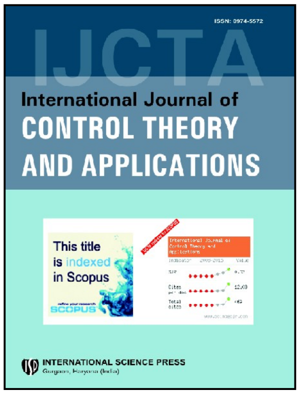

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Studies on the affect of Stress Triaxiality on Strain Energy Density, and CTOD under Plane Stress Condition Subjected to Mixed Mode (I/II) Fracture

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Abstract: In this work, finite element analysis of a rectangular plate with an edge crack was carried out with AL 6061 as material under mixed mode (I/II) loading. Mixed mode stress intensity factors K_I, K_{II}, CTOD, ERR and stress triaxiality are evaluated for the various crack inclination angles from a span of 0° to 55° with a step of 5°, for various crack lengths. Mathematical models relating strain energy density, CTOD and stress triaxiality is developed; Constraints are developed for this model using simple mathematics, later which is optimized to get the crack initiation direction. The effect of CTOD and stress triaxiality on the strain energy density is studied. In addition, the crack initiation directions obtained from the simple SED criterion and the optimization of model-1 are compared. It has been concluded that the effect of CTOD and stress triaxiality is significant. With increase in crack inclination angle, the crack initiation angle is increasing and the mode of failure is shifting to shear. Also at higher crack inclination angles, the strain energy stored is less resulting in lesser energy release rate thereby crack is not opening. Thus, demanding higher loads for the crack to become critical. Also crack lengths beyond 30% of width of the plate are observed to be dangerous. Hence in service conditions, if such conditions are observed, either the component is to be replaced or suitable measures are to be taken for the safe working of the structures.

Key words: Crack, Strain Energy Density, CTOD, ERR, FEM

I. INTRODUCTION

Mixed mode loading conditions will be observed in many mechanical systems because of non-uniformity in loading conditions as well as geometry. In a system with existing crack, this type of mixed mode loading results in propagation of crack at fast rate and transition in failure behavior from ductile to brittle. Hence it is necessary to determine the various crack tip parameters under mixed mode loading conditions.

Firstly, Griffith (1921) proposed a criterion based on energy, which states that the fracture will occur when the energy stored in a structure overcomes the surface energy of the material. Later, Hussain *et al.* (1974)

proposed G-criterion, which states that the crack initiation occurs in the direction of maximum energy release rate. First stress based criterion (MTS-criterion) was proposed by Erdogan and Sih (1963), which states that the crack initiation occurs in the direction of maximum tangential stress along a constant radius around the crack tip. They have assumed that the material is ideally brittle. Shafique and Marwan [2] modified this criterion to make it applicable to ductile materials. Experiments were conducted on various ductile and brittle materials considering all the possible loading conditions to study various crack initiation criteria proposed in the literature. The experimental results do not favour any criteria for low inclination angles but at higher inclinations all the criterions are in good agreement in uniaxial tension. This criterion was later extended by them to anisotropic materials [3].

G. C. Sih [1] proposed another criterion based on strain energy density (S-criterion), which states that the direction of crack initiation will coincides with the direction of minimum strain energy density along a constant radius around the crack tip. This is the first criterion which involves material properties, Poisson's ratio (μ) and shear modulus of elasticity in studying the onset of crack initiation. Theocaris *et al.* [4] modified the S-criterion with a variable radius at the core region as plastic, around the crack tip and proposed maximum dilatational strain energy criterion (T-criterion). Later to cover the strength anisotropy of the material, T-criterion is modified by considering the strength differentiation effect (SDE). Ukadgoanker and Awasare [5] proposed a new criterion to determine the critical radius and angular initiation of crack in ductile materials. In this present criterion the maximum dilatational stress energy density is evaluated along the boundary of the plastic zone, which is the closest to the crack tip. X. M. Kong *et al.* [6] considered the triaxial state of stress as parameter and proposed M-criterion theoretically. It states that the direction of crack initiation coincides with the direction of maximum stress triaxiality ratio along a constant radius around the crack tip. These results are agreeing well with the results of Theocaris et. al [4] especially at low crack inclination angles. S. S. Bhadauria *et al.* [7] performed a comparative study of crack initiation angles calculated using SIFs obtained from numerical analysis and using J-integral. An edge crack at different inclinations is analysed using Finite element analysis. Study reveals that S-criterion predicts the minimum crack initiation angle while M-criterion and T-criterion predicts the maximum initiation angle. Results showed that both approaches are in good agreement with experimental results available in the literature [8].

ShaJiangbo *et al.* [9] performed elastic-plastic finite element analysis of CTS specimen to study the stress triaxiality distribution and deformation at the vicinity of crack tip under mixed mode (I+II) loading in the plane stress and plane strain conditions for various mixed ratios ($K_{I/}K_{II}$) from pure mode-I loading to pure mode –II loading at constant a/w ratio. It was found that the stress triaxiality decreases at the crack tip with an increase in mode-II component for every mixed ratio, with increasing strain hardening exponent, *n*. Anuradha Banerjee and R. Manivasagam [10] developed a new traction separation law (TSL) to study the stress triaxiality dependent cohesive zone model for mode-I fracture in plane strain for the localization of fracture in ductile materials. H. L. Yu and D. Y. Jeong [11] performed non-linear dynamic finite element analysis employing stress triaxiality dependent Bao-Wierzbcki fracture initiation criterion and a linear strain softening law using ABACUS/explicit. It was found that the energy needed to fracture un-notched specimen is twice than the energy required fracturing a notched specimen and it is higher in oblique impact compared to normal impact. Marwan and Shafique [12] adopted stress triaxiality ratio as the parameter which is initially developed for isotropic materials and later extended it to study the anisotropic behavior of materials by considering the linear elastic stress field at the crack tip linked with dynamic anisotropic yield function.

Driemeier *et al.* [13] conducted experiments to investigate the effect of stress intensity, stress triaxiality and load parameter on the material response and failure behavior of aluminum alloys. It has been observed that, in thick plates because of higher stress triaxiality, mode I fracture occurs along the width direction and mode II fracture occurs along the thickness direction. Recho *et al.* [14] proposed a new criterion using numerical and experimental methods for predicting the ductile crack growth in mixed mode fracture using J-integral and plastic mixity parameter M^P as parameters. Three criterions are introduced - TS transition criterion, bifurcation angle criterion for tensile and shear crack, and the critical loading criterion to predict the propagation of a crack under mixed mode loading in a ductile material. Sutton *et al.* [16], [17] proposed a crack initiation criterion under mixed-mode fracture based on crack tip opening displacement (CTOD). It states that the crack will initiate in the direction of either Mode-I or Mode-II depending upon whether the maximum in either the opening component or shearing component of CTOD, measured at a specified distance behind the crack tip, attains maximum value. Sun Jun [15] discussed the importance of developing the relations between local parameter at the crack tip region, stress triaxiality and global characterizing parameters like CTOD, J-integral etc.

