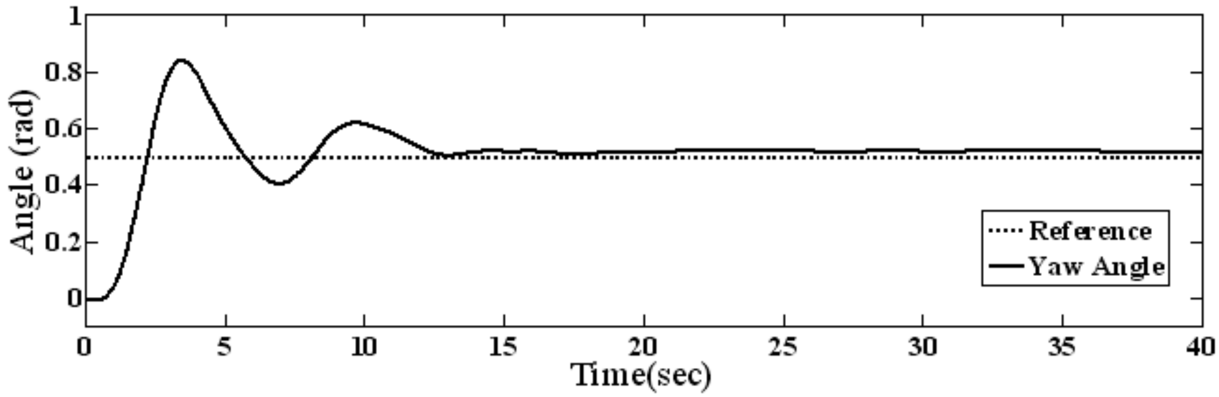

Figure 15: Step response for Tail rotor disturbance applied at 20 sec.

6. CONCLUSION

In this paper, two PID controllers based on the Kharitonov stability theorem have been tuned using PSO technique. The accuracy and stability of the proposed method is verified by the experimental results. The robustness analysis of the system has also been done by adding multiplicative gain with the control signal. The results show that the proposed PID controller technique gives satisfactory performance.

REFERENCES

[1] Twin Rotor MIMO System Control Experiments 33-949S User Manual, Feedback instruments Ltd., East Sussex, U.K.

[2] Mondal S., Mahanta C. , "Adaptive second-order sliding mode controller for a twin rotor multi-input–multi-output system", IET Control Theory and Applications, Vol. 6, Iss. 14, pp. 2157 - 2167, 2012.

[3] Roy T. ,Barai R. K. , "Control Oriented LFT Modeling of a Non Linear MIMO system", International Journal of Electrical, Electronicsand Computer Engineering ,vol.-1(2) ,pp.15-21,2012

[4] Leena G., Ray G., "A Set of decentralized PID controllers for n-link robot manipulator" ,Sadhna,vol-37,part -3,pp.405-423,2012.

[5] Salloum R., Moaveni B., "Robust PID controller design for a Real Electromechanical Actuator, Acta Polytechnica Hungarica" , vol-11,No.-5,pp.125-144,2014.

[6] Qiao X.,Luo F.,Xu Y., "Robust PID Controller tuning for uncertain systems based on Constrained kharitonov theorem and Genetic algorithm", Journal of computational information systems ,vol 11,pp.-3195-3202,2015

[7] Pradhan J.K., Ghosh A., "Design and implementation of decoupled Compensation for a twin rotor multiple-input and multiple-output system" IET Control Theory Appl., Vol. 7, Iss. 2, pp. 282–289, 2013.

[8] Huang Y.J., Wang Y.J. "Robust PID tuning strategy for uncertain plants based on the Kharitonov theorem" ISA Transactions 39,pp.419-431,2000.

[9] Solihin M.I.,Wahyudi, Legowo A.,Akmeliawati R., "Robust PID anti-swing control of automatic gantry crane based on Kharitonov's stability" 4th IEEE Conference on Industrial Electronics and Applications, pp.275-280,2009.

[10] Wen.P.,Lu.W.T. "Decoupling control of a twin rotor MIMO system using robust deadbeat control technique" IET Control Theory Appl.,Vol.2,No.11,pp.999-1007,2008.

[11] Bhattacharyya S.P., Chapellat H., and Keel L., "Robust control the parametric approach" Prentice-Hall, New Jersey, 1995

[12] Hote V.Y., 'Necessary Conditions for stability of Kharitonov Polynomials', IETE technical review,vol.28:5, pp.445-448.2014.

[13] Kharitonov V.L., 'Asymptotic stability of an equilibrium position of a family of systems of linear differential equations', Differential Equations,vol.14,pp.1483-1485,1979