Literature signifies the need for developing relations between various local and global paremeters at the crack tip. Hence in this work, efforts are made to develop relation between CTOD (local parameter) and the global parameters stress triaxiality (M) and strain energy density (SED).

II. MATHEMATICAL MODELING

2.1. Stress and displacement fields:

The stress field equations in the mixed mode fracture at the crack tip in Cartesian co-ordinates is defined as:

$$\left.\begin{aligned} \sigma_x &= \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)-K_{II}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right]\\ \sigma_y &= \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)-K_{II}\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right]\\ \tau_{xy} &= \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right)+K_{II}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right] \end{aligned}\right\} \tag{1}$$

Where σ_x, σ_{yx} and τ_{xy} are stress field defined at the crack tip and K_I and K_{II} are mode 1 and mode 2 SIF, where as $\sigma_z = 0$ in the case.

2.2. Crack tip opening displacement (CTOD)

Displacement field in mixed mode (I/II) at the crack tip is given by,

$$\left.\begin{aligned} u_1 &= \frac{1}{G}\sqrt{\frac{r}{2\pi}}\left[K_I\cos\frac{\theta}{2}\left(\frac{1-\mu}{1+\mu}+\sin^2\frac{\theta}{2}\right)+K_{II}\sin\frac{\theta}{2}\left(\frac{2}{1+\mu}+\cos^2\frac{\theta}{2}\right)\right]\\ u_2 &= \frac{1}{G}\sqrt{\frac{r}{2\pi}}\left[K_I\sin\frac{\theta}{2}\left(\frac{2}{1+\mu}-\cos^2\frac{\theta}{2}\right)-K_{II}\cos\frac{\theta}{2}\left(\frac{1-\mu}{1+\mu}-\sin^2\frac{\theta}{2}\right)\right] \end{aligned}\right\} \tag{2}$$

Where, u_1 and u_2 are the displacement in x and y directions, G is the shear modulus of material and μ is the Poisson's ratio.

CTOD is one of the important parameter which is used to assess the ductile fracture of materials at the crack tip. It is the magnitude of the relative displacement vector between the upper and lower crack faces at a fixed distance behind the crack tip. In particular at a distance of x_c behind the crack tip, it can be written mathematically as

$$\delta = \sqrt{\delta_1^2+\delta_2^2} = \sqrt{\left(u_1\left(x_c,0^+\right)-u_1\left(x_c,0^-\right)\right)^2+\left(u_2\left(x_c,0^+\right)-u_2\left(x_c,0^-\right)\right)^2} \tag{3a}$$

Where, the opening and shear components of CTOD are given by

$$\delta_1 = u_2(x_c, 0^+) - u_2(x_c, 0^-) = \frac{K+1}{G}\sqrt{\frac{x_c}{2\pi}}K_I \tag{3b}$$

$$\delta_2 = u_1(x_c, 0^+) - u_1(x_c, 0^-) = \frac{K+1}{G}\sqrt{\frac{x_c}{2\pi}}K_{II} \tag{3c}$$

On substituting Eq. (3b) and (3c) in Eq. (3a), we get

$$\delta = \frac{K+1}{G}\sqrt{\frac{x_c}{2\pi}}\sqrt{K_I^2 + K_{II}^2} \tag{4}$$

Where, $K = \frac{3-\mu}{1+\mu} \Rightarrow K+1 = \frac{4}{1+\mu}$

Young's modulus and shear modulus are related as $E = 2G(1+\mu)$

Finally equation of CTOD is modified as

$$K_1^2 + K_{11}^2 = \frac{\pi E^2 . \delta^2}{32.x_c} \tag{5}$$

2.3. Energy release rate (G_0)

When a member is loaded, it deforms and is stored in it as strain energy. This energy is to be released for the opening of the crack. It is the energy released per unit increase in area when the crack starts propagating. Mathematically

$$G_0 = \frac{\pi}{E}\left[K_I^2 + K_{II}^2\right] \tag{6}$$

2.4. Stress triaxiality factor (M)

It is the ratio of the hydrostatic stress (σ_h) or means stress to the equivalent von Misses stress (σ_{eq}). Mathematically, it can be represented as

$$M = \frac{\sigma_h}{\sigma_{eq}} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)/3}{\frac{1}{\sqrt{2}}\left(\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}\right)} \tag{7}$$

Where, σ_1, σ_2 and σ_3 re the principal stresses. Converting the above equation of principal stresses in to the normal and shear components of stresses, we get

$$M = \frac{\sigma_h}{\sigma_{eq}} = \frac{\frac{\sigma_x + \sigma_y + \sigma_z}{3}}{\frac{1}{\sqrt{2}}\left(\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2}\right)}$$

In plane stress conditions, stresses along the third direction σ_z, τ_{xz} and τ_{yz} are zero. Here corresponding the strains ε_z, γ_{xz} and γ_{yz} will become zero.

$$M = \frac{\frac{(\sigma_x + \sigma_y)}{3}}{\frac{1}{\sqrt{2}}\sqrt{\sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2 + 6\tau_{xy}^2}}$$

On simplifying we get

$$M = \frac{\frac{(\sigma_x + \sigma_y)}{3}}{\frac{1}{\sqrt{2}}\sqrt{2(\sigma_x + \sigma_y)^2 - 6[\sigma_x\sigma_y - \tau_{xy}^2]}} = \frac{\sqrt{2}(\sigma_{11} + \sigma_{22})}{3\sqrt{2(\sigma_{11} + \sigma_{22})^2 - 6[\sigma_{11}\sigma_{22} - \tau_{12}^2]}}$$

On further simplification, we get

$$[\sigma_x\sigma_y - \tau_{xy}^2] = \frac{(\sigma_x + \sigma_y)^2}{27M^2}\left[9M^2 - 1\right] \tag{8}$$

2.5. Ratio of Principal Stresses

Stress triaxiality in terms of principal stresses is given by eq. (7). In plane stress, stress along the third direction σ_3 is zero. Then eq. (7) reduces to

$$M = \frac{\sigma_h}{\sigma_{eq}} = \frac{(\sigma_1 + \sigma_2)/3}{\frac{1}{\sqrt{2}}\left(\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2}\right)} = \frac{\sigma_1 + \sigma_2}{3\left(\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}\right)}$$

On squaring the above equation and by simplification, we get

$$\sigma_1^2(9M^2 - 1) + \sigma_2^2(9M^2 - 1) - \sigma_1\sigma_2(9M^2 + 2) = 0$$

Dividing the above equation by σ_2^2 and assuming $\sigma = \sigma_1 / \sigma_2$, then above equation can be written as

$$\sigma^2(9M^2 - 1) - \sigma(9M^2 + 2) + (9M^2 - 1) = 0 \tag{9}$$

This is a quadratic equation in terms of ó which is the ratio of σ_1 / σ_2. On substituting the value of M in eq. (9) it is possible to determine σ.

2.5. Strain Energy Density Criterion

G.C. Sih proposed strain energy density criterion. It is the only criterion which includes the material property in studying the criticality of the crack and its initiation. Strain energy density in mixed mode fracture is given as

$$W = \frac{(1+\mu)}{2E}\left[\left(\frac{K+1}{4}\right)(\sigma_{11} + \sigma_{22})^2 - 2\left(\sigma_{11}\sigma_{22} - \tau_{12}^2\right)\right] \tag{10}$$

Where $$K = \frac{3-\mu}{1+\mu}$$

Crack extension direction

Crack will initiate in that direction, along which the strain energy density becomes minimum. For the strain energy density to be minimum,

$$\left.\begin{array}{l} \frac{\partial W}{\partial \theta} = 0 \quad \& \quad \frac{\partial^2 W}{\partial \theta^2} > 0 \\ or \quad \frac{\partial S}{\partial \theta} = 0 \quad \& \quad \frac{\partial^2 S}{\partial \theta^2} > 0 \end{array}\right\} \tag{11}$$

Where S=W.r, r – being distance from the crack tip.

On substituting the expressions for various stress components in eq. (11), and simplification, we get

$$\left.\begin{array}{c}\left[2\cos\theta-(K-1)\right]\sin\theta\, K_I^2+2\left[2\cos 2\theta-(K-1)\cos\theta\right]K_I K_{II}+\\ \left[(K-1)\sin\theta-3\sin 2\theta\right]K_{II}^2=0\\ \left[2\cos 2\theta-(K-1)\cos\theta\right]K_I^2+2\left[(K-1)\sin\theta-4\sin 2\theta\right]K_I K_{II}+\\ \left[(K-1)\cos\theta-6\cos 2\theta\right]K_{II}^2>0\end{array}\right\} \tag{12}$$

2.6.1. Relation between stress triaxiality and Strain energy density

Strain energy density in mixed mode fracture is given as

$$W=\frac{(1+\mu)}{2E}\left[\left(\frac{K+1}{4}\right)(\sigma_{11}+\sigma_{22})^2-2\left(\sigma_{11}\sigma_{22}-\tau_{12}^2\right)\right] \tag{13}$$

Substituting the expression from eq. (8), we can write,

$$W=\left(\frac{1+\mu}{2\mathrm{E}}\right)\left[\left(\frac{1}{1+\mu}\right)(\sigma_{11}+\sigma_{22})^2-2\frac{(\sigma_{11}+\sigma_{22})^2}{27M^2}\left[9M^2-1\right]\right]$$

$$W=\left(\frac{1+\mu}{2\mathrm{E}}\right)(\sigma_{11}+\sigma_{22})^2\left[\frac{27M^2-2.(1+\mu)\left[9M^2-1\right]}{27M^2(1+\mu)}\right]$$

Relation between Strain energy density and stress triaxiality is given by

$$W=\frac{(\sigma_{11}+\sigma_{22})^2}{54\mathrm{E}}\left[9[1-2\mu]+\frac{2(1+\mu)}{M^2}\right] \tag{14}$$

Substituting the expressions for stress field from eq. (1) and on simplification, we get

$$W=\frac{\left[\left[\mathrm{K}_1^2\cos^2\frac{\theta}{2}+\mathrm{K}^2_{11}\sin^2\frac{\theta}{2}-\mathrm{K}_1\mathrm{K}_{11}\sin\theta\right]\right]}{27\pi r\mathrm{E}}\left[9[1-2\mu]+\frac{2(1+\mu)}{M^2}\right] \tag{15}$$

This equation helps us to study the effect of stress triaxiality on the strain energy density as well as crack initiation direction.

2.6.2. Condition for minimum strain energy – Crack initiation

From eq. (15) relation between strain energy density and stress triaxiality is given by

$$W=\frac{1}{27\pi rE}\left(K_I^2\cos^2\frac{\theta}{2}+K_{II}^2\sin^2\frac{\theta}{2}-K_I K_{II}\sin\theta\right)\left(9(1-2\mu)+\frac{2(1+\mu)}{M^2}\right)$$

Writing strain energy density factor, $S=W.r$. Now the above equation becomes

$$S=\frac{(1+\mu)}{27\pi E}\left(K_I^2\cos^2\frac{\theta}{2}+K_{II}^2\sin^2\frac{\theta}{2}-K_I K_{II}\sin\theta\right)\left(9\left(\frac{1-2\mu}{1+\mu}\right)+\frac{2}{M^2}\right)$$

But in plane stress, $K = \frac{3-\mu}{1+\mu}$ and writing $\frac{1-2\mu}{1+\mu} = \frac{3K-5}{4}$

On substituting in above equation, we get

$$S = \frac{(1+\mu)}{27\pi E}\left(K_I^2 \cos^2\frac{\theta}{2} + K_{II}^2 \sin^2\frac{\theta}{2} - K_I K_{II}\sin\theta\right)\left(9\left(\frac{3K-5}{4}\right) + \frac{2}{M^2}\right)$$

$$S = \frac{(1+\mu)}{54\pi E}\left(\left(K_I^2 + K_{II}^2\right) + \left(K_I^2 - K_{II}^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right)\left(9\left(\frac{3K-5}{4}\right) + \frac{2}{M^2}\right)$$

Finally the relation between Strain energy density, and stress triaxiality is given as

$$S = \frac{(1+\mu)}{216\pi E M^2}\left(\left(K_I^2 + K_{II}^2\right) + \left(K_I^2 - K_{II}^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right)\left(9(3K-5)M^2 + 8\right) \quad (16)$$

Above equation is a function of two variables (M and θ). To get the maxima or minima of the above equation, the following conditions are to be satisfied.

$$\left.\begin{aligned} \frac{\partial S}{\partial \theta} &= 0 \\ \frac{\partial S}{\partial M} &= 0 \end{aligned}\right\} \quad (17)$$

On substituting the expression of 'S' in eq. (17), and solving them, we get the values of $\theta_{0.}$

$$\left.\begin{aligned} &\frac{(1+\mu)}{216\pi E M^2}\left(9(3K-5)M^2+8\right)\left(\left(K_{II}^2 - K_I^2\right)\sin\theta - 2K_I K_{II}\cos\theta\right) = 0 \\ &\frac{(1+\mu)}{27\pi E}\left(\frac{-2}{M^3}\right)\left(\left(K_I^2 + K_{II}^2\right) + \left(K_I^2 - K_{II}^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right) = 0 \end{aligned}\right\}$$

Here the first two terms of both the equations cannot be made zero. Hence the second term in both the equations should become zero.

$$\left.\begin{aligned} &\frac{\partial S}{\partial \theta} = 0 \\ &\left(\left(K_{II}^2 - K_I^2\right)\sin\theta - 2K_I K_{II}\cos\theta\right) = 0 \\ &\frac{\partial^2 S}{\partial \theta^2} < 0 \\ &\left(\left(K_I^2 + K_{II}^2\right) + \left(K_I^2 - K_{II}^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right) = 0 \end{aligned}\right\} \quad (18)$$

On solving these equations we get crack initiation angle, θ_0. Now find the following terms:

$$A = \frac{\partial^2 S}{\partial \theta^2}, \quad C = \frac{\partial^2 S}{\partial M^2}, \quad B = \frac{\partial^2 S}{\partial \theta \partial M}$$

On substituting the expression for 'S' in above equation, we get

$$\left.\begin{aligned}
A &= \frac{\partial^2 S}{\partial \theta^2} = \left(\frac{1+\mu}{216\pi E}\right)\left(9(3K-5)M^2+8\right)\left(\left(K_{II}^2-K_I^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right) \\
C &= \frac{\partial^2 S}{\partial M^2} = \left(\frac{2(1+\mu)}{9\pi E M^4}\right)\left(\left(K_I^2+K_{II}^2\right)+\left(K_I^2-K_{II}^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right) \\
B &= \frac{\partial^2 S}{\partial \theta \partial M} = \left(\frac{-2(1+\mu)}{27\pi E M^4}\right)\left(\left(K_{II}^2-K_I^2\right)\sin\theta - 2K_I K_{II}\cos\theta\right)
\end{aligned}\right\}$$

The necessary condition to be satisfied for getting maximum value of the function is $AC - B^2 > 0$. Now Now, finally

$$AC - B^2 = \left(\frac{(1+\mu)^2}{5832\pi^2 E^2 M^6}\right)\begin{pmatrix}
K_{II}^4\left(\left(9M^2(3K-5)+8\right)(6\cos\theta-3\cos 2\theta-3)-16(1-\cos 2\theta)\right) \\
-K_I^4\left(\left(9M^2(3K-5)+8\right)(6\cos\theta+3\cos 2\theta+3)+16(1-\cos 2\theta)\right) \\
+K_I^2K_{II}^2\left(\left(9M^2(3K-5)+8\right)(18\cos 2\theta-6)-16(2+6\cos 2\theta)\right) \\
+K_I K_{II}\begin{pmatrix} K_I^2\left(\left(9M^2(3K-5)+8\right)(12\sin\theta+12\sin 2\theta)-64\sin 2\theta\right)+ \\ K_{II}^2\left(\left(9M^2(3K-5)+8\right)(12\sin\theta-12\sin 2\theta)-64\sin 2\theta\right)\end{pmatrix}
\end{pmatrix}$$

Finally, to get the minimum value of strain energy density factor (S), the following conditions are to be satisfied:

$$\left.\begin{aligned}
&\frac{\partial S}{\partial \theta} = 0 \\
&\frac{\partial S}{\partial M} = 0 \\
&AC - B^2 > 0 \\
&A > 0
\end{aligned}\right\} ------------------ where \quad \left.\begin{aligned}
A &= \frac{\partial^2 S}{\partial \theta^2} \\
C &= \frac{\partial^2 S}{\partial M^2} \\
B &= \frac{\partial^2 S}{\partial \theta \partial M}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\left.\begin{aligned}
&\left(\left(K_{II}^2-K_I^2\right)\sin\theta - 2K_I K_{II}\cos\theta\right) = 0 \\
&\left(\left(K_I^2+K_{II}^2\right)+\left(K_I^2-K_{II}^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right) = 0
\end{aligned}\right\} \\
&\begin{pmatrix}
K_{II}^4\left(\left(9M^2(3K-5)+8\right)(6\cos\theta-3\cos 2\theta-3)-16(1-\cos 2\theta)\right) \\
-K_I^4\left(\left(9M^2(3K-5)+8\right)(6\cos\theta+3\cos 2\theta+3)+16(1-\cos 2\theta)\right) \\
+K_I^2K_{II}^2\left(\left(9M^2(3K-5)+8\right)(18\cos 2\theta-6)-16(2+6\cos 2\theta)\right) \\
+K_I K_{II}\begin{pmatrix} K_I^2\left(\left(9M^2(3K-5)+8\right)(12\sin\theta+12\sin 2\theta)-64\sin 2\theta\right)+ \\ K_{II}^2\left(\left(9M^2(3K-5)+8\right)(12\sin\theta-12\sin 2\theta)-64\sin 2\theta\right)\end{pmatrix}
\end{pmatrix} > 0 \\
&\left(9(3K-5)M^2+8\right)\left(\left(K_{II}^2-K_I^2\right)\cos\theta - 2K_I K_{II}\sin\theta\right) > 0
\end{aligned}\right\} \qquad (19)$$

III. FINITE ELEMENT ANALYSIS

In this work, an edge cracked plate is considered for the analysis. In the case of an inclined crack, the model was not symmetric. Thus, full model of the single edge cracked plate with dimensions 60 mm x 180 mm was analysed using ANSYS 15.0 environment. The orientation of the crack (â) is varied from 0° to 55° with an incremental step of 5°. The crack length to width of the plate (a/b) is varied from 0.1 – 0.5 with an incremental step of 0.1. It is represented diagrammatically with local coordinate system as shown in fig.1. Material considered is Al 6061. Isotropic and elastic behavior of the material is considered in the analysis. The properties are tabulated in table 1.

Table 1
Material properties

S.No	*Material*	*Property*	*Value*
1	AL-6061	Young's modulus	71.2 Gpa
2		Poisson's ratio	0.29
3		Yield strength	344.5Mpa

The problem is modeled as 2-D and PLANE 183 element with plane stress behavior is considered for the generation of mesh. It is presented in fig. 2. It is a 2-D higher order element with 2 degree of freedom at each node (u_x, u_y - translation in x and y) and exhibits quadratic (8 node) and triangular (6 node) behavior. This element is well suited for the modeling of irregular meshes. The element is represented in fig. 2. The meshed view of the plate is presented in fig. 3. During meshing, 5,194 nodes and 2,154 elements are generated.

Mixed mode loading is a combination of tension and shear loads. The crack is modeled as an inclined crack with the uniformly distributed load (pressure) acting perpendicular to the width of the crack. *When the load is resolved into components, the component along the length of the crack gives the shearing effect and the perpendicular component gives the tensile loading or tension effect*. The load applied is equal to the yield strength of the material. Deflection at the base of the plate is arrested in y-direction and the deflection at key point -1 is arrested in x-direction also.

From the FEA, stress intensity factors, stress components, stress triaxiality, CTOD, ERR and strain energy density are evaluated. The obtained results are presented graphically.

IV. OPTIMIZATION

Strain energy density equation is optimized using genetic algorithm in MATLAB for determining the minimum strain energy density and the corresponding crack initiation angle. Genetic algorithm can be applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear.

In this work, constrained optimization is performed. Two different equations are optimized in this work.

i) Eq. (10) representing the actual strain energy criterion proposed by G.C. Sih. It is minimized subject to the constraints in eq. (12). The outcome is the minimum strain energy required for the initiation of crack and the direction in which the existing edge crack will propagate.

ii) Eq. (16) represents the modified strain energy density equation, which helps us to study the effect of stress triaxiality on strain energy density and crack initiation angle. It is minimized subject to the constraints presented in eq. (19).

The following parameters are considered in performing constrained optimization using genetic algorithm. Parameters are tabulated and presented in table 2.

Table 2
Genetic Algorithm Parameters

Population creation function	*Feasible population*
Population type	Double vector
Scaling function	Rank
Selection function	Stochastic uniform
Elite count	2
Cross over function	0.8
Mutation function	Adaptive feasible
Cross over function	Two point
Hybrid function	Pattern search
No. of generations	100
Stall generations	100
Function tolerance	1.00 e-6
Non-linear constraint tolerance	**1.00 e-6**

The results obtained from the optimization are presented graphically in fig (15) to fig. (18).

V. RESULTS AND DISCUSSION

1. Fig. 9 – 14 represents the graphical representation of results from FEA. From fig. 9 and 10, it is observed that with increase in crack inclination angle, K_I decreases & K_{II} increases. This indicates the dominance of shear stress is increasing. In addition, it is observed that K_I maximum is occurring at a/w=0.5 with the sudden increase in magnitude. This represents that the material with larger cracks cannot sustain much higher loads & may result in sudden failure of component.
2. Also with increase in a/w ratio, the values of SIFs are observed to be increasing.

   Note: If the crack length is observed to be greater than 30% of the width, suitable measures are to be taken for the crack closure.
3. From fig. 14, it has been observed that with increase in crack inclination angle, the stress triaxiality is observed to be decreasing. This trend is observed for all a/w ratios and are converging to same value at $\beta = 55^0$. Also decrease in stress triaxiality shows that there is shift in failure mode to shear, which is due to mode mixity. Results indicate that $\beta = 55^0$ becomes the critical crack inclination angle. Hence during the service conditions, if for any a/w ratio, this orientation of crack is observed, the component is to be repaired or replaced.
4. Strain energy density represents the energy stored in the plate due to the applied load per unit volume. From the fig. 12, with an increase in the crack inclination the strain energy density is observed to be decreasing. Lesser the strain energy density results in less release of energy for the formation of two new surfaces. This in turn results in lesser crack tip opening displacement. Results presented in fig. 13 and fig. 11 support this comment.
5. Again from fig. 12, it has been observed that strain energy density for a/w=0.1 is less and for this crack condition higher loads are needed to make the crack critical. Also strain energy density is observed to be increasing with increase in a/w ratio. Hence for higher value of a/w ratio, lesser loads are needed to make the crack critical and cannot sustain higher loads. The same effect is observed from fig. 13 and fig. 11. Strain energy density corresponding to a/w=0.5 is high. This result in larger energy release rate and crack tip opening displacement as presented graphically, which supports the comment-6.

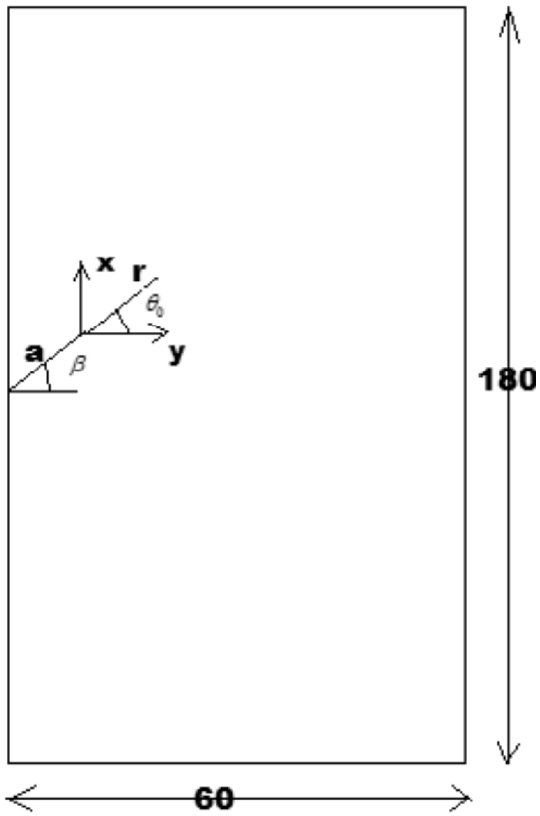

Figure 1: Plate with dimensions and local coordinate system

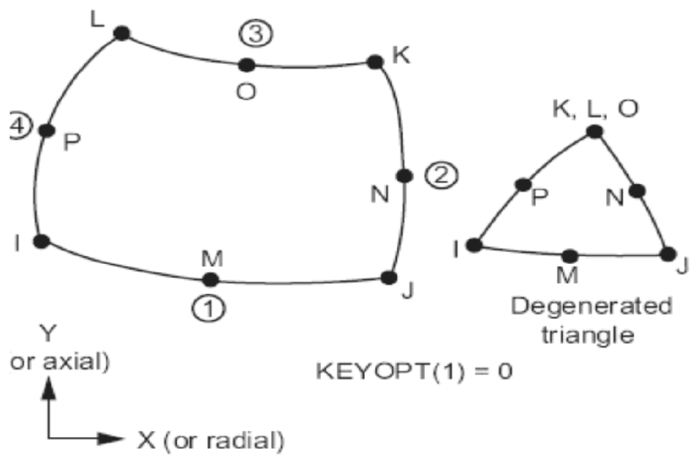

Figure 2: Shape of the element – Plane 183

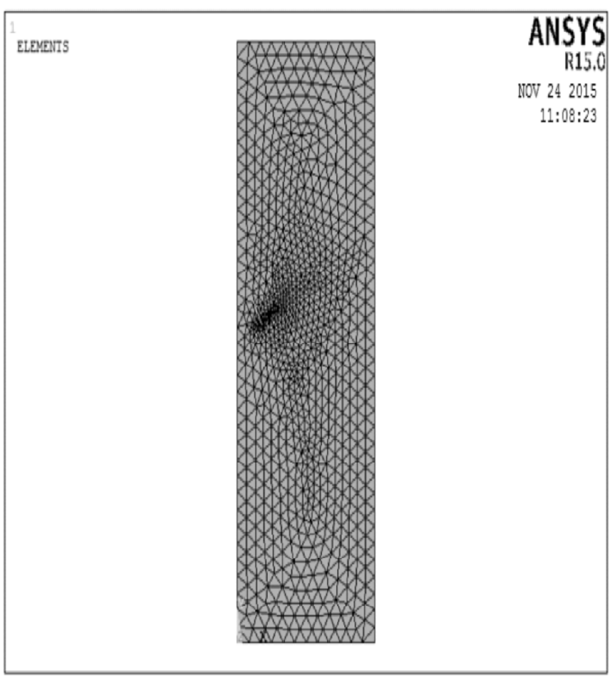

Figure 3: Meshed view of the plate for β = 10^0

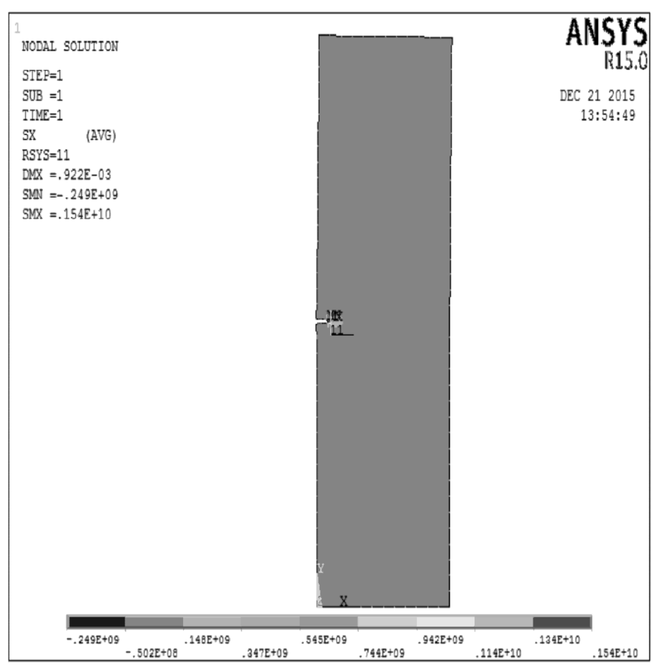

Figure 4: For a/w=0.1, β = 0^0, Stress distribution in x-direction

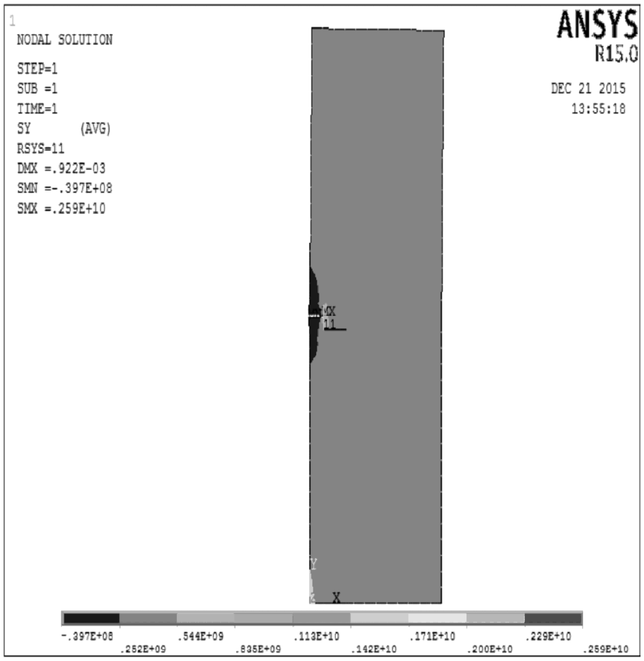

Figure 5: For a/w=0.1, β=0^0, Stress distribution in y-direction

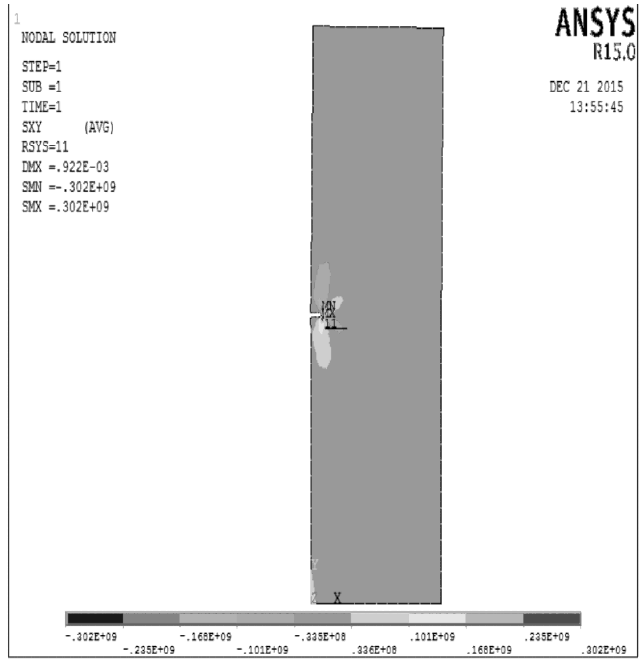

Figure 6: For a/w=0.1, β = 0^0, Shear Stress distribution

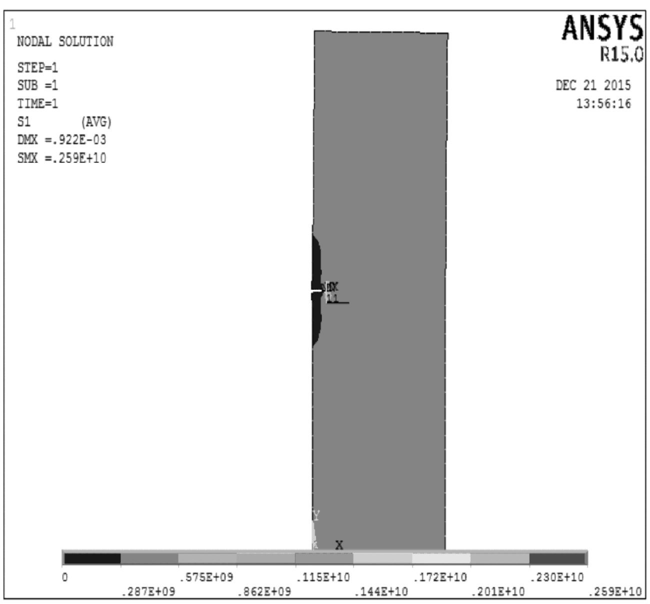

Figure 7: For a/w = 0.1, β = 0^0, First principal Stress distribution

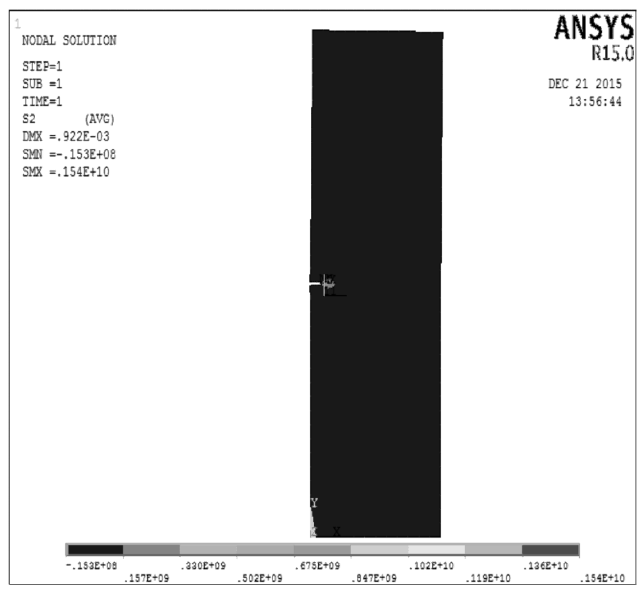

Figure 8: For a/w=0.1, β = 0^0, Second principal Stress distribution

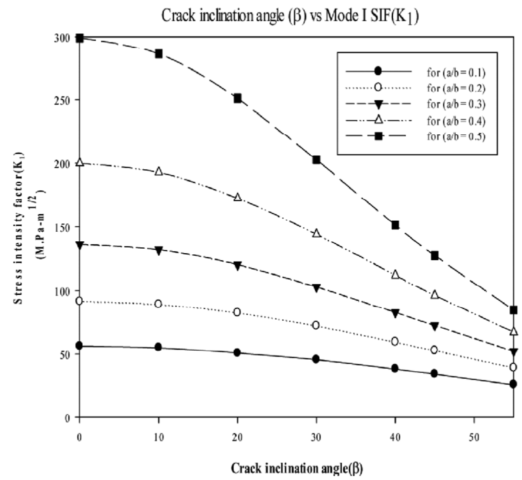

Figure 9: Variation of Mode-I SIF vs Crack inclination angle

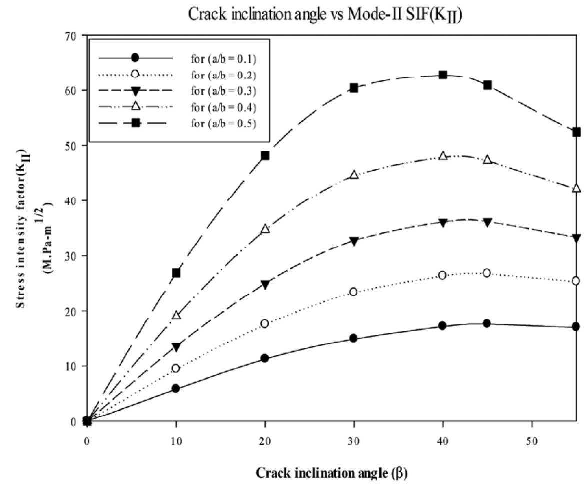

Figure 10: Variation of Mode-II SIF vs Crack inclination angle

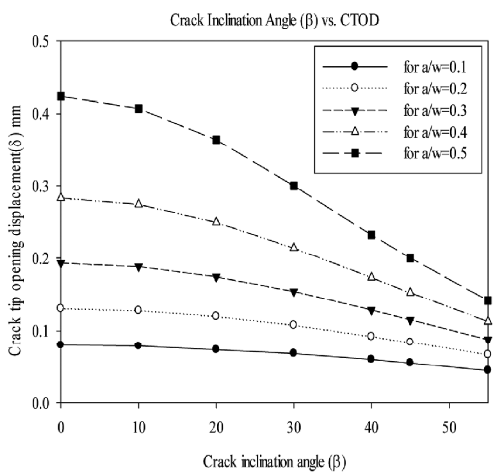

Figure 11: Variation of CTOD vs Crack inclination angle

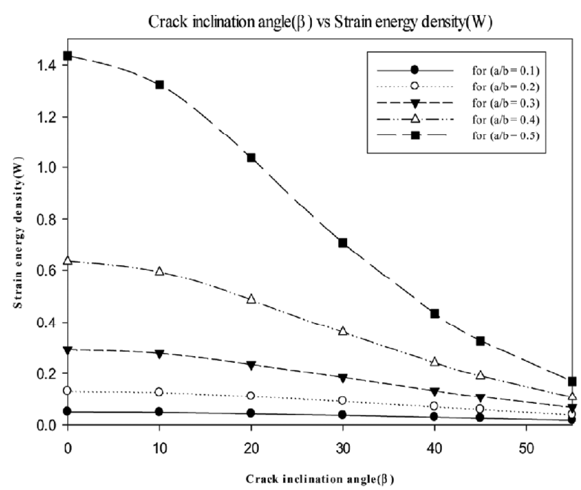

Figure 12: Variation of Strain energy density vs Crack inclination angle

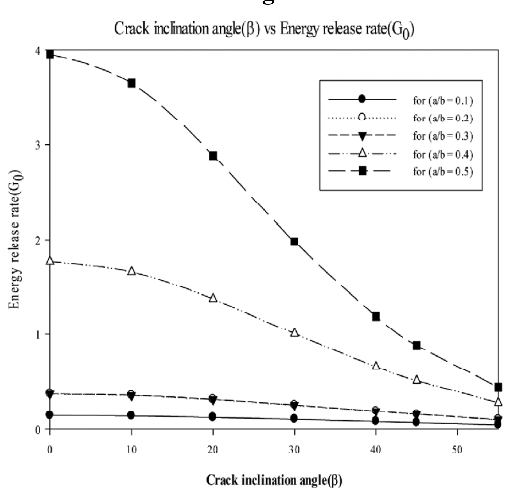

Figure 13: Variation of Energy release rate vs Crack inclination angle

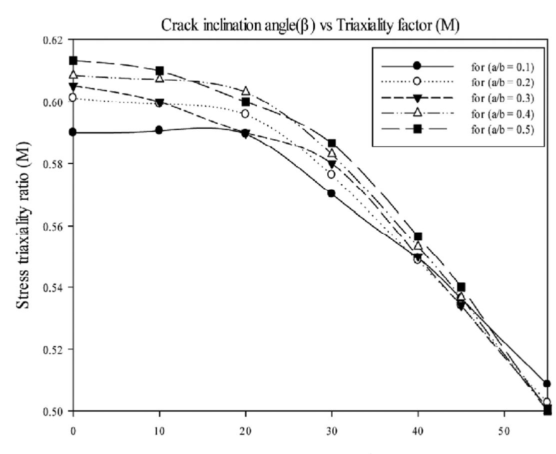

Figure 14: Variation of Stress triaxiality vs Crack inclination angle

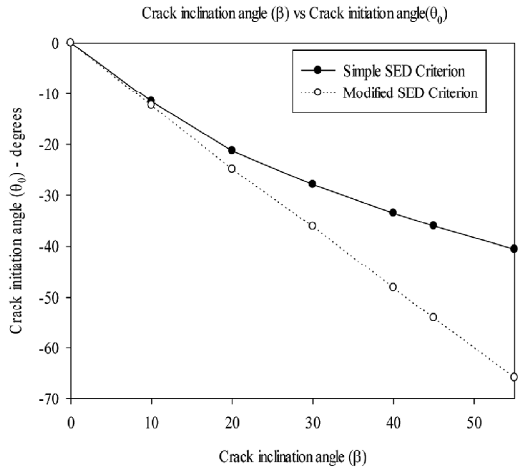

Figure 15: Variation of Energy release rate vs Crack initiation angle

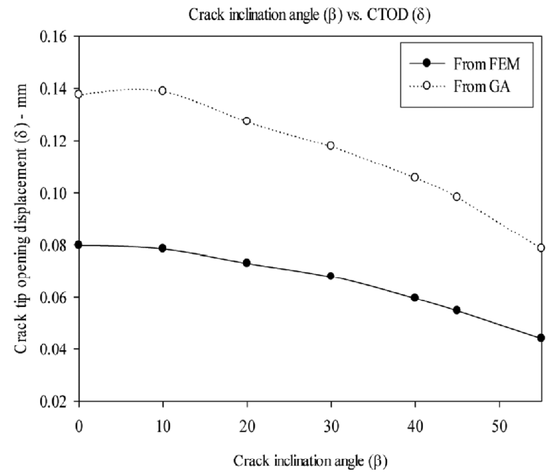

Figure 16: Variation of Energy release rate vs CTOD

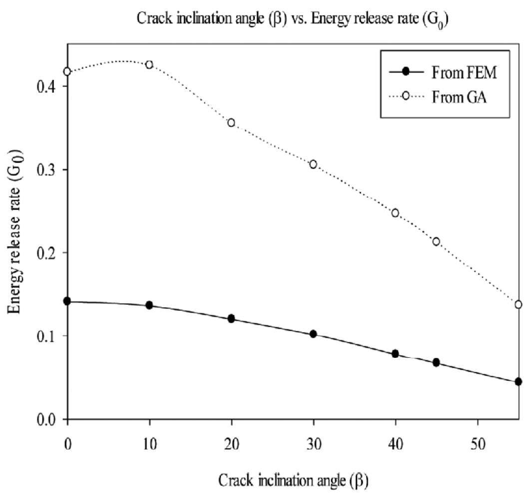

Figure 17: Variation of Energy release rate vs ERR

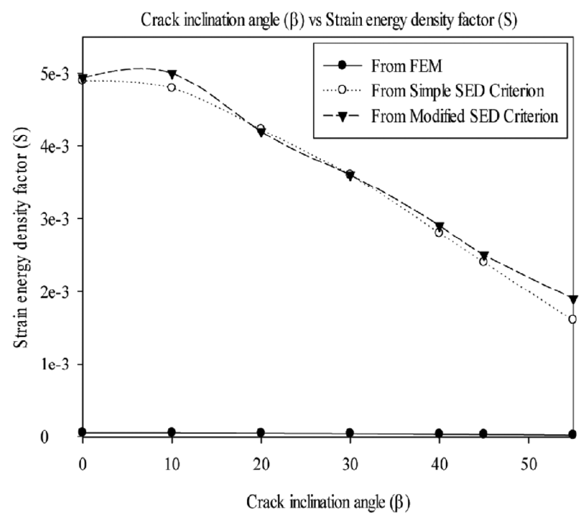

Figure 18: Variation of Energy release rate vs SED Factor

6. With increase in crack inclination angle, the strain energy is observed to be decreasing and for all values of a/w ratio, the curves are converging to a single value for the inclination angle beyond 55^0. This indicates that the strain energy is independent of the crack inclination angle beyond 55^0.

7. Fig. 16 presents the variation of crack initiation angle. With increase in inclination angle, the initiation angle increases. Negative angle indicates that the crack is propagating in downward direction. Also the crack initiation angle obtained from modified SED criterion is higher than that obtained from the simple SED criterion. This indicates that the stress triaxiality is having significant effect on crack initiation. Fig. 18 compares the SED obtained from FEM and optimization; with and without stress triaxiality. It has been observed that the effect of stress triaxiality is significant and SED, ERR and CTOD obtained from the optimization of SED equation with stress triaxiality is higher than that obtained from simple SED criterion and FEM. This indicates that if the stress triaxiality is becoming critical, the strain energy is approaching maximum value. Because of this, more amount of energy will release resulting in larger CTOD. Fig. 6.17 and 6.16 support this comment. In this analysis, the values obtained from FEM are very less.

V. CONCLUSION

An edge cracked plate subject to mixed mode fracture under plane stress condition is considered in this study. Finite element analysis is performed to determine the values of SIFs, stresses, SED, ERR, CTOD and stress triaxiality. The existing SED criterion is modified to study the effect of stress triaxiality on the crack initiation angle, SED, ERR and CTOD. Using the FEM results, the SED equations are optimized using genetic algorithm in MATLAB, to find the critical values. The following conclusions are drawn in this research work.

- With increase in crack inclination angle,
  - the mode-I SIF, ratio of principal stress, stress triaxiality are decreasing and mode-II SIF is increasing. This is due to mode mixity.
  - Strain energy density is decreasing resulting in lesser energy release rate and CTOD.
  - The crack initiation angle is increasing.
- With increase in a/w ratio,
  - the mode-I SIF, ratio of principal stress, stress triaxiality and mode-II SIF are increasing. This is due to decrease in the effective area resting the applied load.
  - Strain energy density is increasing resulting in larger energy release rate and CTOD. This is due to the decrease in the stiffness of the material.

The effect of stress triaxiality on the crack initiation angle, SED, ERR and CTOD is observed to be significant. The result is increase in their values.

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